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A REVIEW ON FUZZY MATHEMATICAL MODELING IN BIOLOGY

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ABSTRACT. Mathematical modeling has played a significant role in recent years in many applications. The Fuzzy model was used to perform specific fields like Ecology, Genetics, Engineers, Psychology, Sociology, Physics and Computer Sciences. The Fuzzy mathematical model enables us to use mathematical tools to examine the system both qualitatively and quantitatively. A fuzzy mathematical model helps explain a method used to study the effects of various elements and make behavioral predictions. The review article deals with various models such as Lotka – Volterra Model and Delay Model. The implementation of the models in different fields is discussed.

1. INTRODUCTION

A mathematical model is a kind of device that utilizes mathematical ideas and language. Mathematical modeling is an experimental proposal to solve a problem and to be more effective, quicker, or accurate over time. Mathematical modeling deals with a mathematical logic or a branch of knowledge which helps us in shaping the real life problems into mathematical models and then it can be solved and gives a solution. The method of drawing up a mathematical model is called mathematical biology, [14].

As early as the 12th century, mathematics was used in biology. Abstract biology aims to accomplish biological behavior at the abstract representation and

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modeling. Fuzzy was introduced by Zadeh in 1975. In a model, he developed a general method for extending mathematical ideas to deal with fuzzy quantities. This method is known as extension principle.

The implementation of the extension problem is difficult but it can be easily solved by using many methods such as appropriate method, perturbation method and iteration method. In this article we are discussed two models such as delay models and Lotka Volterra models. Many researchers have done their work using these two models. Lotka Volterra models and Delay models can be used by many researchers by using differential equations.

2. Factors of Mathematical Modeling

After developing a mathematical model, there are some parameters on which it should be judged for applicability. Few of important parameters are given here.

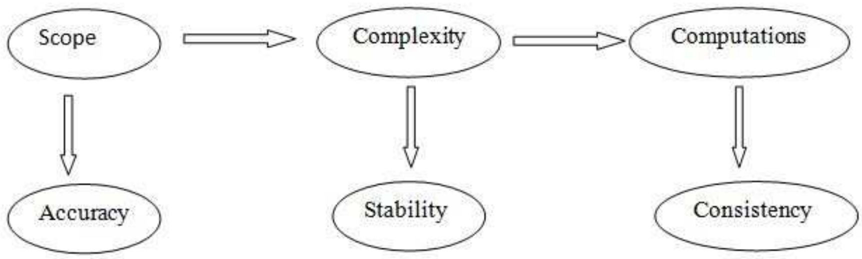

FIGURE 1

2.1. **Processing of modeling cycle.** In mathematics many real applications can be solved. From the below cycle we can analyze that, in the first step we set up a model then analyze the model then we interpret and defining a real world problem by using fuzzy differential equations.

2.2. **Fuzzy differential equations.** Fuzzy differential equations serve as an established way of modeling the distribution of uncertainty of information in a dynamic situation. A fuzzy differential equation has several definitions. First in history one was based on the Hukuhae derivative developed by Peuri Ralescu and researched in several papers including Song-lee, Kalera, Ding-Ma-Kandel, Rodriguez-Lopez, among others. This definition has the drawback that solutions

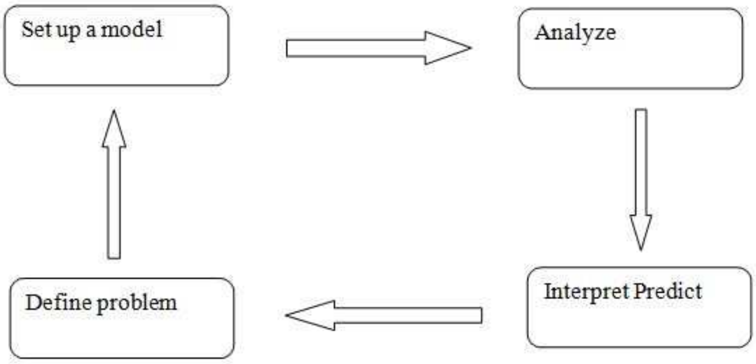

FIGURE 2

of a Fuzzy Differential Equation always have an increasing support length. The reality does not require periodic solutions or asymptotic processes to occur. This is why different ideas and approaches are used to solve troubling differential equations. One of them solves differential equations using the extension theory of Zadeh (Buckley ferring), while another approach interprets fuzzy differential equations by differential structure.

The author discusses the use of fuzzy differential equations in prey and predator population modeling in the paper [1]. A new model called the Fuzzy Predator - the prey model is being implemented. Using fuzzy Euler method this model is then solved numerically. It provides a suitable numerical illustration. Graphically, the outcome can be presented.

In the paper, [2] we will be considering a procedure for evaluating the amount of carbon dioxide in the blood. The model consists of a collection of nonlinear equations for the difference. The linearized model is resolved here. Because several tests and conditions estimating in detail the amount of carbon dioxide in the blood. For these imprecise measurements we must find the ambiguous analog of the linearized model as a method to reduce.

2.3. **Fuzzy delay differential equations.** Li and Liu proposed fuzzy differential equations as a form of differential equation driven by Liu method as well as stochastic differential equations driven by Brownian movement. Liu has proved the existence and uniqueity theorem for homogeneous fuzzy differential equation. Chen and Liu have proved the current outcome for the existence and uniqueness of solution for definite differential equation. But in the paper [9], the author suggested that Liu method solve the existence and uniqueness theorem for fuzzy delay differential equations.

Fuzzy delay differential equation has been involved in many biological process such as HIV dynamics, Predator – Prey Model, Fuzzy tumor model and Medical Cybernetics, [4, 9–11, 13]. Many authors will proposed a system of fuzzy delay differential equations by using symmetric triangular fuzzy number.

2.4. **Population growth model.** A model of population growth is a kind of mathematical model applicable to the study of population dynamics. Population model is used with differential equations to determine the maximum number of research in the field of biology. This helps us understand the complexities of biological invasions and of protection of the environment. Over time it reflects the number of individuals in a society, [6, 7, 12]. A growth of the population will depend on three phases that can be given below.

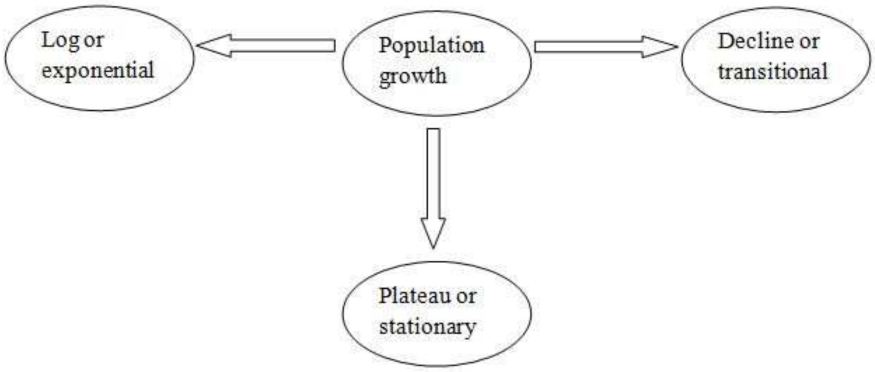

FIGURE 3

The first phase is the lag phase. It shows the unlimited population growth and increase in the rate of intrinsic and it has no predators. The decline phase shows the limiting factors of slow in the population growth. The stationary phase has no growth. Here the limiting factors can have the ability to increase the population capacity. It can be positive or negative.

Population growth are mainly affected by immigration, natality, emigration and mortality. The population growth model will depend on continuous growth model and discrete growth model.

The population size can be calculated by using the formula which is given below. Population size = (Natality + Immigration) − (Mortality + Emmigration)

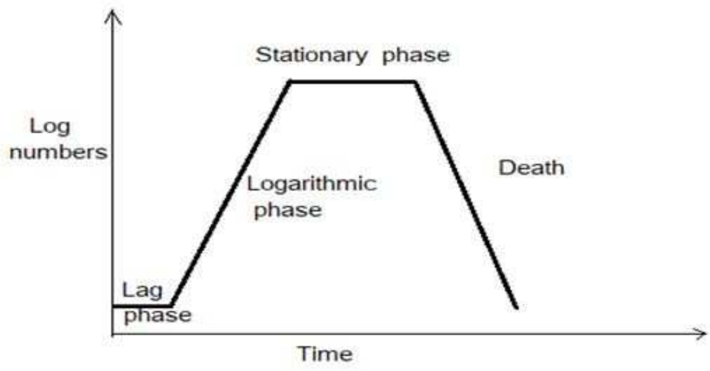

FIGURE 4

2.5. **Exponential growth model.** A Malthusian model of growth is also called as a simple model of exponential growth. It is centered on a constant rate and the curve is J−shaped. A mathematical model of population growth was proposed by Thomos R. Malthus in 1798. In this a population growth rate per capita remains the same irrespective of population size and it makes the population grow faster and faster. The type of an exponential model is

$$\frac{dN}{dt} = rN.$$

It has the solution $N(t) = N_0 e^{rt}$ where r > 0is the intrinsic rate of increase.

2.6. **Logistic growth model.** This model is also known as the Verhulst model. It is a development of the exponential model and was first developed in 1838 by Pierre Verhulst. He indicated that population growth depends not only on the size of the specific group, but on the magnitude of the scale from its upper limit as the carrying capacity taken. The logistical equation might be $\frac{dN}{dt} = rN(1-\frac{N}{k})$ with solution $N(t) = \frac{N_0 K e^{rt}}{K + N_0(e^{rt} - 1)}$. Here, r and k are for positive constants, and k is for carrying capacity. Here per capita birth rate is $r(1-\frac{N}{K})$. when $t \to \infty$ in the solution we get $N(t) \to K$.

2.7. **Lotka –Volterra Model.** The model of Lotka-Volterra is a set of pairs of differential equations representing Predator-Prey. The model is used for defining Predator's complex relationship with Prey. The first to research and apply

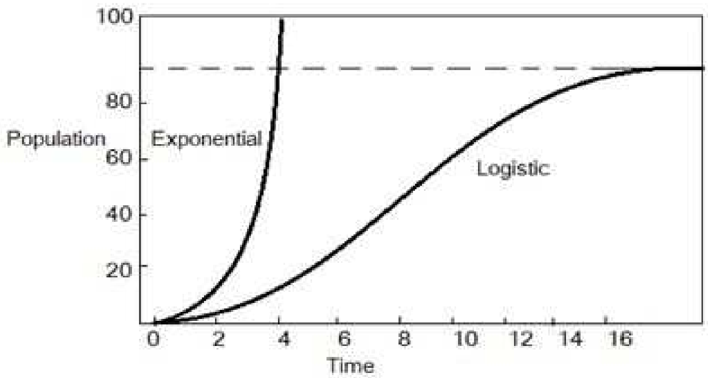

FIGURE 5

this pattern were Lotka in 1920, and Volterra in 1926. It is identified by oscillations in both Predator and Prey population size. Fuzzy differential equations are increasingly being applied in this model for modeling issues in science and engineering. Most of the problems in science and engineering have Fuzzy Differential Equations solutions. Lotka and Volterra's research overlapped in Predator's debate-Prey engagement. The predator prey equations of Lotka Volterra are both first-order and nonlinear differential equations. It's used to forecast two different retail formats, [5,8]. Over time, the population changes according to the pair of equations:

$$\frac{du}{dt} = \lambda u - \beta uv$$
$$\frac{dv}{dt} = \delta uv - \mu v$$

Here u is the number of preys (e.g., rabbits); v is the number of some predators (for instance, foxes). Over time, $\dot{u}$ and $\dot{v}$ represent the growth rates of both populations; T stands for time, and $\lambda, \beta, \delta, \mu$ are positive real parameters defining the relationship between the two organisms.
The model makes several simplifying assumption.

(i) If the predator were not present, the prey population would expand at a natural rate.
(ii) The predator population will decrease or expand at a natural rate in the absence of a prey.

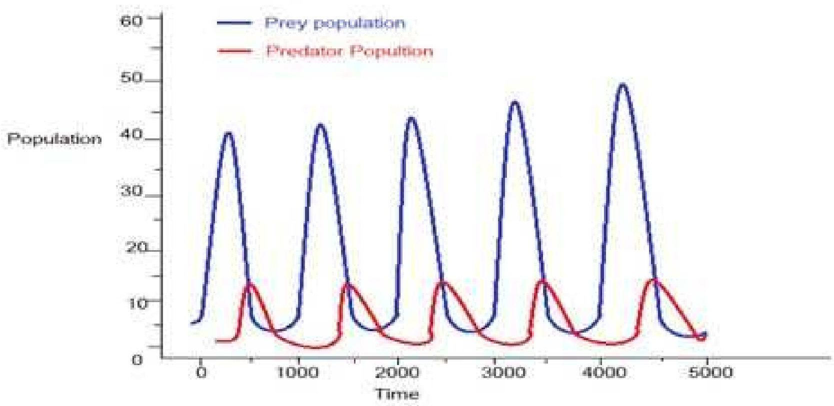

FIGURE 6

(iii) In accordance with these normal rates of growth and decline in the prey population and the rise in the predator population, both predators and prey are present at a rate proportional to the frequency of such interactions between individuals of the two populations.

2.8. **Delay model.**

(i) Delay models are increasingly common for many branches of biological modeling to appearIn mathematics, delay differential equations is a type of differential equation in which, at a given time, the derivative of the unknown function is given in terms of the values of the previous function. The delays or lags may reflect gestational times, maturation cycles, transportation delays or may simply block complicated living processes. The study of models of chemostat, circadin rhythms, epidemiology, the respiratory system, tumor growth, and neural networks is continued.

(ii) Delay differential equations include a time lag that makes the structures more biologically rational and accurate. Delay differential equations include the time delays in biological processes that actually occur. It also shows that Delay differential equations are more reliable in life sciences modeling.

(iii) The majority of the population model did not take into account maturity and gestation. But in reality these factors play an important part in the dynamics of the population.

(iv) Time delay is the time to reach maturity and the finite gestation period that is considered in the population dynamics and the corresponding differential equation is known as delay differential equation which is of the form,

$$\frac{dN}{dt} = f(N(t), N(t-T)),$$

where $T > 0$, the delay is a parameter.

(v) Delay differential equation can be solved without delay by using a system's initial value problem. It can be used to treat cancer and growth of tumours.

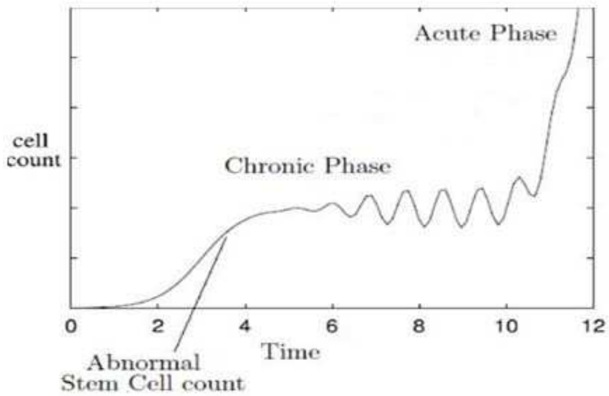

FIGURE 7

For instance, in [3], the authors prospect an impulsive stochastic infected predator-prey system with levy jumps and delays. The main purpose of this paper is to explore the effects of time delays and stochastic impetus inferences on predator dynamics-prey model. The authors suggested some of the properties and theorems that will be associated with certain conditions and investigated using differential equations of stochastic and delay. They verify effects of some simulation and describes the biological consequences.

3. CONCLUSION

Mathematical modeling has now been involved in many areas of research in particular in the field of biology. This lets us study the effects of different components, and make behavior predictions. Finally the problem of the real world can be simplified into a real model, then we calculate into a mathematical model and we analyze and interpret a model to a real situation in life. In this article we have discussed about the Lotka-Volterra and Delay models by using fuzzy differential equation. For instance fuzzy delay differential equation have involved in some biological fields such as HIV dynamics, Predator-Prey Interaction, Fuzzy tumor model and Medical cybernetics.

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