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COEFFICIENT ESTIMATES FOR CERTAIN SUBCLASSES OF BI-PSEDUO- MOCANU-CONVEX FUNCTIONS

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ABSTRACT. In this paper, we introduce and investigate a new subclass Mocanu-Pseduo-bi-convex functions of the class Σ in the open unit disk,satisfy Ma and Minda subordination conditions. Furthermore, we find estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in the new subclass introduced here. Several (known or new) consequences of the results are also pointed out.

1. INTRODUCTION

Let A denote the class of analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

normalized by the conditions $f(0) = 0 = f'(0) - 1$ defined in the open unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. The Koebe one quarter theorem [4] ensures that the image of $\mathbb{U}$ under every univalent function $f \in$ A contains a disk of radius $\frac{1}{4}$. Thus every univalent function f has an inverse f^{-1} satisfying $f^{-1}(f(z)) = z,\ (z \in \mathbb{D})$ and $f(f^{-1}(\omega)) = \omega\ (|\omega| < r_0(f),\ r_0(f) \geq \frac{1}{4})$. A function $f \in$ A is said to be bi-univalent in $\mathbb{D}$ if both f and f^{-1} are univalent in $\mathbb{D}$. Let Σ denote the class of bi-univalent functions defined in the unit disk $\mathbb{D}$. Since $f \in \Sigma$ has the

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Maclaurian series given by (1.1), a computation shows that its inverse $g = f^{-1}$ has the expansion

(1.2) $$g(\omega) = f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 + \cdots$$

An analytic function f is subordinate to an analytic function g, written $f(z) \prec g(z)$, provided there is an analytic function w defined on $\mathbb{D}$ with $w(0) = 0$ and $|\omega(z)| < 1$ satisfying $f(z) = g(\omega(z))$.

The class $\mathrm{L}_\vartheta(\mu)$ of ϑ-pseudo-starlike functions of order $\mu(0 \leq \mu < 1)$ were introduced and investigated by Babalola [2] whose geometric conditions satisfy $\Re\left(\frac{z(f'(z))^\vartheta}{f(z)}\right) > \mu$. He showed that all pseudo-starlike functions are Bazilevič of type $\left(1 - \frac{1}{\vartheta}\right)$ order $\beta^{\frac{1}{\vartheta}}$ and univalent in open unit disk $\mathbb{D}$. If $\vartheta = 1$, we have the class of starlike functions of order μ, which in this context, are 1-pseudo-starlike functions of order μ. Further, for $\vartheta = 2$ we note that functions in $\mathrm{L}_2(\mu) \equiv \mathrm{G}(\mu)$ are defined by $\Re\left(f'(z)\frac{zf'(z)}{f(z)}\right) > \mu, (z \in \mathbb{D})$

Recently there has been triggering interest to study bi-univalent functions (see [8, 10, 11].Motivated by the works of Ali et al. [1] and Goyal and Goswami [5] and Srivastava et al [7]in this paper we introduce a new subclass $\mathrm{M}_\Sigma^\gamma(\vartheta, \mu, h)$ of bi-univalent functions to estimate the coefficients $|a_2|$ and $|a_3|$ for the functions in the class $\mathrm{M}_\Sigma^\tau(\vartheta, \mu, h)$.

Definition 1.1. *Let $h : \mathbb{D} \to \mathbb{C}$ be a convex univalent function such that $h(0) = 1$ and $\Re(h(z)) > 0, z \in \mathbb{D}$. A function $f(z)$ is said to be in the class $\mathrm{M}_\Sigma^\gamma(\vartheta, \mu, h)$ if the following conditions are satisfied:*

(1.3)
$$e^{i\tau}\left[(1-\mu)\frac{z(f'(z))^\vartheta}{f(z)} + \mu\left(\frac{z[(f'(z))']^\vartheta}{f'(z)}\right)\right] \prec h(z)\cos\tau + i\sin\tau,\ (f \in \Sigma,\ z \in \mathbb{D})$$

and

(1.4) $$e^{i\tau}\left[(1-\mu)\frac{\omega(g'(\omega))^\vartheta}{g(\omega)} + \mu\left(\frac{\omega[(g'(\omega))']^\vartheta}{g'(\omega)}\right)\right] \prec h(\omega)\cos\tau + i\sin\tau,\ (\omega \in \mathbb{D})$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2}), \vartheta \geq 0$ and $g = f^{-1}$.

Remark 1.1. *If we set $h(z) = (1 + Cz)/(1 + Dz),\ -1 \leq D < C \leq 1$, then the class $\mathrm{M}_\Sigma^\tau(\vartheta, \mu, h)$ reduces to $\mathrm{M}_\Sigma^\tau(\vartheta, \mu, C, D)$ which is defined as*

(1.5)
$$e^{i\tau}\left[(1-\mu)\frac{z(f'(z))^\vartheta}{f(z)} + \mu\left(\frac{z[(f'(z))']^\vartheta}{f'(z)}\right)\right] \prec \frac{1+Cz}{1+Dz}\cos\tau + i\sin\tau,\ (f \in \Sigma,\ z \in \mathbb{D})$$

and

(1.6)
$$e^{i\tau}\left[(1-\mu)\frac{\omega(g'(\omega))^{\vartheta}}{g(\omega)}+\mu\left(\frac{\omega[(g'(\omega))']^{\vartheta}}{g'(\omega)}\right)\right]\prec\frac{1+C\omega}{1+D\omega}\cos\tau+i\sin\tau,\ (\omega\in\mathbb{D})$$

where $\tau\in(-\frac{\pi}{2},\frac{\pi}{2}),\mu\geq 0$ *and* $g=f^{-1}$.

In particular, by setting $h(z)=\frac{1+(1-2\beta)z}{1-z},\ 0\leq\beta<1$, the class $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,h)$ reduces to $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,1-2\beta,-1)\equiv\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,\beta)$. Further, we note that $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,0,\beta)\equiv\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\beta)$. and $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,1,\beta)\equiv\mathrm{N}_{\Sigma}^{\tau}(\vartheta,\beta)$.

Remark 1.2. *Taking* $\mu=0$ *in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,C,D)$ *we have* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,C,D)$ *and if* $f\in\mathrm{M}_{\Sigma}^{\tau}(\vartheta,C,D)$, *then*

$$e^{i\tau}\frac{z(f'(z))^{\vartheta}}{f(z)}\prec\frac{1+Cz}{1+Dz}\cos\tau+i\sin\tau,\ (f\in\Sigma,\ z\in\mathbb{D}) \tag{1.7}$$

and

$$e^{i\tau}\frac{\omega(g'(\omega))^{\vartheta}}{g(\omega)}\prec\frac{1+C\omega}{1+D\omega}\cos\tau+i\sin\tau,\ (\omega\in\mathbb{D}) \tag{1.8}$$

where $\tau\in(-\frac{\pi}{2},\frac{\pi}{2})$ *and* $g=f^{-1}$.

We note that by taking $\vartheta=1,\mu=0$, and $\tau=0$, we get the known class $\mathrm{M}_{\Sigma}^{0}(1,0,1-2\beta,-1)\equiv\mathrm{S}_{\Sigma}^{*}(\beta)$ [3].

Remark 1.3. *Taking* $\mu=1$ *in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,C,D)$, *we have* $\mathrm{N}_{\Sigma}^{\tau}(\vartheta,C,D)$ *and if* $f\in\mathrm{N}_{\Sigma}^{\tau}(\vartheta,C,D)$, *then*

$$e^{i\tau}\frac{z[(f'(z))']^{\vartheta}}{f'(z)}\prec\frac{1+Cz}{1+Dz}\cos\tau+i\sin\tau,\ (f\in\Sigma,\ z\in\mathbb{D}) \tag{1.9}$$

and

$$e^{i\tau}\left(\frac{w[(g'(\omega))']^{\vartheta}}{g'(\omega)}\right)\prec\frac{1+C\omega}{1+D\omega}\cos\tau+i\sin\tau,\ (w\in\mathbb{D}) \tag{1.10}$$

where $\tau\in(-\frac{\pi}{2},\frac{\pi}{2})$ *and* $g=f^{-1}$.

We also note that $\mathrm{M}_{\Sigma}^{0}(1,1,1-2\beta,-1)\equiv\mathrm{K}_{\Sigma}(\beta)$ [3].

The object of the paper is to estimate the coefficients $|a_2|$ and $|a_3|$ for the functions in the class $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,h)$. Further we define a new generalized subclass of $\mathrm{M}_{\Sigma}^{\tau}(\vartheta,\mu,h)$ involving Hohlov operator and obtained the Maclaurians coefficients for f in this new generalized class.

2. Coefficients estimates for the function class $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, h)$

In order to prove our main result for the functions class $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, h)$, we recall the following lemma.

Lemma 2.1. *Let the function* $\Phi(z)$ *given by* $\Phi(z) = \sum_{n=1}^{\infty} B_k z^k$ *be convex in* $\mathbb{D}$. *Suppose also that the function* $h(z)$ *given by* $h(z) = \sum_{k=1}^{\infty} h_k z^k$, *is holomorphic in* $\mathbb{D}$. *If* $h(z) \prec \Phi(z)(z \in \mathbb{D})$, *then* $|h_k| \leq |B_1|(k \in \mathbb{N})$.

Theorem 2.1. *Let* f *given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, h)$. *Then*

$$|a_2| \leq \sqrt{\frac{|B_1| \cos\tau}{\vartheta(2\vartheta - 1) + 2\mu(6\vartheta^2 + 6\vartheta - 1)}} \tag{2.1}$$

and

$$|a_3| \leq \left(\frac{1}{(1+2\mu)(3\vartheta - 1)} + \frac{|B_1| \cos\tau}{(2\vartheta - 1)^2 (1+\mu)^2} \right) |B_1| \cos\tau \tag{2.2}$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$ *and* $\mu \geq 0$.

Proof. Let $f \in \mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, h)$ and $g = f^{-1}$. Then from (1.3) and (1.4) we have

$$e^{i\tau} \left[(1-\mu) \frac{z(f'(z))^{\vartheta}}{f(z)} + \mu \left(\frac{z[(f'(z))']^{\vartheta}}{f'(z)} \right) \right] \prec p(z) \cos\tau + i \sin\tau, \ (z \in \mathbb{D}) \tag{2.3}$$

and

$$e^{i\tau} \left[(1-\mu) \frac{w(g'(\omega))^{\vartheta}}{g(\omega)} + \mu \left(\frac{w[(g'(\omega))']^{\vartheta}}{g'(\omega)} \right) \right] \prec q(w) \cos\tau + i \sin\tau, \ (w \in \mathbb{D}) \tag{2.4}$$

where $p(z) \prec h(z)$ and $q(w) \prec h(\omega)$ and have the following forms:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots, \ z \in \mathbb{D} \tag{2.5}$$

and

$$q(\omega) = 1 + q_1 \omega + q_2 \omega^2 + q_3 \omega^3 + \cdots, \ \omega \in \mathbb{D}. \tag{2.6}$$

On other hand we have

$$e^{i\tau} \left[(1-\mu) \frac{z(f'(z))^{\vartheta}}{f(z)} + \mu \left(\frac{z[(f'(z))']^{\vartheta}}{f'(z)} \right) \right] = e^{i\tau} \{ 1 + (1+\mu)(2\vartheta - 1) a_2 z$$

$$+ [(2\vartheta^2 - 4\vartheta + 1)(1+3\mu) a_2^2 + (1+2\mu)(3\vartheta - 1) a_3] z^2 + \cdots \} \prec p(z) \cos\tau + i \sin\tau \tag{2.7}$$

and

$$e^{i\tau}\left[(1-\mu)\frac{w(g'(\omega))^{\vartheta}}{g(\omega)}+\mu\left(\frac{\omega[(g'(\omega))']^{\vartheta}}{g'(\omega)}\right)\right]=e^{i\tau}\{1-(1+\mu)(2\vartheta-1)a_2\omega+\big[[(2\vartheta^2+2\vartheta-1)$$

$$+\mu(6\vartheta^2-1)]a_2^2-(1+2\mu)(3\vartheta-1)a_3\big]\omega^2+\cdots\}\prec q(\omega)\cos\tau+i\sin\tau,\ (\omega\in\mathbb{D}) \tag{2.8}$$

Now,equating the coefficients of z and z^2 in (2.7) and (2.8) we get

$$e^{i\tau}(1+\mu)(2\vartheta-1)a_2 = p_1\cos\tau, \tag{2.9}$$

$$e^{i\tau}[2\vartheta^2-4\vartheta+1)(1+3\mu)a_2^2+(1+2\mu)(3\vartheta-1)a_3] = p_2\cos\tau, \tag{2.10}$$

$$-e^{i\tau}(1+\mu)(2\vartheta-1)a_2 = q_1\cos\tau \tag{2.11}$$

and

$$e^{i\tau}\{[(2\vartheta^2+2\vartheta-1)+\mu(6\vartheta-1)]a_2^2-(1+2\mu)(3\vartheta-1)a_3\}=q_2\cos\tau. \tag{2.12}$$

From (2.9) and (2.11) it follows that

$$p_1=-q_1 \tag{2.13}$$

and

$$2e^{2i\tau}(1+\mu)^2(2\vartheta-1)^2a_2^2=(p_1^2+q_1^2)\cos^2\tau. \tag{2.14}$$

That is

$$a_2^2=\frac{(p_1^2+q_1^2)\cos^2\tau}{2(1+\mu)^2(2\vartheta-1)^2}e^{-2i\tau}$$

Adding (2.10) and (2.12) it follows that

$$a_2^2=\frac{(p_2+q_2)}{2\{\vartheta(2\vartheta-1)+2\mu(6\vartheta^2+6\vartheta-1)\}}e^{-i\tau}\cos\tau. \tag{2.15}$$

Since $p(z),q(z)\in h(\mathbb{D})$, applying Lemma 2.1 for the coefficients p_2 and q_2, we get

$$|a_2|^2=\frac{|B_1|\cos\tau}{\vartheta(2\vartheta-1)+2\mu(6\vartheta^2+6\vartheta-1)} \tag{2.16}$$

which gives the estimate on $|a_2|$ as asserted in (2.1). Subtracting (2.12) from (2.10) we get

$$2(1+2\mu)(3\vartheta-1)a_3-2(1+2\mu)(1-3\vartheta)a_2^2=(p_2-q_2)e^{-i\tau}\cos\tau \tag{2.17}$$

Substituting the value of a_2^2 from (2.14) in (2.17) we get

$$a_3=\frac{(p_2-q_2)e^{-i\tau}\cos\tau}{2(1+2\mu)(3\vartheta-1)}+\frac{(p_1^2+q_1^2)e^{-2i\tau}\cos^2\tau}{2(2\vartheta-1)^2(1+\mu)^2}.$$

Applying Lemma 2.1 once again for the coefficients p_1, p_2, q_1 and q_2, we get

$$|a_3| \leq \frac{|B_1|\cos\tau}{(1+2\mu)(3\vartheta-1)} + \frac{|B_1|^2\cos^2\tau}{(2\vartheta-1)^2(1+\mu)^2}$$

which gives the estimate on $|a_3|$ as asserted in (2.2). □

By taking $\mu = 0$ we state the following:

Corollary 2.1. *Let* f *given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, h)$. *Then*

$$|a_2| \leq \sqrt{\frac{|B_1|\cos\tau}{\vartheta(2\vartheta-1)}} \tag{2.18}$$

and

$$|a_3| \leq \left(\frac{1}{3\vartheta-1} + \frac{|B_1|\cos\tau}{(2\vartheta-1)^2}\right)|B_1|\cos\tau \tag{2.19}$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$ *and* $\mu \geq 0$.

By taking $\mu = 1$ we state the following:

Corollary 2.2. *Let* f *given by* (1.1) *be in the class* $\mathrm{N}_{\Sigma}^{\tau}(\vartheta, h)$. *Then*

$$|a_2| \leq \sqrt{\frac{|B_1|\cos\tau}{\vartheta(2\vartheta-1)+2(6\vartheta^2+6\vartheta-1)}} \tag{2.20}$$

and

$$|a_3| \leq \left(\frac{1}{3(3\vartheta-1)} + \frac{|B_1|\cos\tau}{4(2\vartheta-1)^2}\right)|B_1|\cos\tau \tag{2.21}$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$ *and* $\mu \geq 0$.

By setting $h(z) = \frac{1+Cz}{1+Dz}, -1 \leq D < C \leq 1$ in Theorem 2.1, we get the following corollary:

Corollary 2.3. *Let* f *given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, C, D)$. *Then*

$$|a_2| \leq \sqrt{\frac{(C-D)\cos\tau}{\vartheta(2\vartheta-1)+2\mu(6\vartheta^2+6\vartheta-1)}} \tag{2.22}$$

and

$$|a_3| \leq \left(\frac{1}{(1+2\mu)(3\vartheta-1)} + \frac{(C-D)\cos\tau}{(2\vartheta-1)^2(1+\mu)^2}\right)(C-D)\cos\tau, \tag{2.23}$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$ *and* $\mu \geq 0$.

If we take $\mu = 0$ in corollary 2.3 we obtain

Corollary 2.4. *Let f given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, C, D)$. *Then*

$$|a_2| \leq \sqrt{\frac{(C-D)\cos\tau}{\vartheta(2\vartheta-1)}}$$

and

$$|a_3| \leq \left(\frac{1}{3\vartheta-1} + \frac{(C-D)\cos\tau}{(2\vartheta-1)^2}\right)(C-D)\cos\tau,$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

If we take $\mu = 1$ in corollary 2.3 we obtain

Corollary 2.5. *Let f given by* (1.1) *be in the class* $\mathrm{N}_{\Sigma}^{\tau}(\vartheta, C, D)$. *Then*

$$|a_2| \leq \sqrt{\frac{(C-D)\cos\tau}{\vartheta(2\vartheta-1)+2(6\vartheta^2+6\vartheta-1)}}$$

and

$$|a_3| \leq \left(\frac{1}{3(3\vartheta-1)} + \frac{(C-D)\cos\tau}{4(2\vartheta-1)^2}\right)(C-D)\cos\tau,$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Further, by setting $h(z) = \frac{1+(1-2\beta)z}{1-z},\ 0 \leq \beta < 1$ in Theorem 2.1 we get the following corollary:

Corollary 2.6. *Let f be given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, \beta)$. *Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)\cos\tau}{\vartheta(2\vartheta-1)+2\mu(6\vartheta^2+6\vartheta-1)}}$$

and

$$|a_3| \leq \left(\frac{1}{(1+2\mu)(3\vartheta-1)} + \frac{4(1-\beta)\cos\tau}{(2\vartheta-1)^2(1+\mu)^2}\right)(1-\beta)\cos\tau,$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$ *and* $\mu \geq 0$.

If we take $\mu = 0$ in corollary 2.6 we obtain

Corollary 2.7. *Let f given by* (1.1) *be in the class* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \beta)$. *Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)\cos\tau}{\vartheta(2\vartheta-1)}}$$

and

$$|a_3| \leq \left(\frac{1}{3\vartheta-1} + \frac{4(1-\beta)\cos\tau}{(2\vartheta-1)^2}\right)(1-\beta)\cos\tau,$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

If we take $\mu = 1$ in corollary 2.6 we obtain

Corollary 2.8. *Let* f *given by* (1.1) *be in the class* $\mathrm{N}_{\Sigma}^{\tau}(\vartheta, \beta)$. *Then*

$$|a_2| \leq \sqrt{\frac{(1-\beta)\cos\tau}{\vartheta(2\vartheta-1) + 2(6\vartheta^2 + 6\vartheta - 1)}}$$

and

$$|a_3| \leq \left(\frac{1}{3(3\vartheta-1)} + \frac{(1-\beta)\cos\tau}{4(2\vartheta-1)^2}\right)(1-\beta)\cos\tau,$$

where $\tau \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Remark 2.1. *On taking* $\tau = 0$, *and* $\vartheta = 1$ *we observe that the classes* $\mathrm{M}_{\Sigma}^{\tau}(\vartheta, \mu, h)$, $\mathrm{M}_{\Sigma}^{\tau}(0, h)$ *and* $\mathrm{M}_{\Sigma}^{\tau}(1, h)$ *becomes the familiar classes* $\mathrm{M}_{\Sigma}^{0}(\mu, h) \equiv \mathrm{M}_{\Sigma}(\mu, h)$, $\mathrm{M}_{\Sigma}^{0}(0, h)$ $\equiv \mathrm{S}_{\Sigma}^{*}(h)$ *and* $\mathrm{M}_{\Sigma}^{0}(1, h) \equiv \mathrm{K}_{\Sigma}(h)$ *respectively.*

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