Advances in Mathematics: Scientific Journal **9** (2020), no.8, 5927–5934
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.8.60 Special Issue on ICMA-2020

APPLICATIONS OF KAMAL TRANSFORM IN TWO TANK MIXING PROBLEMS

S. KALAIARASI[1], S. KEERTHANA, N. NITHIYAPRIYA, AND M. ANITA

ABSTRACT. The solutions of problems involving differential equations are obtained by various techniques. In case of typical problems solving the equations using transforms are very appropriate as they simplify the problems. In this paper, we have made use of the Kamal transform method for solving the Two Tank Mixing Problems which is an application of first order linear differential equations.

1. INTRODUCTION

Differential and integral equations plays an important role in science and engineering, [4,5]. Integral transforms belong to the basic subjects of mathematical analysis, in the theory of differential and integral equations and too many other areas of pure and applied mathematics, [2,3].

Kamal transform of the function $f(t)$ is defined by, [1],

$$[f(t)] = \int_0^\infty f(t)e^{\frac{-t}{v}}dt = G(v), t > 0, k_1 \leq v \leq k_2. \tag{1.1}$$

The aim of this work is to calculate the amount of salt content in the two tank mixing problems by using Kamal transform without large computational work.

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 60K10, 68M115.

Key words and phrases. integral transform, Kamal transform, inverse Kamal transform, two tank mixing problems.

2. Kamal Transform of Various Functions

In this part, we derive the Kamal transform of some standard functions and its first derivatives in solving problems involving first order differential equations.

i) Let $f(t) = 1$, by the definition we have,

$$K[1] = G(v) = \int_0^\infty e^{\frac{-t}{v}}\, dt = [-ve^{\frac{t}{v}}]_0^\infty = v.$$

ii) Let $f(t) = t$, then

$$K[t] = \int_0^\infty te^{\frac{-t}{v}}\, dt$$

Integrating by parts, we get $K[t] = v^2$.
In the general case if $n \geq 0$ is integer number, then

$$K[t^n] = n!v^{n+1}.$$

iii) Let $f(t) = e^{at}$ then,

$$K[e^{at}] = \int_0^\infty e^{at} e^{\frac{-t}{v}}\, dt = \frac{v}{1-av}.$$

iv) Let $f(t) = e^{-at}$ then,

$$K[e^{-at}] = \int_0^\infty e^{-at} e^{\frac{-t}{v}}\, dt = \frac{v}{1+av}.$$

Theorem 2.1. *Let $G(v)$ is the Kamal transform of $f(t)[Kf(t)] = G(v)$ then:*

i) $K[f'(t)] = \frac{1}{v}G(v) - f(0)$;
ii) $K[f''(t)] = \frac{1}{v^2}G(v) - \frac{1}{v}f(0) - f'0$;
iii) $K[f^{(n)}(t)] = v^{-n}G(v) - \sum_{k=0}^{n-1} v^{k-n-1} f^{(k)}(0)$.

Theorem 2.2 (Inverse Kamal Transform)**.** *If $K[F(t)] = G(v)$, then $F(t)$ is called the inverse kamal transform of $G(v)$ and mathematically it is defined as $F(t)K^{-1}[G(v)]$, where K^{-1} is the inverse Kamal transform operator.*

3. Application and analysis

Illustration 1:
Let Tank-I and Tank-II be connected with pipes. 50 liters of water and 20 grams of salts be contained in Tank-I and Tank-II has twice the amount of water in Tank-I and 1.5 times more salt. Amount of fresh water entering Tank-I and leaving to Tank-II be 3 liters per min and solution from Tank-II is pumped to Tank-I at the same rate. Find as a function of time the distribution of salt in Tank-I and Tank-II.

Solution:

Let us assume that the two tanks are Tank-I and Tank-II.

Let $x(t)$ and $y(t)$ represents the amount of salt content in Tank-I and Tank-II at time t.

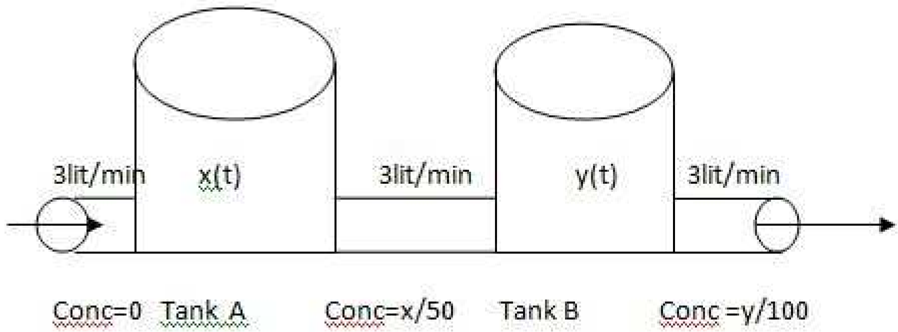

FIGURE 1. Connected tanks

For Tank I we have,

$\frac{dx}{dt}$=(Rate of salt entering into tank-I)-(Amount of salt leaving from tank-II)

$\frac{dx}{dt}=[(0kg/liters)(3liters/minute)] - [(\frac{x(t)}{50}kg/liter)(3liters/min)]$

$$\Rightarrow \frac{dx}{dt} + \frac{3x}{50} = 0 \tag{3.1}$$

At the initial time $t = 0$, $x(0) = 20$.

Taking Kamal transform on equation (3.1),

$$K\left[\frac{dx}{dt}+\frac{3x}{50}\right]=K(0).$$

$$\Rightarrow \overline{x}(l)=\frac{20l}{1+\frac{3}{50}l}. \tag{3.2}$$

Taking the inverse Kamal transform on equation (3.2), we get,

$$x(t)=20e^{\frac{-3}{50}t}.$$

Similarly, for Tank II we have: At the initial time $t=0$, $y(0)=30$. Taking Kamal transform on equation, [4] it is obtained:

$$K[\frac{dy}{dt}+\frac{3}{100}y]=K[\frac{3x}{50}]$$

$$\Rightarrow \overline{y}(l)=\frac{6}{5}\frac{l^2}{(1+\frac{3}{50})(1+\frac{3}{100})}+30\frac{l}{(1+\frac{3}{50})} \tag{3.3}$$

For simplification we consider,

$$\frac{6}{5}\frac{l^2}{(1+\frac{3}{50})(1+\frac{3}{100})}=\frac{6}{5}\left\{\frac{-100/3l}{(1+\frac{3}{50})}\right\}+\left\{\frac{100/3l}{(1+\frac{3}{100})}\right\}$$

$$\Rightarrow \overline{y}(l)=40\left[\frac{-l}{(1+\frac{3}{50})}+\frac{l}{(1+\frac{3}{100})}\right]. \tag{3.4}$$

Now substituiting (3.4) in (3.3), we get,

$$\overline{y}(l)=40\left[\frac{-l}{(1+\frac{3}{50})}+\frac{l}{(1+\frac{3}{100})}\right]+30\frac{l}{(1+\frac{3}{50})}.$$

Taking inverse Kamal transform on both sides, we get,

$$y(t)=40[-e^{\frac{-3}{50}t}+e^{\frac{-3}{100}t}]+30e^{\frac{-3}{100}t}$$

$$y(t)=70e^{(\frac{-3}{100})t}-40e^{(\frac{-3}{50})t}.$$

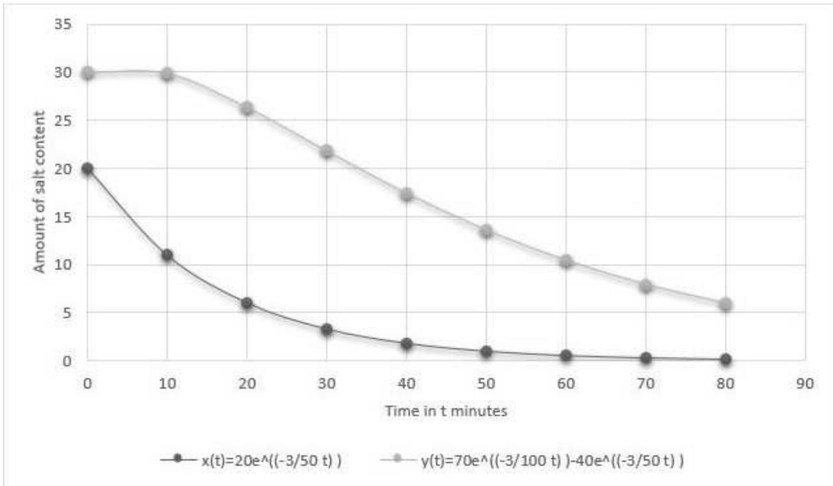

FIGURE 2. The dotted curve shows the quantity of the salt content in tank-I and the line curve shows the quantity of the salt content present in the tank-II

Illustration 2:
Let us consider Tank-I and Tank-II are 50 quarts($2pints = 1quart$). Initially Tank-I contains salt water at concentration 0.25 liters per pint, Tank-II is filled with pure water. Water from Tank-II enters Tank-I at a rate of 4 pints per min and complete mixture of pure and salt water from Tank-I leaves to Tank-II at the same rate. Water at the rate of 4 pints per min leaves the system from Tank-II. Find the salt distribution in both tanks.
Solution:
Let us assume that the amount of water in the pipe between the tank is negligible. Thus, the concentration in tank I and tank II are $x/100$ and $y/100$ respectively (Figure 3).
Amount of salt in Tank-I and Tank-II calculated in pounds be represented by $x(t)$ and $y(t)$.
The differential equation describing the amount of salt in Tank-I.

For Tank I we have,
$\frac{dx}{dt}$ =(Rate of salt entering into tank-I)-(Rate of salt leaving from tank-I).

$$\frac{dx}{dt} = R_1 - R_2 \tag{3.5}$$

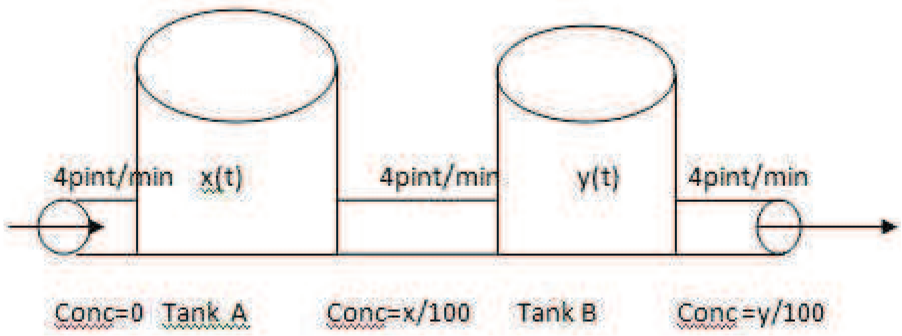

FIGURE 3. 100 Pint tanks

$$\frac{dx}{dt} = [(0lb/pint)(4pint/min)] - \left[\left(\frac{x(t)}{100}lb/pint\right)(4pint/min)\right] = \frac{-4}{100}x(t) = \frac{-x(t)}{25}.$$

From (3.5),

$$\frac{dx}{dt} + \frac{x(t)}{25} = 0. \tag{3.6}$$

Initially, the concentration of salt content in tank-I$=(0.25)(100) = 25$.
i.e. at $t = 0, x(0) = 25$
Taking Kamal transform on both sides, we have,

$$K[\frac{dx}{dt} + \frac{x}{25}] = K(0).$$

$$\Rightarrow \overline{x}(l) = \frac{25l}{1 + \frac{1}{25}l}. \tag{3.7}$$

Taking the inverse Kamal transform on equation (3.7), we get,

$$K^{-1}[\overline{x}(l)] = K^{-1}\left(\frac{25l}{1 + \frac{1}{25}l}\right)$$

$$\Rightarrow x(t) = 25e^{\frac{-1}{25}t}.$$

Similarly, we obtain for tank-II.
Taking inverse kamal transform on equation we have,

$$K^{-1}[\overline{y}(l)] = K^{-1}\left(\frac{l^3}{\left[1+\dfrac{l}{25}\right]^2}\right)$$

$$\Rightarrow y(t) = \frac{t^2}{2!}e^{\frac{-1}{25}t}.$$

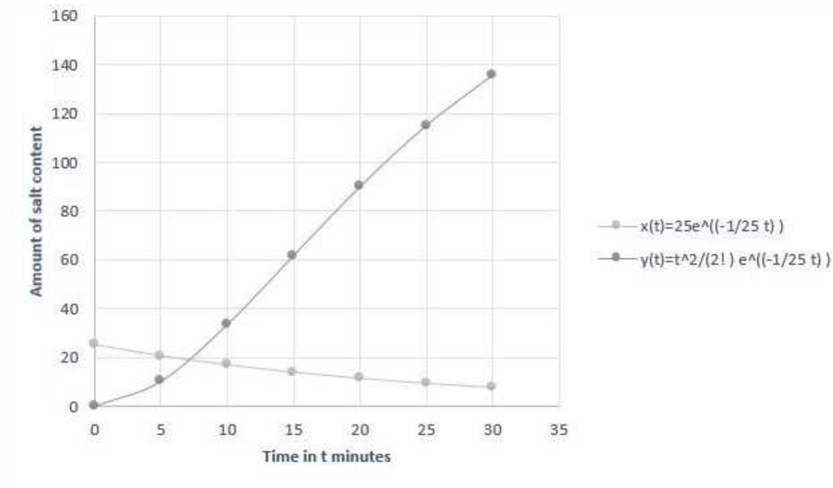

FIGURE 4. Amount of salts in two tanks

The schematic diagram of the given graph represents that the amount of salt in $x(t)$ decreases, but in $y(t)$ increases.

4. CONCLUSION:

In this paper, we have applied Kamal transforms to find the system of linear differential equations using initial conditions, which is very well used to calculate the amount of salt content in two tank mixing problems and also in greenhouse industries to control the agricultural pests to promote plant growth.

REFERENCES

[1] S. AGGARWAL: *Kamal Transform Of Bessel's Functions*, International Journal of Research and Innovation In Applied Science, **3**(7) (2018), 1–4.

[2] S. AGGARWAL, N. SHARMA, R. CHAUHAN: *Applications Of Kamal Transforms for Solving Linear Volterra Integral Equations of First Kind*, International Journal of Research in Advent Technology, **6**(8) (2018), 2081–2088.

[3] A. KASHURI, A. FUNDO: *A New Integral Transform*, Advances in Theoretical and applied Mathematics, **8**(1) (2003), 27–43.

[4] H. ELTAYEB, A. KILICMAN: *On Some Applications of a New Integral Transform*, Int. Journal of Math. Analysis, **4**(3) (2010), 123–132.

[5] W. S. WEIGELHOFER, K. A. LINDSAY: *Ordinary Differential Equations and Applications*, Mathematical Methods for Applied Mathematicians, Physicists, Engineers and Bioscientists, Woodhead Publishing, 1999.

DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE
TIRUPATTUR-635601 ADDRESS
Email address: `skalaiarasi2@gmail.com`

DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE
TIRUPATTUR-635601 ADDRESS
Email address: `keerthanas1722@gmail.com`

DEPARTMENT OF MATHEMATICS
D.K.M COLLEGE FOR WOMEN
VELLORE-635601 ADDRESS
Email address: `nityaselva41@gmail.com`

DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE
TIRUPATTUR-635601 ADDRESS
Email address: `mercelineanita@gmail.com`