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RADIO ANALYTIC MEAN D-DISTANCE LABELING OF SOME BASIC GRAPHS

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ABSTRACT. A Radio Analytic Mean D-distance labeling of a connected graph G is an injective function f from the vertex set V(G) to N such that for two distinct vertices u and v of G. $d^D(u,v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$. Where d^D (u,v) denotes the D- distance between u and v and $diam^D$(G) denotes the D-diameter of G. The radio Analytic Mean D-distance number of f, $ramn^D(f)$ is the maximum label assigned to any vertex of G. The radio Analytic mean D-distance number of G, $ramn^D(G)$ is the minimum value of $ramn^D(f)$ taken over all radio Analytic mean D-distance labeling f of G. In this paper we compute the radio analytic mean D-distance number of some basic graphs.

1. INTRODUCTION

By a graph G=(V,E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Radio labeling (multi-level distance labeling) can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [1]. Chartrand et al [2] introduced the concept of radio labeling of graph. Chartrand [3] gave the upper bound for the radio number of path.

The concept of D-distance was introduced by D. Reddy Babu et.al [7]. If u,v are vertices of a connected graph G, the D-length of a connected u-v path s

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is defined as $l^D(s) = l(s) + deg(v) + deg(u) + \sum deg(w)$ where the sum runs over all intermediate vertices w of s and l(s) is length of the path. The D-distance , $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{l^D(s)\}$ where the minimum is taken over all u-v path s in G. In other words , $d^D(u, v) = \min\{l^D(s) = l(s) + deg(v) + deg(u) + \sum deg(w)\}$ where the sum runs over all intermediate vertices w in s and minimum is taken over all u-v paths in G.

In [8], Radio Analytic mean labeling was introduced by P.Poomalai et al [12] . A radio analytic mean labeling is a one to one mapping f from V(G) to N satisfying the condition $d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam(G)$. for every u,v $\in$ G. The span of labeling f is the maximum integer that f maps to a vertex of G. The radio analytic mean number of G , ramn (G) is the lowest span taken over all radio analytic mean labeling of the graph G. The above condition is called radio analytic mean condition.

Further we are introduced the concept of radio analytic mean D-distance.The radio analytic mean D-distance coloring is a function $f : V(G) \rightarrow NU\{o\}$ such that $d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$. It is denoted by $ramn^D(G)$. where $ramn^D(G)$ is called the radio analytic mean D- distance number . The radio Analytic Mean D-distance number of f, $ramn^D(f)$ is the maximum label assigned to any vertex of G. The radio Analytic mean D-distance number of G, $ramn^D(G)$ is the minimum value of $ramn^D(f)$ taken over all radio Analytic mean D-distance labeling f of G. In this paper we determine the radio analytic mean D-distance number of some basic graphs.

2. Premilinears

Definition 2.1. *A Radio Analytic Mean D-distance labeling of a connected graph G is an injective function f from the vertex set V[G] to N such that for two distinct vertices u and v of G.* $d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$*, Where* $d^D(u, v)$ *denotes the D-distance between u and v and* $diam^D(G)$ *denotes the D-diameter of G.*

Proposition 2.1. *The following basic graphs definition and proof, illustrations in [15] and [16]:*

1) *Any path graph* (P_n) *is admitted Radio analytic mean graph.*
2) *Ary cycle graph* (C_n) *is admitted Radio analytic mean graph.*

3) *Any star graph* $(k_{1,n})$ *is admitted Radio analytic mean graph.*
4) *Any Bistar* $(B_{n,n})$ *is admitted Radio analytic mean graph.*
5) *Any complete bipartite graph* $(B_{m,n})$ *is admitted Radio analytic mean graph.*
6) *Any subdivision of complete graphs* $S(k_n)$ *is admitted Radio analytic mean graph.*
7) *Any subdivision of spoke wheel graph* $S(w_n)$ *is admitted Radio analytic mean graph.*

3. MAIN RESULTS

Theorem 3.1. *The Radio analytic mean D- distance number of a path,* $ramn^D(P_n) = 5n-6, n \geq 3$.

Proof. Let $(V(p_n) = \{v_1, v_2, \ldots, v_n\}$. Define the function f as follows. We will realize the Radio Analytic mean D-distance condition

$$d^D(u,v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G).$$

When n is odd:

$$f(v_1) = 5n-6,$$
$$f(v_{2i}) = 4i+4, 1 \leq i \leq \tfrac{n-1}{2},$$
$$f(v_{2i+2}) = 4n-5-2i,\ 1 \leq i \leq \tfrac{n-1}{2}.$$

When n is even:

$$f(v_2) = 5n-6,$$
$$f(v_{2i}) = 4i+4, 1 \leq i \leq \tfrac{n}{2},$$
$$f(v_{2i+1}) = 4n-5-2i,\ 1 \leq i \leq \tfrac{n}{2}-1.$$

The $diam^D(p_n)$ of the path graph is 3(n-1).

Case (i): when n is odd.

Case (a): compute the pair (v_1, v_{2i}), $1 \leq i \leq \frac{n-1}{2}$:

$$d^D(v_1, v_{2i}) + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \geq 1 + diam^D(G) = 1 + 3(n-1) = 3n-2$$
$$6i - 2 + \lceil \frac{|(5n-6)^2 - (4i+4)^2|}{2} \rceil \geq 3n-2$$

Case (b): compute the pair (v_2, v_{2i+1}), $1 \leq i \leq \frac{n-1}{2}$

$$61 + 1 + \lceil \frac{((5n-6)^2 - (4n-5-2i)^2}{2} \rceil \geq 3n-2$$

Case (c): compute the pair (v_1, v_{2i+2}) both end vertices, $1 \leq \frac{n-1}{2}$

$$6i + \lceil \frac{|(f(v_1)^2, f(v_{2i+1}^2}{2}) \rceil \geq 3n-2$$

Case(d): If any two intermedate adjacent vertices (v_i, v_j)

$$5 + \lceil \frac{|(f(v_i)^2, f(v_j)^2}{2}) \rceil \geq 1 + diam^D(G)$$

Case (ii): when n is even.
Case (a): compute the pair (v_1, v_{2i}) , $1 \le i \le \frac{n}{2}$
$d^D(v_1, v_{2i}) + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \ge 1 + diam^D(G)$ = 1+ 3(n-1) = 3n-2
$6i - 2 + \lceil \frac{|(5n-6)^2 - (4i+4)^2|}{2} \rceil \ge 3n - 2$
Case (b): compute the pair (v_1, v_{2i+1}) , $1 \le i \le \frac{n}{2} - 1$
$d^D(v_1, v_{2i+1}) + \lceil \frac{|f(v_1)^2 - f(v_{2i+1})^2|}{2} \rceil \ge 1 + diam^D(G)$
$6i + 1 + \lceil \frac{|(5n-6)^2 - (4n-5-2i)^2|}{2} \rceil \ge 3n - 2$
Case (c): If both end vertices (v_1, v_{2i}) $d^D(v_i, v_j) + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \ge 1 + diam^D(G)$
$6i - 3 + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \ge 1 + diam^D(G)$
Case (d): If any two intermediate adjacent vertices (v_i, v_j)
$5 + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \ge 1 + diam^D(G)$

Hence $ramn^D(p_n)$ is 5n-6 , $n \ge 3$. □

Theorem 3.2. *The Radio Analytic mean D-distance number of cycle,* $ramn^D(C_n) = 3n, n \ge 2$.

Proof. Let $v(c_n) = \{v_1, v_2, \ldots, v_n\}$. Define the function f as follows.
When n is odd:
Let $f(v_1) = 3n, f(v_{2i}) = 2n + 2i$, $1 \le i \le \frac{(n-1)}{2}$, $f(v_{2i+1}) = 2n - i$, $1 \le i \le \frac{(n-1)}{2}$.
It is $diam^D(C_n) = 3(\frac{n}{2}) + 2$ (n is even) and $diam^D(C_n) = 3\frac{n-1}{2} + 2$ (n is odd).
We will realize the Radio Analytic mean D-distance condition:
$d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \ge 1 + diam^D(G)$
Case (i): when n is odd.
Case (a): Compute the pair (v_1, v_{2i}), $1 \le i \le \frac{n-1}{2}$:
$d^D(v_1, v_{2i}) + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \ge 1 + diam^D(G)$
$6i - 1 + \lceil \frac{|(3n)^2 - (2n+2i)^2|}{2} \rceil \ge 1 + diam^D(G)$
Case(b): Compute the pair (v_1, v_{2i+1}), $1 \le i \le \frac{n}{2}$
$d^D(v_1, v_{2i+1}) + \lceil \frac{|f(v_1)^2 - f(v_{2i+1})^2|}{2} \rceil \ge 1 + diam^D(G)$
$6(1 + i) - 4 + \lceil \frac{|(3n)^2 - (2n-i)^2|}{2} \rceil \ge 1 + diam^D(G)$
Case(c): Compute the two adjacent vertices (v_i, v_j).
$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \ge 1 + diam^D(G)$
$5 + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \ge 1 + diam^D(G)$
Case(d): Compute the remaining non adjacent vertices (v_i, v_j).
$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \ge 1 + diam^D(G)$

$3\frac{(n-1)}{2} + 2 + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (ii): when n is even.
Let $f(v_1) = 3n$, $f(v_{2i}) = 2n + i$, $1 \leq i \leq \frac{n}{2}$, $f(v_{2i+1}) = 2n - i$, $1 \leq i \leq \frac{n}{2} - 1$.
Its $diam^D(C_n) = 3(\frac{n}{2}) + 2$ (n is even).
Case(a): Compute the pair (v_1, v_{2i}), $1 \leq i \leq \frac{n-1}{2}$.

$d^D(v_1, v_{2i}) + \lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \rceil \geq 1 + diam^D(G)$

$6i - 1 + \lceil \frac{|(3n)^2 - (2n+i)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (b): Compute the pair (v_1, v_{2i+1}), $1 \leq i \leq \frac{n}{2}$.

$d^D(v_1, v_{2i+1}) + \lceil \frac{|f(v_1)^2 - f(v_{2i+1})^2|}{2} \rceil \geq 1 + diam^D(G)$

$6(1 + i) - 4 + \lceil \frac{|(3n)^2 - (2n-i)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (c): Compute the two adjacent vertices (v_i, v_j).

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$5 + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (d): Compute the remaining non adjacent vertices (v_i, v_j)

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$3\frac{n}{2} + 2 + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Hence, $ramn^D(C_n) = 3n$, $n \geq 2$ □

Theorem 3.3. *The Radio Analytic mean D-distance number of star graph* $(k_{1,n})$*,* $ramn^D = n + 1, n \geq 3$.

Proof. We Define the function f as $f(v_o) = n + 1$. Let $f(v_i) = i, 1 \leq i \leq n$. We shall check the radio analytic mean D-distance

$d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$.

The $diam^D(G)$ of the star graph is $n + 4$.
Case (i): compute the pair $(v_o, v_i), 1 \leq i \leq n$.

$d^D(v_o, v_i) + \lceil \frac{|f(v_0)^2 - f(v_i)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 2 + \lceil \frac{|(n+1)^2 - (i)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (ii): compute the pair (v_i, v_j), $1 \leq i \leq n$, $i + 1 \leq i \leq n$

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 4 + \lceil \frac{|(i)^2 - (j)^2|}{2} \rceil \geq 1 + diam^D(G)$.

Therefore, $ramn^D = n + 1, n \geq 3$ □

Theorem 3.4. *The Radio Analytic mean D-distance number of Bistar graph is* (B_n, n) *is* $4 + 3n$*,* $n \geq 2$.

Proof. Let $v(B_n, n) = \{u_1, u_2, , \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set, and u,v be the central vertex. Let $E(B_n, n) = \{uu_i, uv, vv_i : 1 \leq i \leq n\}$ be the edge set. The $diam^D(G)$ of the Bistar graph is $7 + 2n$. We define the vertex label as $f(u) = 4 + 2n - 1$, $f(u_i) = n + 2 + i$, $1 \leq i \leq n$, $f(v) = 4 + 3n$, $f(v_i) = 4 + 3n - i$, $1 \leq i \leq n$.

We shall check the radio analytic mean D-distance $d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$. Then $d^D(u, u_i) = d^D(v, v_i) = n + 3$, $n \geq 2$. $d^D(u, v) = 2n + 3, n \geq 2$, $d^D(u_i, u_j) = d^D(v_i, v_j) = 5 + n, n \geq 2$.

Case (a): compute the pair (u, u_i), $1 \leq i \leq n$.

$$d^D(u, u_i) + \lceil \frac{|f(u)^2 - f(u_i)^2|}{2} \rceil \geq 1 + diam^D(G)$$
$$n + 3 + \lceil \frac{|(4+2n+1)^2 - (n+2+i)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (b): compute the pair (u_i, u_j), $1 \leq i \leq n, i + 1 \leq i \leq n$

$$5 + n + \lceil \frac{|(n+2+i)^2 - (n+2+j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (c): compute the pair (v, v_i), $1 \leq i \leq n$

$$d^D(v, v_i) + \lceil \frac{|f(v)^2 - f(v_i)^2|}{2} \rceil \geq 1 + diam^D(G)$$
$$n + 3 + \lceil \frac{|(4+3n)^2 - (4+3n-i)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (d): compute the pair (v_i, v_j), $1 \leq i \leq n$, $i + 1 \leq i \leq n$

$$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$$
$$5 + n + \lceil \frac{|(4+3n-i)^2 - (4+3n-j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (e): compute the pair (u_i, v_i), $1 \leq i \leq n$

$$d^D(u_i, v_i) + \lceil \frac{|f(u_i)^2 - f(v_i)^2|}{2} \rceil \geq 1 + diam^D(G)$$
$$7 + 2n + \lceil \frac{|(n+2+i)^2 - (4+3n-i)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (f): compute the pair (u,v)

$$d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$$
$$2n + 3 + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (g): compute the pair (u, v_i), $1 \leq i \leq n$

$$6 + 2i - 1 + \lceil \frac{|f(u)^2 - f(vi)^2|}{2} \rceil \geq 1 + diam^D(G).$$

Hence, $ramn^D = 4 + 3n$, $n \geq 2$ □

Theorem 3.5. *The Radio analytic mean D-distance number of a complete Bipartite graph,*

$$ramn^D(k_{m,n}) = \begin{cases} 2m + n, m \text{ is even}, & n > m \\ 7\frac{m-1}{2} + 2, m \text{ is odd}, & n > m \end{cases}.$$

Proof. Let $v_1, v_2, \ldots, v_m$ and $u_1, u_2, \ldots, .u_n$ are the partite set. Here $diam^D(k_{m,n}) = m + 2(n+1)$. We will realize the Radio Analytic mean D-distance condition

$$d^D(u,v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (i): When m is even , and $n > m$ we construct the function f as

$$f(v_i) = \lceil \frac{m+n}{2} \rceil + i - 1, 1 \leq i \leq m, \text{ and}$$

$$f(u_i) = m + n + i - 1, 1 \leq i \leq n$$

Case (a): Compute the pair $(v_i, v_j) 1 \leq i \leq n, i+1 \leq j \leq n$

$$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

$$m + 2n + 2 + \lceil \frac{|(\lceil \frac{m+n}{2} \rceil + i - 1)^2 - (\lceil \frac{m+n}{2} \rceil + j - 1)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (b): compute the pair $(v_i, u_j), 1 \leq i \leq m, 1 \leq j \leq n$

$$d^D(v_i, u_j) + \lceil \frac{|f(v_i)^2 - f(u_j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

$$m + n + 1 + \lceil \frac{|(\lceil \frac{m+n}{2} \rceil + i - 1)^2 - (m+n+j-1)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (c): compute the pair $(u_i, u_j), 1 \leq i \leq n, i+1 \leq i \leq n$

$$2m + n + 2 + \lceil \frac{|(m+n+i-1)^2 - (m+n+j-1)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (ii): When m is odd , and $n > m$

We construct the function f as $f(v_i) = \frac{m-1}{2} + n - 3 + i$ and $f(u_i) = 2\{\frac{m-1}{2}\} + n + i - 1$:

Case (a): Compute the pair $(v_i, v_j) 1 \leq i \leq m, i+1 \leq j \leq m$

$$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

$$m + 2n + 2 + \lceil \frac{|(\frac{m-1}{2} + n - 3 + i)^2 - (\frac{m-1}{2} + n - 3 + j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (b): compute the pair $(v_i, u_j), 1 \leq i \leq m, 1 \leq j \leq n$

$$d^D(v_i, u_j) + \lceil \frac{|f(v_i)^2 - f(u_j)^2|}{2} \rceil \geq 1 + diam^D(G)$$

$$m + n + 1 + \lceil \frac{|(\frac{m-1}{2} + n - 3 + i)^2 - (2\frac{m-1}{2} + n + j - 1)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Case (c): compute the pair $(u_i, u_j), 1 \leq i \leq n, i+1 \leq i \leq n$

$$2m + n + 2 + \lceil \frac{|(2\frac{m-1}{2} + n + i - 1)^2 - (2\frac{m-1}{2} + n + j - 1)^2|}{2} \rceil \geq 1 + diam^D(G)$$

Therefore, $ramn^D(k_{m,n}) = \begin{cases} 2m + n, \text{m is even}, & n > m \\ 7\frac{m-1}{2} + 2, \text{m is odd}, & n > m \end{cases}.$ □

Theorem 3.6. *The Radio Analytic mean D-distance number of a wheel graph*

$$ramn^D(w_n) = \begin{cases} 5(\frac{n}{2}) \text{ if n is even}, & n \geq 4 \\ 5(\frac{n-1}{2}) + 2 \text{if n is odd}, & n \geq 3 \end{cases}.$$

Proof. Let $v(w_n) = \{v_1, v_2, \dots, v_n\}$ be the boundary vertex set of w_n. Then x_o is the central vertex .The $diam^D(wn) = n + 8 (n \geq 2)$. We construct the function f:

When n is even

$$f(v_i) = \begin{cases} x_i = 3 & i = 0 \\ \frac{n}{2} + 2i & 1 \leq i \leq n \end{cases}$$

We will realize the Radio Analytic mean D-distance condition

$d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$ for all the pair of vertices.

Case (i): compute the pair x_0, v_i are adjacent for $1 \leq i \leq n$

$d^D(x_0, v_i) + \lceil \frac{|f(x_0)^2 - f(v_i)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 4 + \lceil \frac{|(3)^2 - (\frac{n}{2} + 2i)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (ii): compute the pair v_i, v_j are adjacent for $1 \leq i \leq n, i + 1 \leq j \leq n$

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$7 + \lceil \frac{|(\frac{n}{2} + 2i)^2 - (\frac{n}{2} + 2j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (iii): compute the pair $(v_i, v_j) \leq \frac{n}{2}$

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 8 + \lceil \frac{|(\frac{n}{2} + 2i)^2 - (\frac{n}{2} + 2j)^2|}{2} \rceil \geq 1 + diam^D(G)$

When n is odd

$$f(v_i) = \begin{cases} x_i = 1, & i = 0 \\ \frac{n-1}{2} + 2i & 1 \leq i \leq n \end{cases}$$

Case (i): when n is odd we will realize the Radio Analytic mean D- distance condition $d^D(u, v) + \lceil \frac{|f(u)^2 - f(v)^2|}{2} \rceil \geq 1 + diam^D(G)$ for all the pair of vertices.

Case (ii): compute the pair x_0, v_i are adjacent for $1 \leq i \leq n$

$d^D(x_0, v_i) + \lceil \frac{|f(x_0)^2 - f(v_i)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 4 + \lceil \frac{|(3)^2 - (\frac{n-1}{2} + 2i)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (iii): compute the pair v_i, v_j are adjacent for $1 \leq i \leq n, i + 1 \leq j \leq n$

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$7 + \lceil \frac{|(\frac{n-1}{2} + 2i)^2 - (\frac{n-1}{2} + 2j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Case (iv): compute the pair $(v_i, v_j) \leq \frac{n}{2}$

$d^D(v_i, v_j) + \lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \rceil \geq 1 + diam^D(G)$

$n + 8 + \lceil \frac{|(\frac{n-1}{2} + 2i)^2 - (\frac{n-1}{2} + 2j)^2|}{2} \rceil \geq 1 + diam^D(G)$

Hence,

$$ramn^D(w_n) = \begin{cases} 5(\frac{n}{2})\text{if n is even}, n & \geq 4 \\ 5(\frac{(n-1)}{2}) + 2\text{if n is odd}, & n \geq 3 \end{cases}.$$

□

4. Conclusion

In this paper we studied the Radio analytic mean D-distance graphs, which involves D-distance and diameter. We computed the Radio analytic mean D-distance number by using in some basic graphs and radio analytic mean number depends on the distance constraints.

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