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RESULTS ON BAZILEVIC SAKAGUCHI TYPE OF FUNCTIONS

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ABSTRACT. In the present research paper, a new subclass of bi univalent functions pertaining to pseudo starlike functions has been introduced and investigated using Sakaguchi function in the disc $|z| < 1$. Furthermore, using this new subclass, the bounds pertaining to the initial coefficients for these classes are found.

1. INTRODUCTION

Consider A as to the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are normalized by the conditions $f(0) = 0$ and $f'(0) = 1$and analytic in the unit disc $D = \{z : |z| < 1\}$.

Let S mean the subclass of A consisting of univalent functions in D. Besides consider $S^*(\alpha)$ as well as $K(\alpha)$ denote the familiar classes of starlike and convex functions of the order α, $(0 \leq \alpha < 1)$ (refer [4]).

Koebe's one quarter theorem [4] confirms that the image of D under every univalent function $f \in S$ contains a disk of the radius $1/4$. Hence, it is

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understood that every univalent function, that is $f \in S$ virtually posses an inverse of f defined by $f^{-1}[f(z)] = z, (z \in U)$ and

$$f[f^{-1}(w)] = w, \quad (|w| < r_0(f), \quad r_0(f) \geq \frac{1}{4}),$$

where

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots . \tag{1.2}$$

When function $f \in A$ is biunivalent, both $f$and f^{-1} are univalent in D. The class of bi univalent functions in $D$is denoted and assigned by (1.1) by $\sum$, (see [18]).

The class $\sum$ of bi univalent functions was first analyzed by Lewin [13] and it confirmed that $|a_2| < 1.51$. Brannan and Clunie (c.f. [2]) further developed Lewin's concept and assumed $|a_2| \leq \sqrt{2.}$, as well as in [11], Kedzierawski confirmed this conjecture for the starlike functions. Recently, Netanyahu [10], proved that $\max\limits_{f\in\sum} |a_2| = \frac{4}{3}$. Further, Tan [19] showed the pre-eminent estimate $|a_2| \leq 1.485$ for functions in $\sum$. The problem of coefficient estimate, i.e., bound of $|a_n| \, (n \in \mathrm{N}/\{1.2\})$ for a subclass of $\sum$was given in 2013 by Jahangiri and Hamidi [8] by applying Faber's expansions.

Brannan and Taha [4] also propounded certain subclasses $S^*_{\sum}(\alpha)$ and $K_{\sum}(\alpha)$ of strongly bi starlike and bi convex functions of order α and identified the initial coefficient bounds $|a_2|$ and $|a_3|$ (c.f. [3]).

Of late, many a researcher investigated and propounded various subclasses of bi univalent functions and fixed the initial coefficient bounds $|a_2|$ and $|a_3|$ (refer [6–10, 14, 15, 18]).

K.O. Babalola [1] explained the class $L_\lambda(\beta)$of λ pseudo starlike functions of order β and detected that all pseudo starlike functions are Bazilevic of kind $\left(1 - \frac{1}{\lambda}\right)$, order $\beta^{(\frac{1}{\lambda})}$and univalent in open unit disk D.

Frasin [5] investigated the inequalities of coefficient for certain classes of Sakaguchi type functions satisfying geometrical condition as

$$Re\left\{\frac{(s-t)z(f'(z))}{f(sz) - f(tz)}\right\} > \alpha \quad (z \in D) \tag{1.3}$$

for complex numbers s, t but $s \neq t$ and $0 \leq \alpha < 1$, denoted by $S(\alpha, s, t)$.

The purpose of this paper is to introduce the subclasses $\mathrm{S}P^{\mu}_{\sum}(\beta,\delta,\tau)$ as well as $SP_{\sum}(\mu,\delta,\tau,\alpha)$ of $\sum$ associated with Sakaguchi type functions derive the initial coefficient estimates $|a_2|$ and $|a_3|$.

In order to find out the main results, the following Lemma can be recalled here.

Lemma 1.1. *[17] Let $c\in P$ be the family of all functions c analytic in D for that $Re\{c(z)\}>0$ and have the form $c(z)=1+c_1z+c_2z^2+c_3z^3+...$ for $z\in D$. Then $|c_n|\leq 2$, for each n.*

2. Main Results

Estimates of Coefficient for the functions in the class $SP^{\mu}_{\sum}(\beta,\delta,\tau)$.

Definition 2.1. *A function f given by* (1.1) *is in the class $SP^{\mu}_{\sum}(\beta,\delta,\tau)$ if it fulfills the following conditions: $f\in\sum$,*

$$\left|\arg\left(\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}}\right)\right|<\frac{\beta\pi}{2},\quad (z\in D) \tag{2.1}$$

and

$$\left|\arg\left(\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}}\right)\right|<\frac{\beta\pi}{2},\quad (z\in D) \tag{2.2}$$

where $0<\beta\leq 1$, $0\leq\delta<1,\mu\geq 1,|\tau|\leq 1$ but $\tau\neq 1$, g is extension of f^{-1} to D explained in (1.2).

In the ensuing theorems, the initial Taylor coefficients $|a_2|$ and $|a_3|$ for the function $SP^{\mu}_{\sum}(\beta,\delta,\tau)$ and $SP_{\sum}(\mu,\delta,\tau,\gamma)$ are obtained.

Theorem 2.1. *If the function f given by* (1.1) *is in the class $SP^{\mu}_{\sum}(\beta,\delta,\tau)$,*

$$|a_2|\leq\frac{2\beta}{\sqrt{(2\mu-(1-\delta)(1+\tau))^2+\left[\begin{array}{l}4\mu^2+2\mu\\ +(1-\delta)\left\{\begin{array}{c}(1+\tau)^2(2-\delta)\\ -4\mu(1+\tau)\\ -2(1+\tau+\tau^2)\end{array}\right\}\\ -(2\mu-(1-\delta)(1+\tau))^2\end{array}\right]\beta}} \tag{2.3}$$

and

$$|a_3| \leq \frac{2\beta}{3\mu-(1-\delta)(1+\tau+\tau^2)} + \frac{4\beta^2}{(2\mu-(1-\delta)(1+\tau))^2}. \tag{2.4}$$

Proof. From equations (2.1) and (2.2) as

$$\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}} = [p(z)]^{\beta} \tag{2.5}$$

and

$$\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}} = [q(w)]^{\beta}, \tag{2.6}$$

where the functions $p(z), q(w) \in P$ and they are of the form

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$$

and

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \cdots .$$

Clearly,

$$[p(z)]^{\beta} = 1 + \beta p_1 z + \left(\beta p_2 + \frac{\beta(\beta-1)}{2}p_1^2\right) z^2 + ...$$

and

$$[q(w)]^{\beta} = 1 + \beta q_1 w + \left(\beta q_2 + \frac{\beta(\beta-1)}{2}q_1^2\right) w^2 + ...$$

Also

$$\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}} = 1 + (2\mu-(1-\delta)(1+\tau))a_2 z +$$

$$\left(\begin{array}{l} (3\mu-(1-\delta)(1+\tau+\tau^2))a_3 \\ + \left(\begin{array}{l} 2\mu(\mu-1-(1-\delta)(1+\tau)) \\ +\frac{\delta(1-\delta)(1+\tau)^2}{2} \\ +(1+\tau)^2(1-\delta)^2 \end{array}\right) a_2^2 \end{array}\right) z^2 + ...$$

and

$$\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}} = 1 - (2\mu-(1-\delta)(1+\tau))a_2 w +$$

$$\left[\begin{array}{l}\left(\begin{array}{l}2\mu^2+4\mu+\frac{\delta(1-\delta)(1+\tau)^2}{2}\\+(1-\delta)^2(1+\tau)^2\\-2(1-\delta)(1+\tau+\tau^2)\\-2\mu(1-\delta)(1+\tau)\end{array}\right)a_2^2\\+((1-\delta)(1+\tau+\tau^2)-3\mu)a_3\end{array}\right]w^2+...$$

By equating the coefficients in (2.5) and (2.6), it is obtained that

$$(2\mu-(1-\delta)(1+\tau))a_2=\beta p_1, \tag{2.7}$$

$$(3\mu-(1-\delta)(1+\tau+\tau^2))a_3+\left(\begin{array}{l}2\mu(\mu-1-(1-\delta)(1+\tau))\\+\frac{\delta(1-\delta)(1+\tau)^2}{2}+(1+\tau)^2(1-\delta)^2\end{array}\right)a_2^2 =\beta p_2+\frac{\beta(\beta-1)}{2}p_1^2 \tag{2.8}$$

$$-(2\mu-(1-\delta)(1+\tau))a_2=\beta q_1 \tag{2.9}$$

(2.10)
$$\left(\begin{array}{l}2\mu^2+4\mu+\frac{\delta(1-\delta)(1+\tau)^2}{2}+(1-\delta)^2(1+\tau)^2-2(1-\delta)(1+\tau+\tau^2)\\-2\mu(1-\delta)(1+\tau)\end{array}\right)a_2^2 +((1-\delta)(1+\tau+\tau^2)-3\mu)a_3=\beta q_1+\frac{\beta(\beta-1)}{2}q_1^2$$

From (2.7) and (2.9), we get

$$p_1=-q_1 \tag{2.11}$$

and

$$a_2^2=\frac{(p_1^2+q_1^2)\beta^2}{2(2\mu-(1-\delta)(1+\tau))^2}. \tag{2.12}$$

By summing up the equations (2.8) and (2.10), the following is obtained

$$\left(4\mu^2+2\mu+(1-\delta)\left\{(1+\tau)^2(2-\delta)-4\mu(1+\tau)-2(1+\tau+\tau^2)\right\}\right)a_2^2 =\beta(p_2+q_2)+\frac{\beta(\beta-1)}{2}(p_1^2+q_1^2).$$

Using (2.12), it is obtained that

$$\left(4\mu^2+2\mu+(1-\delta)\left\{(1+\tau)^2(2-\delta)-4\mu(1+\tau)-2(1+\tau+\tau^2)\right\}\right)a_2^2 =\beta(p_2+q_2)+\frac{\beta(\beta-1)}{2}\left(\frac{2(2\mu-(1-\beta)(1+\tau))^2}{\beta^2}\right)$$

$$\Rightarrow a_2^2 = \frac{\beta^2 (p_2 + q_2)}{\begin{pmatrix} 2\mu \\ -(1-\delta)(1+\tau) \end{pmatrix}^2 + \left\{ \begin{array}{l} 4\mu^2 + 2\mu \\ +(1-\delta) \begin{pmatrix} (1+\tau)^2 (2-\delta) \\ -4\mu (1+\tau) \\ -2(1+\tau+\tau^2) \end{pmatrix} \\ -(2\mu - (1+\tau)(1-\delta))^2 \end{array} \right\} \beta}$$

By using Lemma (1.1), for p_2 and q_2, the following is obtained.

$$|a_2| \leq \frac{2\beta}{\sqrt{\begin{pmatrix} 2\mu \\ -(1-\delta)(1+\tau) \end{pmatrix}^2 + \left\{ \begin{array}{l} 4\mu^2 + 2\mu \\ +(1-\delta) \begin{pmatrix} (1+\tau)^2 (2-\delta) \\ -4\mu (1+\tau) \\ -2(1+\tau+\tau^2) \end{pmatrix} \\ -(2\mu - (1+\tau)(1-\delta))^2 \end{array} \right\} \beta}}.$$

It is obtained on $|a_2|$as given in (2.3).

Next, to find$|a_3|$, (2.10) should be subtracted from (2.8), then it is obtained

$$\begin{aligned} 2\left((1-\delta)(1+\tau+\tau^2) - 3\mu\right) a_3 + 2\left(3\mu - (1-\delta)(1+\tau+\tau^2)\right) a_2^2 \\ = \beta (p_2 - q_2) + \tfrac{\beta(\beta-1)}{2} (p_1^2 - q_1^2). \end{aligned}$$

Using the equation (2.11), we obtain $p_1^2 = q_1^2$ and also (2.12) we have

$$a_3 = \frac{\beta (p_2 - q_2)}{2\left(3\mu - (1-\delta)(1+\tau+\tau^2)\right)} + \frac{\beta^2 p_1^2}{(2\mu - (1-\delta)(1+\tau))^2}.$$

Again by applying Lemma (1.1) the following is obtained:

$$|a_3| \leq \frac{2\beta}{3\mu - (1-\delta)(1+\tau+\tau^2)} + \frac{4\beta^2}{(2\mu - (1-\delta)(1+\tau))^2}.$$

We derived the bound on $|a_3|$ as given in (2.4) and this proved the Theorem 2. □

If $\delta = 0$ in the above theorem, the following initial Taylor coefficients $|a_2|$ and $|a_3|$ for the function class $SP_{\Sigma}^{\mu}(\beta, \delta, \tau)$ are arrived.

Corollary 2.1. *Let f given by* (1.1) *in the class* $SP^{\mu}_{\sum}(\beta, \delta, \tau)$. *Therefore*

$$|a_2| \leq \frac{2\beta}{\sqrt{(2\mu-(1+\tau))^2+\left\{\begin{array}{c} 4\mu^2+2\mu+2(1+\tau)^2-4\mu(1+\tau) \\ -2(1+\tau+\tau^2)-(2\mu-(1+\tau))^2 \end{array}\right\}\beta}}$$

and

$$|a_3| \leq \frac{2\beta}{3\mu-(1+\tau+\tau^2)}+\frac{4\beta^2}{(2\mu-(1+\tau))^2}$$

When the parameter $\tau = 0$ *is taken in the above Corollary 2.1, the following initial Taylor coefficients bounds* $|a_2|$ *and* $|a_3|$ *for the function of the class* $SP^{\lambda}_{\sum}(\alpha, \tau)$ *are arrived in [11] Joshi et al.*

Corollary 2.2. *Let f given by* (1.1) *in the class* $SP^{\mu}_{\sum}(\beta)$. *Therefore*

$$|a_2| \leq \frac{2\beta}{\sqrt{4\mu^2-4\mu+1+\beta(2\mu-1)}}$$

and

$$|a_3| \leq \frac{2\beta}{3\mu-1}+\frac{4\beta^2}{(2\mu-1)^2}$$

3. Coefficients estimate for the function in the class $SP_{\sum}(\mu, \delta, \tau, \alpha)$

Definition 3.1. *Let the function f given by* (1.1) *be in the class* $SP_{\sum}(\mu, \delta, \tau, \alpha)$ *when it fulfills the ensuing condition:* $f \in \sum$,

$$\Re\left(\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}}\right) > \alpha \quad (z \in D) \tag{3.1}$$

and

$$\Re\left(\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}}\right) > \alpha \quad (z \in D). \tag{3.2}$$

where $z \in D$, $w \in D$, $0 \leq \alpha < 1$, $0 \leq \delta < 1$, $\mu \geq 1$, $|\tau| \leq 1$, $\tau \neq 1$ *and* g *are defined in equation* (1.2).

The following estimates of coefficient stands for bi-univalent functions in the class $SP_{\sum}(\mu, \delta, \tau, \alpha)$.

Theorem 3.1. *If f given by* (1.1) *is in the class* $SP_{\sum}(\mu,\delta,\tau,\alpha)$,

$$|a_2| \leq \sqrt{\frac{4(1-\alpha)}{\begin{array}{l}4\mu^2+2\mu-4\mu(1-\delta)(1+\tau)\\ +\delta(1-\delta)(1+\tau)^2+2(1-\delta)^2(1+\tau)^2\\ -2(1-\delta)(1+\tau+\tau^2)\end{array}}} \tag{3.3}$$

and

$$|a_3| \leq \frac{2(1-\alpha)}{(3\mu-(1-\delta)(1+\tau+\tau^2))}+\frac{4(1-\alpha)^2}{(2\mu-(1-\delta)(1+\tau))^2}. \tag{3.4}$$

Proof. From the equations (3.1) and (3.2),

$$\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}}=\alpha+(1-\alpha)p(z) \tag{3.5}$$

and

$$\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}}=\alpha+(1-\alpha)q(w), \tag{3.6}$$

respectively.

In case of $p(z), q(w) \in P$, they are of the form

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots$$

and

$$q(w)=1+q_1w+q_2w^2+q_3w^3+\cdots.$$

Clearly,

$$\alpha+(1-\alpha)p(z)=1+(1-\alpha)p_1z+(1-\alpha)p_2z^2+\cdots$$

and

$$\alpha+(1-\alpha)q(w)=1+(1-\alpha)q_1w+(1-\alpha)q_2w^2+\cdots.$$

Also

$$\begin{aligned}\frac{((1-\tau)z)^{1-\delta}(f'(z))^{\mu}}{(f(z)-f(\tau z))^{1-\delta}} &= 1+(2\mu-(1-\delta)(1+\tau))a_2z\\ &\quad+\left(\begin{array}{l}(3\mu-(1-\delta)(1+\tau+\tau^2))a_3\\ +\left(\begin{array}{l}2\mu(\mu-1-(1-\delta)(1+\tau))+\\ \frac{\delta(1-\delta)(1+\tau)^2}{2}\\ +(1+\tau)^2(1-\delta)^2\end{array}\right)a_2^2\end{array}\right)z^2+\ldots\end{aligned}$$

and

$$\frac{((1-\tau)w)^{1-\delta}(g'(w))^{\mu}}{(g(w)-g(\tau w))^{1-\delta}} = 1-(2\mu-(1-\delta)(1+\tau))\,a_2 w + \left[\begin{array}{l}\left(\begin{array}{l} 2\mu^2+4\mu+\frac{\delta(1-\delta)(1+\tau)^2}{2} \\ +(1-\delta)^2(1+\tau)^2 \\ -2(1-\delta)(1+\tau+\tau^2) \\ -2\mu(1-\delta)(1+\tau)\end{array}\right) a_2^2 \\ +((1-\delta)(1+\tau+\tau^2)-3\mu)\,a_3\end{array}\right] w^2+\ldots$$

Equating the coefficients in (3.5) and (3.6), is obtained

$$(2\mu-(1-\delta)(1+\tau))\,a_2=(1-\alpha)\,p_1 \tag{3.7}$$

$$\left(\begin{array}{l}3\mu-(1-\delta)\\(1+\tau+\tau^2)\end{array}\right)a_3+\left(\begin{array}{l}2\mu(\mu-1-(1-\delta)(1+\tau))\\+\frac{\delta(1-\delta)(1+\tau)^2}{2}\\+(1+\tau)^2(1-\delta)^2\end{array}\right)a_2^2=(1-\alpha)\,p_2 \tag{3.8}$$

$$-(2\mu-(1-\delta)(1+\tau))\,a_2=(1-\alpha)\,q_1 \tag{3.9}$$

$$\begin{array}{l}\left(\begin{array}{l}2\mu^2+4\mu+\frac{\delta(1-\delta)(1+\tau)^2}{2}+(1-\delta)^2(1+\tau)^2\\-2(1-\delta)(1+\tau+\tau^2)-2\mu(1-\delta)(1+\tau)\end{array}\right)a_2^2\\+((1-\delta)(1+\tau+\tau^2)-3\mu)\,a_3=(1-\alpha)\,q_2.\end{array} \tag{3.10}$$

From equations (3.7) and (3.9), is arrived

$$p_1=-q_1 \tag{3.11}$$

and

$$2\,(2\mu-(1-\delta)(1+\tau))^2\,a_2^2=(1-\alpha)^2\left(p_1^2+q_1^2\right). \tag{3.12}$$

By summing (3.8) and (3.10), is obtained

$$\left(\begin{array}{l}4\mu^2+2\mu-4\mu(1-\delta)(1+\tau)+\delta(1-\delta)(1+\tau)^2\\+2(1-\delta)^2(1+\tau)^2-2(1-\delta)(1+\tau+\tau^2)\end{array}\right)a_2^2=(1-\alpha)(p_2+q_2)$$

$$\Rightarrow\left|a_2^2\right|\le\frac{(1-\alpha)(|p_2|+|q_2|)}{\left(\begin{array}{l}4\mu^2+2\mu-4\mu(1-\delta)(1+\tau)+\delta(1-\delta)(1+\tau)^2\\+2(1-\delta)^2(1+\tau)^2-2(1-\delta)(1+\tau+\tau^2).\end{array}\right)}$$

Now, by using Lemma (1.1), for both p_2 and q_2, the following is arrived:

$$|a_2| \leq \sqrt{\frac{4(1-\alpha)}{\left(\begin{array}{l} 4\mu^2 + 2\mu - 4\mu(1-\delta)(1+\tau) + \delta(1-\delta)(1+\tau)^2 \\ +2(1-\delta)^2(1+\tau)^2 - 2(1-\delta)(1+\tau+\tau^2) \end{array}\right)}}$$

It is really the bound on $|a_2|$ as given in (3.3).

Next, to get$|a_3|$, we subtract (3.10) from (3.8):

$$2\left((1-\delta)(1+\tau+\tau^2) - 3\mu\right)a_3 + 2\left(3\mu - (1-\delta)(1+\tau+\tau^2)\right)a_2^2 = (1-\alpha)(p_2 - q_2)$$

$$a_3 = \frac{(1-\alpha)(p_2-q_2)}{2\left(3\mu - (1-\delta)(1+\tau+\tau^2)\right)} + a_2^2.$$

From (3.11), it is assumed that $p_1^2 = q_1^2$ and also using (3.12), is obtained

$$a_3 = \frac{(1-\alpha)(p_2-q_2)}{2\left(3\mu - (1-\delta)(1+\tau+\tau^2)\right)} + \frac{(1-\alpha)^2(p_1^2+q_1^2)}{2\left(2\mu - (1-\delta)(1+\tau)\right)^2}.$$

Again applying Lemma forp_1, p_2 and q_2, the following is got:

$$|a_3| \leq \frac{2(1-\alpha)}{3\mu - (1-\delta)(1+\tau+\tau^2)} + \frac{4(1-\alpha)^2}{\left(2\mu - (1-\delta)(1+\tau)\right)^2}.$$

This is the required bound on $|a_3|$ as given in (3.4). It proved the Theorem 3. □

If $\delta = 0$ in the above theorem, the following initial Taylor coefficients $|a_2|$ and $|a_3|$ for the function class $SP_{\sum}(\mu, \delta, \tau, \alpha)$ are arrived.

Corollary 3.1. *If f is given by* (1.1) *in the class* $SP_{\sum}(\mu, \delta, \tau, \alpha)$

$$|a_2| \leq \sqrt{\frac{4(1-\alpha)}{\left(4\mu^2 + 2\mu - 4\mu(1+\tau) + 2(1+\tau)^2 - 2(1+\tau+\tau^2)\right)}}$$

and

$$|a_3| \leq \frac{2(1-\alpha)}{3\mu - (1+\tau+\tau^2)} + \frac{4(1-\alpha)^2}{\left(2\mu - (1+\tau)\right)^2}.$$

If the parameter $\tau = 0$ *is taken in the above Corollary 3.1, the following initial*

Taylor coefficient estimates on $|a_2|$ *and* $|a_3|$ *which of the function of the class* $SP_{\sum}(\mu,\tau,\alpha)$ *is obtained by [11] Joshi et al.*

Corollary 3.2. *If f is given by* (1.1) *in the class* $SP_{\sum}(\mu,\alpha)$*,*

$$|a_2| \leq \sqrt{\frac{2(1-\alpha)}{2\mu^2-\mu}}$$

and

$$|a_3| \leq \frac{2(1-\alpha)}{3\mu-1} + \frac{4(1-\alpha)^2}{(2\mu-1)^2}.$$

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