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UNFUZZY DELINEATION OF KNOTS

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ABSTRACT. In this paper one and two thread simple knots are studied. An unambiguous matrix representation of knots is provided for planar, non - planar, normal crossection and self cross section types of single and double thread knots. Further with the help of the matrix representation the properties of knots are studied.

1. INTRODUCTION

A study of the geometrical aspects of knots is known as Knot theory. Basic knots are studied and they are classified accordingly. Using a minimum of 40 cm long rope or thread it is attempted to study knots. The terms and definitions used in this paper are from the book [1] 'Knot Theory and Its Applications' written by Kunio Murasugi translated by Bohdan Kurpita.

Knots are man-made. Physiologically a knot is equilibrium. The important fact of biological system is that all living organism exist in a steady state characterised by concentrations of each of these biomolecules. These biomolecules are in a metabolic flux. Any chemical or physical process moves spontaneously to equilibrium. The steady state is a non-equilibrium state. Remember from physics, the system at equilibrium cannot perform work. As living organisms work continuously, they cannot afford to reach equilibrium. Hence the living

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state is a non-equilibrium steady state to able to perform work; living process is a constant effort to prevent falling into equilibrium.

Few more applications and results of knot theory can be studied from [2, 3] [2,3]. [1] A knot is an entwined circle. The non - planar knots are the class of 3-dimension and that of the 2- dimensions are the planar knots. Further knots are also classified as oriented and not oriented. Additionally the knots are sorted into open knots and closed knots.

2. Open and Closed Knots

If the ends of the rope are joined after knotting then it is said to be the closed knot. Moreover the entwined closed circle is also a closed knot. Closed knot can also be termed as no end knots.
As well if the ends are untied then the knots are called open knots. The open knots can also be called as a 2n end knots where the n represents the number of rope or thread involved in the knot.

Result: Using the fundamental unknotting steps; all the open knots are bounded to be unknotted. But not any of the closed knot can be unknotted.

3. Matrix Delineation of Single Thread Knots

The knots discussed here are single rope and oriented. To start with the knot should be labelled, primarily label the ends and starting from one end label all the places where the rope undergoes a cross section.

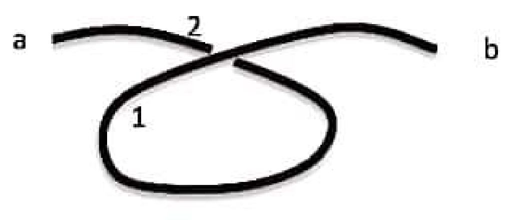

Figure 1. Open Twist

Considering the above twist, calling it a 'twist' would be more preferable, both the ends are untied, therefore it is open. Matrix representation is,

$$T = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Where, the first row of T represents the underlying position in the cross section and the second row represents the overlapping position of the rope in the cross section. The number of columns in T represents the number of cross sections in the knot. Let us consider a closed twist for matrix representation, consider the Figure 2.

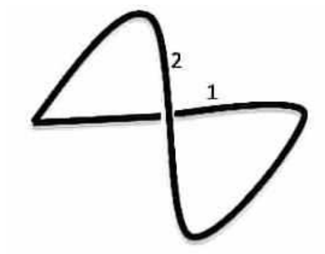

FIGURE 2. Closed Twist

Figure 2 can be represented as,

$$T = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{(1,2)},$$

where 1 is underlying and two is overlapping, the suffix to the matrix (1,2) reveals that the ends 1 and 2 are tied. Hence it is closed.

Theorem 3.1. *The two knots K_1 and K_2 given in figure 3 are isomorphic but not equivalent, hence does not belong to the Corbordism Group.*

Proof. The representation of K_1 and K_2 are given as follows,

$$K_1 = \begin{pmatrix} 4 & 2 & 6 \\ 1 & 5 & 3 \end{pmatrix}_{(1,6)} \qquad K_2 = \begin{pmatrix} 1 & 5 & 3 \\ 4 & 2 & 6 \end{pmatrix}_{(1,6)}$$

From the representation the following facts are unambiguous,

(i) The two knots K_1 and K_2 are inverse or mirror image of each other.
(ii) They are isomorphic, since the number of rows, number of columns and the numbers of faces are equal.

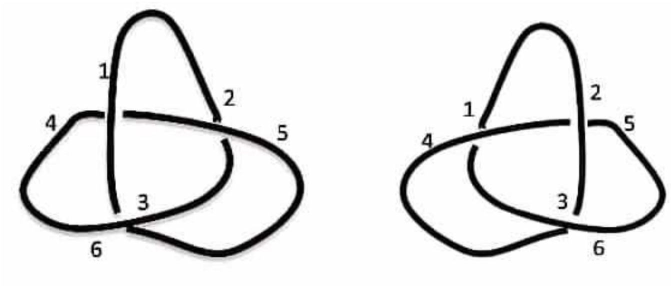

FIGURE 3. Isomorphic Knots

Apart from the similarities it is also clear that using the elementary knot moves [1] K_1 and K_2 are not equivalent. Since their does not exist an equivalence relation they do not belong to the Corbordism group. □

Proposition 3.1. *Stepwise unknotting of the Tangle T_6 and its matrix delineation.*

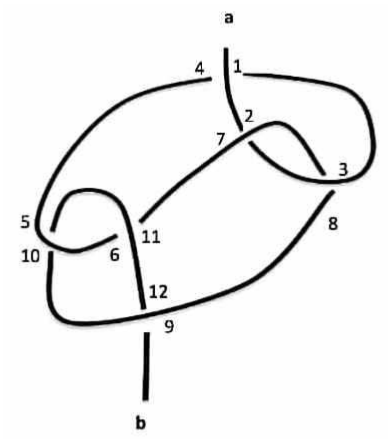

FIGURE 4. Tangle T_6

T_6 is tangle with six cross sections. The matrix delineation of the above tangle is given below.

$$T_6 = \begin{pmatrix} 4 & 2 & 8 & 10 & 6 & 9 \\ 1 & 7 & 3 & 5 & 11 & 12 \end{pmatrix}$$

Step 1: Choose the end b and removing it will give the figure 5.
The matrix representation of tangle T_5 is given below.

$$T_5 = \begin{pmatrix} 4 & 2 & 8 & 9 & 6 \\ 1 & 7 & 3 & 5 & 10 \end{pmatrix}$$

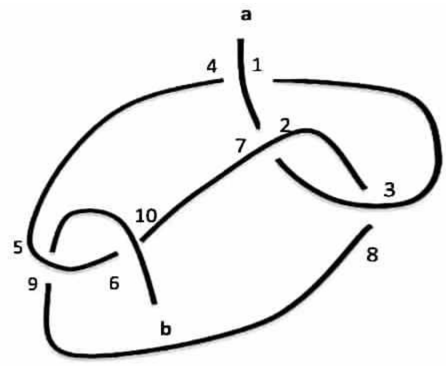

FIGURE 5. Tangle T_5

Performing the first step repeatedly, the following matrices are derived,

$$T_4 = \begin{pmatrix} 4 & 2 & 7 & 8 \\ 1 & 6 & 3 & 5 \end{pmatrix}$$

$$T_3 = \begin{pmatrix} 4 & 2 & 6 \\ 1 & 5 & 3 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$T_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Hence the tangle is unknotted.

4. DOUBLE THREAD KNOT'S MATRIX DELINEATION

Case: 1 Planar Knots.

Let $K(2,n)$ denote knots of two threads with n cross sections, if all the cross sections involves both the knots. Then matrix delineation of $K(2,n)$ is given below.

$$K(2,n) = \begin{matrix} & 1_2 & \cdots & n_2 \\ 1_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ n_1 & 0 & \cdots & 1 \end{matrix}$$

Here, $1_1 \cdots n_1$ represents the cross sections in the first thread and $1_2 \cdots n_2$ represents the cross sections in the second thread in the above matrix. The entries in the matrix are 1 and 0. The entry 1 denotes that the row element and the column element intersect; where else 0 if they do not intersect.

Example 1. *Consider the following double thread knot. It has four cross section, hence it can be denoted as K(2,4). The matrix delineation for the knot is also given.*

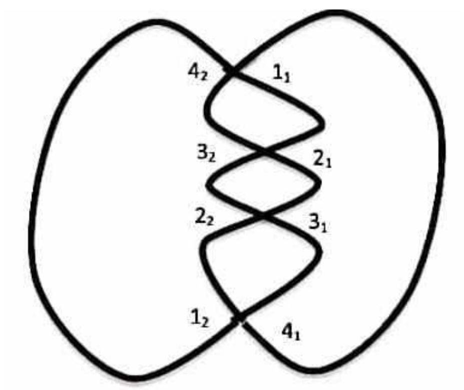

FIGURE 6. $K(2,4)$

$$K(2,4) = \begin{array}{c} \\ 1_1 \\ 2_1 \\ 3_1 \\ 4_1 \end{array} \begin{array}{c} \begin{array}{cccc} 1_2 & 2_2 & 3_2 & 4_2 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Case: 2 Non Planar Knots.
The entries in the non-planar matrix represent the thread that underlies the cross section. Let us consider the following example for the further clarification.

Example 2. *The knot given in this example is a non-planar double knot, which also has a self-crossing as well as the normal cross section. This type of knots is denoted as* $K(2, n, n_1, n_2)$ *. Where, K and 2 denotes the knot with two threads respectively.*

- n *denotes the number of cross sections that involves both the threads.*
- n_1 *denotes the number of cross section that involves thread 1 alone.*

- n_2 denotes the number of cross section that involves thread 2 alone.
Here in the given knot, let the circle be assumed as thread 1 and the other one as thread 2. There are totally 5 cross sections and among them four cross sections involves both the thread and one involves thread 2 alone. Hence the knot can be denoted as $K(2,4,0,1)$. So the number of row (number of crossings that involves thread 1) is 4 and similarly the number of column is 6 (Each self-crossing section counts 2).

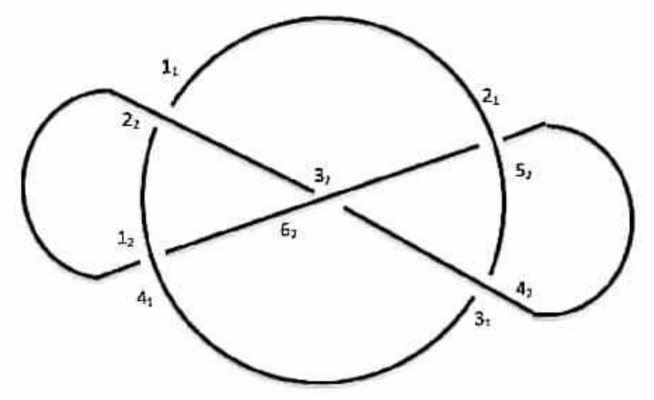

FIGURE 7. $K(2,4,0,1)$

The matrix delineation for the above knot is given below.

$$K(2,4,0,1) = \begin{array}{c} \\ 1_1 \\ 2_1 \\ 3_1 \\ 4_1 \end{array} \begin{array}{c} \begin{array}{cccccc} 1_2 & 2_2 & 3_2 & 4_2 & 5_2 & 6_2 \end{array} \\ \begin{pmatrix} 1_1 & 0 & 0 & 0 & .2_2 & 0 \\ 0 & 0 & 0 & 4_2 & 0 & 0 \\ 0 & 0 & 3_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6_2 \end{pmatrix} \end{array}$$

Now consider the first row and the first column entry in the matrix, it is $(1_1, 1_2) = 1_1$. This implies that 1_1 and 1_2 intersects and among them 1_1 is the underlying in the intersection. And the first row fifth column entry is $.2_2$. The "." represents this is a self crossing. This means that 5_2 and 2_2 intersects and 2_2 is the underlying. This is the self-crossing of the second knot.

Hence the matrix delineation of two thread knot for both planar and non-planar knots that has different types of cross section has been explained.

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