

ON FUZZY SOFT WEAK BAIRE SPACES

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ABSTRACT. The aim of this paper is to introduce and study the concept of fuzzy soft weak Baire spaces are introduced, and their characterizations and properties are investigated in this work. Several examples are given to illustrate the concepts introduced in this paper.

1. INTRODUCTION

The concept of fuzzy set where launched by Lotfi A. Zadeh in 1965 [12]. C.L. Chang [2] used fuzzy sets and defined by fuzzy topological space also C.L. Chang found the way for the Massive extension of the countless fuzzy topological concepts. In classical topology, the concept of σ -boundary sets was defined by J. Martinez and W.W. Mc Govern [7]. The concept of fuzzy soft set fss, for short is introduced and studied [3–6, 8] a more generalized concept which is a combination of fuzzy set and fss.

In this paper the idea of fuzzy soft σ -boundary set used fuzzy soft topological spaces.

2. PRELIMINARIES

Definition 2.1. [3] The fss $F_\phi \in FS(U, E)$ is said to be null fss and it is denoted by ϕ , if for all $e \in E$, $F(e)$ is the null fss $\bar{0}$ of U , where $\bar{0}(x) = 0$ for all $x \in U$.

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2010 Mathematics Subject Classification. 54A40, 03E72.

Key words and phrases. Fuzzy soft σ -boundary sets, fuzzy soft weak Baire spaces.

Definition 2.2. [3] Let $F_E \in FS(U, E)$ and $F_E(e) = \bar{1}$ all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fss. It is denoted by $\bar{E}$.

Definition 2.3. [3] A fss F_A is said to be a fuzzy soft subset of a fss G_B over a common universe U if $A \subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e \in A$, i.e., if $\mu_{F_A}^e(x) \leq \mu_{G_B}^e(x)$ for all $x \in U$ and for all $e \in E$ and denoted by $F_A \subseteq G_B$.

Definition 2.4. [3] Two fsss F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A .

Definition 2.5. [3] The union of two fsss F_A and G_B over the common universe U is the fss H_C , defined by $H_C(e) = \mu_{H_C}^e = \mu_{F_A}^e \cup \mu_{G_B}^e$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \check{\vee} G_B$.

Definition 2.6. [3] Let F_A and G_B be two fsss. Then the intersection of F_A and G_B is a fss H_C , defined by $H(e) = \mu_{H_C}^e = \mu_{F_A}^e \cap \mu_{G_B}^e$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \check{\wedge} G_B$.

Lemma 2.1. [1] For a family $A = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X , $\bigvee (cl(\lambda_\alpha)) \leq cl(\bigvee (\lambda_\alpha))$. In case A is a finite set, $\bigvee (cl(\lambda_\alpha)) = cl(\bigvee (\lambda_\alpha))$. Also $\bigvee (int(\lambda_\alpha)) \leq int(\bigvee (\lambda_\alpha))$.

Definition 2.7. [11] Let $F_A \in FS(U, E)$ be a fss. Then the complement of F_A , denoted by F_A^C , defined by

$$F_A^C(e) = \begin{cases} \bar{1} - \mu_{F_A}^e, & \text{if } e \in A \\ \bar{1}, & \text{if } e \notin A \end{cases}$$

Definition 2.8. [10] Let ψ be the collection of fsss over U . Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

- (i) $\Phi, \bar{E}$ belong to ψ .
- (ii) The union of any number of fsss in ψ belongs to ψ .
- (iii) The intersection of any two fsss ψ belongs to ψ .

The triplet (U, E, ψ) is called a fuzzy soft topological space over U . The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U .

Definition 2.9. [9] The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Theorem 2.1. [9] $\text{int}^{fs}(F_A \check{\wedge} G_B) = \text{int}^{fs}(F_A) \check{\wedge} \text{int}^{fs}(G_B)$.

Definition 2.10. [9] Let $F_A \in FS(U, E)$ be a fss. Then the intersection of all closed sets, each containing F_A , is called the closure of F_A and is denoted by $\text{cl}^{fs}(F_A)$.

Remark 2.1. [9]

- (1) For any fss F_A in a fuzzy soft topological space (U, E, ψ) , it is easy to see that $((F_A))^c = \text{int}^{fs}(F_A^C)$ and $(\text{int}^{fs}(F_A))^c = \text{cl}^{fs}(F_A^C)$.
- (2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ) , we define the fuzzy soft subspace topology. ψ on F_A by $K_D \in \psi_{F_A}$ if $K_D = F_A \check{\wedge} G_B$ for some $G_B \in \psi$.
- (3) For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $\text{int}_{F_A}^{fs}(H_C)$ and $\text{cl}_{F_A}^{fs}(H_C)$, respectively.

3. FUZZY SOFT σ -BOUNDARY SET

Definition 3.1. A fss F_A in a FSTS (U, E, ψ) is called a fs σ -boundary set if $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fuzzy soft regular open sets (fsro, for short) in (U, E, ψ) .

Example 1. Let $U = \{x_1, x_2, x_3\}$. The fuzzy sets F_E, G_E, H_E are defined on U as follows. Let

$$F_E = \begin{bmatrix} 0.6 & 0.3 & 0.3 \\ 0.4 & 0.4 & 0.3 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}, \quad G_E = \begin{bmatrix} 0.4 & 0.5 & 0.6 \\ 0.5 & 0.6 & 0.7 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}, \quad H_E = \begin{bmatrix} 0.4 & 0.6 & 0.5 \\ 0.5 & 0.5 & 0.6 \\ 0.7 & 0.7 & 0.8 \end{bmatrix}.$$

Then $\psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fst on (U, E, ψ) . The fs σ -boundary set in (U, E, ψ) are $G_{B_1} \vee G_{B_2} \vee G_{B_3} = 1 - F_{E_i}$ implies $1 - F_E$ is a fs σ -boundary set in (U, E, ψ) .

Proposition 3.1. If F_A is a fs σ -boundary set in a FSTS (U, E, ψ) , then F_A is a fs F_{σ} -set in (U, E, ψ) .

Proof. Let F_A be a fs σ -boundary set in a FSTS (U, E, ψ) . Then $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$ where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsro sets in (U, E, ψ) . Since (F_{A_i}) 's are fsro sets in (U, E, ψ) , $(1 - F_{A_i})$ is a fuzzy soft regular closed set (fsrcs, for short) in (U, E, ψ) . This implies that $(1 - F_{A_i})$ is a fsrcs in (U, E, ψ) and hence

$cl^{fs}(1 - F_{A_i}) = (1 - F_{A_i})$ in (U, E, ψ) . Then $G_{B_i} = cl^{fs}(F_{A_i}) \wedge cl^{fs}(1 - F_{A_i})$ is a fscs in (U, E, ψ) . Thus $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where (G_{B_i}) 's are fscs in (U, E, ψ) , implies that F_A is a fs F_{σ} -set in (U, E, ψ) . $\square$

Proposition 3.2. *If $G_B = cl^{fs}(F_A) \wedge (1 - F_A)$, where F_A is a fsro sets in a FSTS (U, E, ψ) , then G_B is a fuzzy soft pre-closed set (fspcs, for short) in (U, E, ψ) .*

Proof. Let F_A be a fsro sets in (U, E, ψ) , then $int^{fs}[cl^{fs}(F_A)] = F_A$ in (U, E, ψ) . Now, $G_B = cl^{fs}(F_A) \wedge (1 - F_A) = cl^{fs}(int^{fs}[cl^{fs}(F_A)]) \wedge (1 - int^{fs}[cl^{fs}(F_A)]) = cl^{fs}(int^{fs}[cl^{fs}(F_A)]) \wedge cl^{fs}[int^{fs}(1 - F_A)] \geq cl^{fs}[(int^{fs}[cl^{fs}(F_A)])] = cl^{fs}int^{fs}[cl^{fs}(F_A) \wedge (1 - F_A)] = cl^{fs}(G_B)$. Thus $cl^{fs}int^{fs}(G_B) \leq G_B$ in (U, E, ψ) implies that G_B is a fspcs in (U, E, ψ) . $\square$

Proposition 3.3. *If $G_B = cl^{fs}(F_A) \wedge (1 - F_A)$, where F_A is a fsro sets in (U, E, ψ) , then $int^{fs}(G_B) = F_A \wedge int^{fs}(1 - F_A)$ in (U, E, ψ) .*

Proof. Suppose that $G_B = Cl(F_A) \wedge (1 - F_A)$. Then $int^{fs}(G_B) = int^{fs}cl^{fs}(F_A) \wedge (1 - F_A) = int^{fs}cl^{fs}(F_A) \wedge int^{fs}(1 - F_A) = int^{fs}cl^{fs}(F_A) \wedge [1 - cl^{fs}(F_A)] = F_A \wedge [1 - cl^{fs}(F_A)]$, Since F_A is a fsro sets in (U, E, ψ) , $int^{fs}cl^{fs}(F_A) = F_A$. Then $int^{fs}(G_B) = F_A \wedge [1 - cl^{fs}(F_A)] = F_A \wedge [int^{fs}(1 - F_A)]$ in (U, E, ψ) . $\square$

Proposition 3.4. *If $G_B = cl(F_A) \wedge (1 - F_A)$, where F_A is a fsro sets in (U, E, ψ) , then $cl^{fs}(G_B) \leq fsbd(F_A)$ where $fsbd(F_A)$ is the fs boundary of F_A in (U, E, ψ) .*

Proof. Suppose that $G_B = CL(F_A) \wedge (1 - F_A)$, where F_A is a fsro sets in (U, E, ψ) . Then $cl^{fs}(G_B) = cl^{fs}[cl^{fs}(F_A) \wedge (1 - F_A)] \leq cl^{fs}cl^{fs}(F_A) \wedge (1 - F_A) = cl^{fs}(F_A) \wedge cl^{fs}(1 - F_A)$. This implies that $cl^{fs}(G_B) \leq [cl^{fs}(F_A) \wedge cl^{fs}(1 - F_A)] = fsbd(F_A)$. Thus $cl^{fs}(G_B) \leq fsbd(F_A)$ in (U, E, ψ) . $\square$

Proposition 3.5. *If $fsbd(F_A)$ is a fuzzy soft nowhere dense set (fsnds, for short), for a fsro set F_A in (U, E, ψ) , then $G_B = CL^{fs}(F_A) \wedge (1 - F_A)$ is a fsnds in (U, E, ψ) .*

Proof. Let F_A be a fsro set in (U, E, ψ) such that $int^{fs}cl^{fs}[fsbd(F_A)] = 0$. Suppose that $G_B = CL^{fs}(F_A) \wedge (1 - F_A)$ in (U, E, ψ) . Then by proposition 3.4, $cl(G_B) \leq fsbd(F_A)$, in (U, E, ψ) . Then $int^{fs}cl^{fs}[cl(G_B)] \leq int^{fs}cl^{fs}[fsbd(F_A)]$ in (U, E, ψ) and hence $int^{fs}cl^{fs}(G_B) \leq int^{fs}cl^{fs}[fsbd(F_A)]$ in (U, E, ψ) . Since $int^{fs}cl^{fs}[fsbd(F_A)] = 0$, $int^{fs}cl^{fs}(G_B) \leq 0$ in (U, E, ψ) . That is, $int^{fs}cl^{fs}(G_B) = 0$ in (U, E, ψ) and hence G_B is a fsnds in (U, E, ψ) . $\square$

Proposition 3.6. *If F_A is a fs σ -boundary set in a FSTS (U, E, ψ) , then $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where (G_{B_i}) 's are fspcs in (U, E, ψ) .*

Proof. Let F_A be a fs σ -boundary set in (U, E, ψ) . Then $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where $G_{B_i} = CL^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsro set in (U, E, ψ) . By proposition 3.2, (G_{B_i}) 's are fpcs in (U, E, ψ) , and hence $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where (G_{B_i}) 's are fspcs in (U, E, ψ) . $\square$

Definition 3.2. *A fss F_A in a Fsts (U, E, ψ) is called a fsp F_{σ} -set if $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where G_{B_i} 's are fspcs in (U, E, ψ) .*

Proposition 3.7. *If F_A is a fs σ -boundary set in a FSTS (U, E, ψ) , then F_A is a fsp F_{σ} -set in (U, E, ψ) .*

Proof. The proof follows from definition 3.2 and proposition 3.6. $\square$

Definition 3.3. *A fs set H_E in a FSTS in (U, E, ψ) is called a fuzzy soft co- σ -boundary set, if $H_E = \bigwedge_{i=1}^{\infty} (H_{E_i})$ where $H_{E_i} = int^{fs}(1 - F_{A_i}) \vee F_{A_i}$ and (F_{A_i}) 's are fsro sets in (U, E, ψ) .*

Proposition 3.8. *If H_E is a fs co- σ -boundary set in (U, E, ψ) , then $1 - H_E$ is a fs σ -boundary set in (U, E, ψ) .*

Proof. Let H_E be a fs co- σ -boundary set in (U, E, ψ) . Then $H_E = \bigwedge_{i=1}^{\infty} (H_{E_i})$, where $H_{E_i} = int^{fs}(1 - F_{A_i}) \vee F_{A_i}$ and (F_{A_i}) 's are fsro sets in (U, E, ψ) . Now $1 - H_E = 1 - \bigwedge_{i=1}^{\infty} (H_{E_i}) = \bigvee_{i=1}^{\infty} (1 - H_{E_i})$, in (U, E, ψ) . Also $1 - H_{E_i} = 1 - [int^{fs}(1 - F_{A_i}) \vee (F_{A_i})] = [1 - int^{fs}(1 - F_{A_i})] \wedge (1 - F_{A_i}) = \{1 - [1 - cl^{fs}(F_{A_i})]\} \wedge (1 - F_{A_i})$ and hence $1 - H_{E_i} = cl^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$. Let $G_{B_i} = 1 - H_{E_i}$, then $1 - H_E = \bigvee_{i=1}^{\infty} (G_{B_i})$ where $G_{B_i} = cl^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsros in (U, E, ψ) . Thus $1 - H_E$ is a fs σ -boundary set in (U, E, ψ) . $\square$

4. FUZZY SOFT WEAK BAIRE SPACES

Definition 4.1. *A FSTS (U, E, ψ) is called a fswbs if $int^{fs}[\bigvee_{i=1}^{\infty} (G_{B_i})] = 0$, where $G_{B_i} = cl^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsro set in (U, E, ψ) .*

Example 2. Let $U = \{x_1, x_2, x_3\}$. The fuzzy sets F_E, G_E, H_E , are defined on U as follows. Let

$$F_E = \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.5 & 0.3 & 0.7 \\ 0.5 & 0.3 & 0.4 \end{bmatrix} \quad G_E = \begin{bmatrix} 0.4 & 0.7 & 0.7 \\ 0.3 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.8 \end{bmatrix} \quad H_E = \begin{bmatrix} 0.6 & 0.5 & 0.7 \\ 0.6 & 0.4 & 0.6 \\ 0.5 & 0.3 & 0.4 \end{bmatrix}.$$

Then, $\Psi = \{0, F_E, G_E, H_E, 1\}$ is clearly fst on (U, E, Ψ) . Now consider the fs σ -boundary set in (U, E, ψ) are $(1 - G_E \vee H_E)1 - F_E$. Now we prove

$$\begin{aligned} \text{Int}\left(\bigvee_{i=1}^{\infty} H_{B_i}\right) &= G_{B_1} \vee G_{B_2} \vee G_{B_3} \\ &= \text{int}(1 - G_E \vee H_E) \\ &= 0. \end{aligned}$$

Hence (U, E, ψ) is a fswbs.

Proposition 4.1. Let (U, E, ψ) be a fsts. Then, the following are equivalent;

- (i) (U, E, ψ) is a fuzzy soft weak Baire space (fswbs, for short).
- (ii) $\text{int}^{fs}(F_A) = 0$, for each fs σ -boundary set F_A in (U, E, ψ) .
- (iii) $\text{cl}^{fs}(H_E) = 1$, for each fs co- σ -boundary set H_E in (U, E, ψ) .

Proof.

(i) $\Rightarrow$ (ii) Let F_A be a fuzzy σ -boundary set in (U, E, ψ) . Then $F_A = \bigvee_{i=1}^{\infty} (G_{B_i})$, where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsro set in (U, E, ψ) . Then $\text{int}^{fs}(F_A) = \text{int}^{fs}[\bigvee_{i=1}^{\infty} (G_{B_i})] = 0$, since (U, E, ψ) is a fswbs. Thus, for the fs - boundary set F_A in (U, E, ψ) , $\text{int}^{fs}(F_A) = 0$ in (U, E, ψ) .

(ii) $\Rightarrow$ (iii) Let H_E be a fs co- σ -boundary set in (U, E, ψ) , then by proposition 3.8, $1 - H_E$ is a fuzzy soft σ -boundary set in (U, E, ψ) . Then, by hypothesis, $\text{int}^{fs}(1 - H_E) = 0$ in (U, E, ψ) . Now $\text{cl}^{fs}(H_E) = 1 - [1 - \text{cl}^{fs}(H_E)] = 1 - [\text{int}^{fs}(1 - H_E)] = 1 - 0 = 1$. Thus $\text{cl}^{fs}(H_E) = 1$, for the fs co- σ -boundary set H_E in (U, E, ψ) .

(iii) $\Rightarrow$ (i) Let H_E be a fs co- σ -boundary set in (U, E, ψ) . Then $1 - H_E$ is a fs σ -boundary set in (U, E, ψ) and hence $1 - H_E = \bigvee_{i=1}^{\infty} (G_{B_i})$ where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fsro sets in (U, E, ψ) . Then $\text{int}^{fs}(1 - H_E) = \text{int}^{fs}[\bigvee_{i=1}^{\infty} (G_{B_i})]$. Now, by hypothesis $\text{cl}^{fs}(H_E) = 1$ in (U, E, ψ) . Since $\text{int}^{fs}(1 - H_E) = 1 - \text{cl}^{fs}(H_E) = 1 - 1 = 0$, $\text{int}^{fs}[\bigvee_{i=1}^{\infty} (G_{B_i})] = 0$, where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 -$

F_{A_i}) and (F_{A_i}) 's are fsro sets in (U, E, ψ) . This implies that (U, E, ψ) is a fswbs. $\square$

Proposition 4.2. *If (U, E, ψ) is a fswbs, then for any fsro set F_A in (U, E, ψ) , $[F_A \wedge \text{int}^{fs}(1 - F_A)] = 0$, in (U, E, ψ) .*

Proof. Let (U, E, ψ) be a fswbs. Then $\text{int}^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})] = 0$, where $G_{B_i} = \text{cl}^{fs}(F_{A_i}) \wedge (1 - F_{A_i})$ and (F_{A_i}) 's are fuzzy soft regular open sets in (U, E, ψ) . By lemma 2.1, $\bigvee_{i=1}^{\infty} \text{int}^{fs}(G_{B_i}) \leq \text{int}^{fs}[\bigvee_{i=1}^{\infty}(G_{B_i})]$ in (U, E, ψ) . Then $\bigvee_{i=1}^{\infty} \text{int}^{fs}(G_{B_i}) \leq 0$ in (U, E, ψ) . That is., $\bigvee_{i=1}^{\infty} \text{int}^{fs}(G_{B_i}) = 0$ in (U, E, ψ) . This implies that $\text{int}^{fs}(G_{B_i}) = 0$ in (U, E, ψ) . Then from proposition 3.3, $[F_{A_i} \wedge \text{int}^{fs}(1 - F_{A_i})] = 0$, in (U, E, ψ) . Thus, if F_A is a fsro set in (U, E, ψ) , then $F_A \wedge \text{int}^{fs}(1 - F_A) = 0$ in (U, E, ψ) . $\square$

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