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COMPARISON ANALYSIS BETWEEN ADOMAIN DECOMPOSITION METHOD AND RUNGE-KUTTA FOURTH-ORDER METHOD USING MATLAB

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ABSTRACT. In this paper, we present a comparative analysis between Runge Kutta Method (RKM) and Adomain Decomposition method (ADM). The study outlines the significant features of two approaches. We use these methods to solve the biological linear differential equation. In this article,the examples were computed by both the method and compared with the exact solution.Using Matlab it is noted that RKM is considered to be the best method that is approximately equal to the exact solution. The RKM is perceived to be particularly suitable for initial value problem.

1. INTRODUCTION

Differential Equation is an effective tool in scientific modelling.In many disciplines, differential equation plays an important role. In many fields such as physics, chemistry and biology, it has been used as a tool.Many methods have solved the differential equations. Here we will discuss two methods to solve the differential equation. The methods were Adomain Decomposition method and Runge-Kutta method of fourth-order. The Adomain Decomposition method (ADM) was first introduced by G. Adomain [1, 2]. This method can be used to any kind of differential equation. Evans and Raslan [11] used this method to study the initial value problem. A.M.Wazwaz [16] developed this method

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for nonlinearity and found a new technique to calculate adomain polynomials. Biazar et. al [6–8] have found the simplicity of the adomain polynomials to be efficient for the non-linear problems. The Adomain method can be applied to any kind of differential, integral, linear, non-linear, homogeneous and non-homogeneous equation [13]. The Runge-Kutta method is established on the taylor series by K.E. Aktinson [4]. This method is Popular because of its simplicity and efficiency. The fourth-order Runge-Kutta Method is an extremely popular method for solving initial value ordinary differential equation. Mathews et.al [12] have used a technique to get the accuracy of the solution. This method is easily computed and the accuracy has been effectively achieved [14]. The Runge-kutta fourth order is mainly used for linear equations.

2. ADOMAIN DECOMPOSITION METHOD

Consider the differential equation

$$Lv + Rv + Nv = g(x), \tag{2.1}$$

where N represents the non-linear operator [1, 2, 13]. Finding for Lv,

$$Lv = g(x) - R(v) - N(v). \tag{2.2}$$

Here L is invertible such that (2.2) becomes $L^{-1}\ Lv = L^{-1}\ g -\ L^{-1}\ Rv -\ L^{-1}\ Nv$. For Initial Value Problem L^{-1} can be written as $\frac{d^n}{dx^n}$ for n-fold definite integration from 0 to x. If L is a second order operator, L^{-1} can be a two fold integral so we get

$$v = a + bx + L^{-1}g(x) - L^{-1}Rv - L^{-1}Nv, \tag{2.3}$$

where a and b are constant of integration and it can be found to be initial or boundary conditions.

The non-linear term Nv can be equated to $\sum\limits_{n=0}^{\infty} A_n$ where A_n are adomain polynomials [6–8, 16] and it is given as $A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[f(\sum\limits_{k=0}^{\infty} v_k\lambda^k)\right]_{\lambda=0}$ $n = 0, 1, 2$. The term v will be decomposed into $\sum\limits_{n=0}^{\infty} v_n$ with v_0 identified as $a + bx + L^{-1}g$. Now (2.3) becomes

$$\sum_{n=0}^{\infty} v_n = a\ +\ bx\ +\ L^{-1}g\left(x\right)\ \ L^{-1}R\left(\sum_{n=0}^{\infty} v_n\right)\ \ L^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{2.4}$$

The equation (2.4) has an recursive relationship [3, 10] and is found to be

$$\left.\begin{aligned} v_0 &= g(x) \\ v_{n+1} &= -L^{-1}Ru_n - L^{-1}A_n \end{aligned}\right\}$$

From the above equation we can conclude the solution v as v = $n \xrightarrow{lt} \infty\, \psi_n(v)$
Where $\psi_n(v) = \sum_{i=0}^{n} v_i$.

3. Runge-Kutta Method

Runge-Kutta method (RKM) is one of the methods used to numerically solve the initial value problem. This method can be designed for different orders. The Fourth-order approach is the most widely used method [5, 9]. There are different types of fourth-order RK methods are available, the most popular is the classical fourth-order Runge-Kutta method. The formula for the classical Runge-Kutta method is given as

$$\begin{aligned} m_1 &= f(x_i, y_i), \\ m_2 &= f\left(x_i + \frac{h}{2}, y_i + \frac{m_1 h}{2}\right) \\ m_3 &= f\left(x_i + \frac{h}{2}, y_i + \frac{m_2 h}{2}\right), \\ m_4 &= f(x_i + h, y_i + m_3 h) \\ y_{i+1} &= y_i + \left(\frac{m_1 + 2m_2 + 2m_3 + m_4}{6}\right) h \end{aligned}$$

Example 1. *Consider the system* $\frac{dy}{dt} = y$, $y(0) = 1$ *with the [13] analytical solution is given as* $y(t) = e^t$ *The system* $\frac{dy}{dt} = y$, $y(0) = 1$ *can be solved by using the Adomain Decomposition and Runge Kutta method.*

Adomain Decomposition method:

The general procedure for finding the solution using ADM for the given system can be obtained as follows

$$\psi_0 = 1$$
$$\psi_1 = 1 + t$$
$$\psi_2 = 1 + t + \frac{t^2}{2}$$
$$\psi_3 = 1 + t + \frac{t^2}{2} + \frac{t^3}{6}$$
$$\psi_4 = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24}.$$

The solution can be nearly to the following as $y(t) = \psi_n(t) = \sum_{n=0}^{\infty} y_n(t)$. *For this problem we can have the solution with order four. Therefore we have*

$$y(t) = \psi_4(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24}.$$

For different values of t *we have the solution which is given in the below Table 1.*

Runge-Kutta method:

From the system $y' = y$, $y(0) = 1$ *we can find the following and can be used in the formula of Runge-Kutta method of order four. f(t,y) = y ,*$t_0 = 0$, $y_0 = 1$ *we are fixing the step size as* $h = 0.1$. *Now we substitute these values in (3.1-3.5) and for various values of i=0 to 9 we get the solution which is given in the Table 1.*

TABLE 1. Solution of the Example 1

t	*Exact*	*ADM*	*RK4*
0	*1*	*1*	*1*
0.1	*1.1052*	*1.1052*	*1.1052*
0.2	*1.2214*	*1.2214*	*1.2214*
0.3	*1.3499*	*1.3498*	*1.3499*
0.4	*1.4918*	*1.4917*	*1.4918*
0.5	*1.6487*	*1.6484*	*1.6487*
0.6	*1.8221*	*1.8214*	*1.8221*
0.7	*2.0138*	*2.0122*	*2.0138*
0.8	*2.2255*	*2.2255*	*2.2255*
0.9	*2.4596*	*2.4596*	*2.4596*

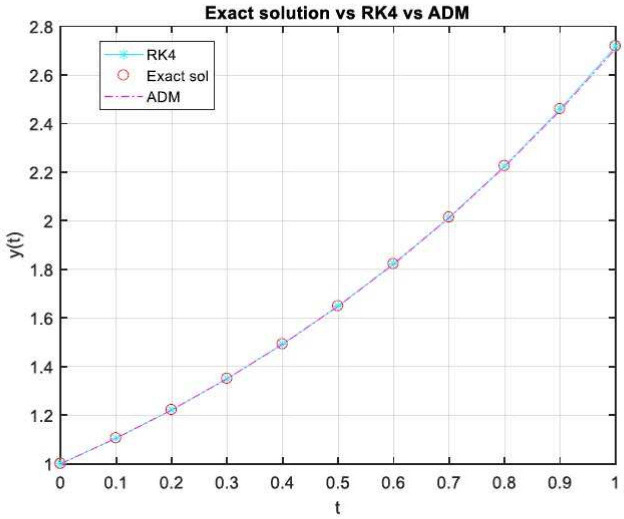

FIGURE 1. Exact Vs RK4 Vs ADM

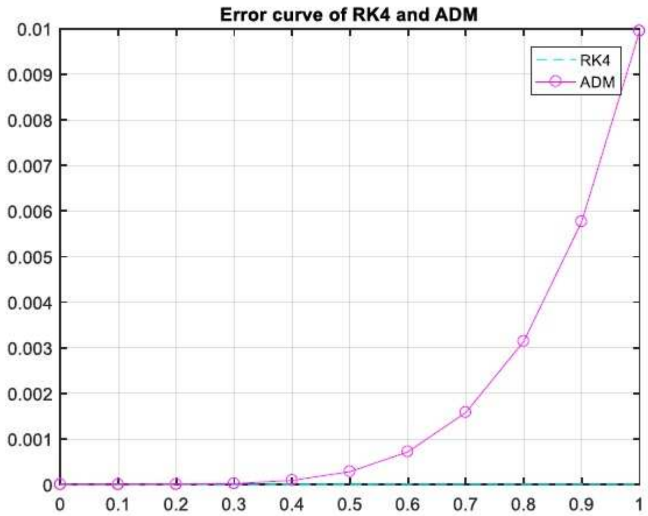

FIGURE 2. Error Curve of RK4 and ADM

Example 2. *Let us consider the [13]differential equation* $\frac{dN}{dt} = rN$*, N(0) = 4454 which is the initial population of the mid year and the rate of growth is given as 0.017. The modelled population generallyhas the exponential growth which is given as* $\frac{dN}{dt} = rN, N(0) = N_0$ *(1) Where N is the population at time t, r > 0 is a constant rate of growth,* N_0 *is population size at time t = 0. Th solution of (1) is*

$N = N_0 e^{rt}$.*The system* $\frac{dN}{dt} = rN$, $N(0) = 4454$ *can be solved by using the Adomain Decomposition and Runge Kutta method.*

Adomain Decomposition method:

The general procedure for finding the solution using ADM for the given system can be obtained as follows

$\psi_0 = 4454$

$\psi_1 = 4454\,[1 + tk]$

$\psi_2 = 4454\left[1 + tk + \frac{t^2k^2}{2}\right]$

$\psi_3 = 4454\left[1 + tk + \frac{t^2k^2}{2} + \frac{t^3k^3}{6}\right]$.

The solution can be nearly to the following as $y(t) = \psi_n(t) = \sum_{n=0}^{\infty} y_n(t)$. *For this problem we can have the solution with order three. Therefore we have* $y(t) = \psi_3(t) = 4454\left[1 + tk + \frac{t^2k^2}{2} + \frac{t^3k^3}{6}\right]$ *For different values of twe have the solution which is given in the below Table 2.*

Runge-kutta method:

From the system $\frac{dN}{dt} = 0.017N$, $N(0) = 4454$ *we can find the following and can be used in the formula of Runge-Kutta method of order four:* $f(t, N) = 0.017N$, $t_0 = 0$, $N_0 = 4454$ *we are fixing the step size as* $h = 0.5$. *Now we substitute these values in (3.1-3.5) and for various values of i=0 to 9 we get the solution which is given in the Table 2.*

TABLE 2. Solution of the example2

t	*Exact*	*ADM*	*RK4*
0	*4454*	*4454*	*4454*
0.5	*4492*	*4492*	*4492*
1	*4530*	*4530*	*4530*
1.5	*4569*	*4569*	*4569*
2	*4608*	*4608*	*4608*
2.5	*4647*	*4647*	*4647*
3	*4687*	*4687*	*4687*
3.5	*4727*	*4727*	*4727*
4	*4767*	*4767*	*4767*
4.5	*4808*	*4808*	*4808*

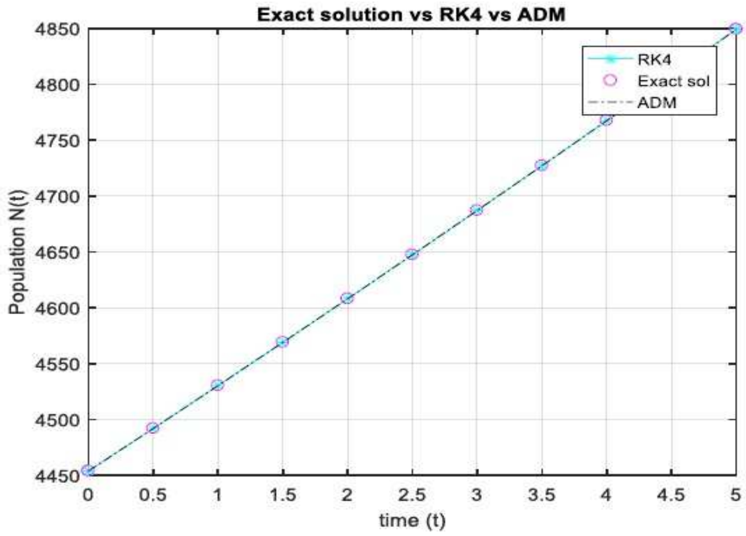

FIGURE 3. Exact Vs RK4 Vs ADM

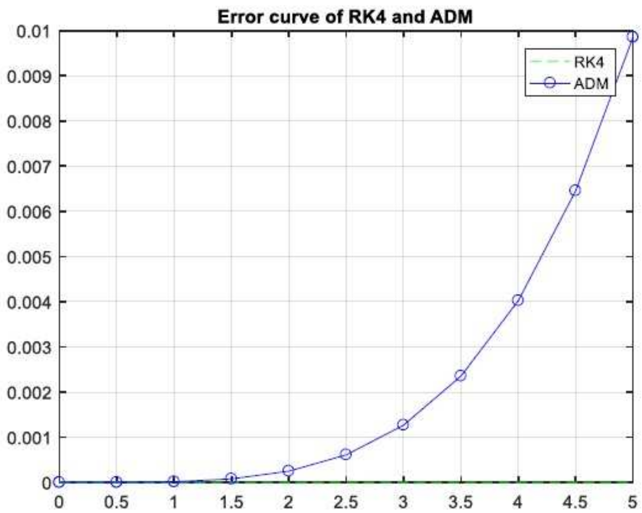

FIGURE 4. Error Curve of RK4 and ADM

4. CONCLUSION

The results of the above two examples show that the Runge-Kutta method is the best technique for solving the linear equation of the initial value problem that is frequently found in biology. Table 1 shows that there is a small error in

ADM compared to the exact, but there are no such errors in Table 2 due to the approximation point.Using Matlab the error can be seen in Figure 2 and Figure 4 is that there is no error between exact and RK4, but there is small error between exact and ADM. The method approximately equal to the analytical solution is the fourth-order Runge-Kutta method.

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