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SINGLE SERVER REPAIRABLE RETRIAL QUEUEING SYSTEM WITH MODIFIED BERNOULLI VACATION, OPTIONAL RE-SERVICE

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ABSTRACT. This paper deals with the steady state behavior of single server retrial queue with modified Bernoulli vacations, where each type of service is voluntary service subject to breakdowns and repairs. In the system if there are no customers at the end of each service, while the next customer arrives, the server waits with probability 1-a and probability a, the server takes vacation. We construct the mathematical model and derive the stationary probability distribution number of customer in the system by using the supplementary variable method. Some system performance measures and special cases are obtained.

1. INTRODUCTION

In the retrial theory, when the arriving customer finds the server is engaged and there is no availability of waiting space then the entire group join in the retrial group and it is defined an orbit, also repeats the service later on. It has been applied in communication and computer networks. To know the detailed study of the fundamental concepts of retrial queues, we refer the reader to read the books which are Falin and Templeton (1997) and Artalejo and Gomez-Corral (2008).

In the current study we have seen an interest in queueing system with modified Bernoulli vacation. In the retrial queueing system the server may not

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sure about the behavior of the customers in the system. For analysis of modified Bernoulli vacation, may refer to G. Choudhury and K.C. Madan (2005), Madhu Jain and Praveen Kumar Agarwal (2010), Pavai Madheswari and Suganthi (2016) and Pavai Madheswari, Krishnakumar, and Suganthi (2017).

2. MATHEMATICAL ASSUMPTIONS OF THE MODEL AND ERGODICITY CONDITION

Customers arrive according to a Poisson stream with rate λ. The server's retrial time follows general distribution with the distribution function $A(x)$ and density function $\theta(x)$ and Laplace-Stieljie's transform (LST) $A^*(x)$.

The arriving customer undergoes normal service provided by a single server on a first come first served basis. On completing normal service some of the arriving customer may opt re-service of the same service taken with probability r or leaves the system with probability $\bar{r} = 1 - r$.

The server's service time follows general distribution function with the distribution $S_1(t)$ for normal service and $S_2(t)$ for optional re-service and LST $S_1^*(t)S_2^*(t)$ We assume that the server takes modified Bernoulli vacation, The vacation time of the server follows a general distribution function $V(t)$ and LST $V^*(x)$.

The server is subject to breakdown. The life times are assumed to occur according to exogeneous Poisson stream with mean breakdown rates α_1, α_2 normal service and optional re-service. The server's repair time follows a general distribution function$G_i(t)$ and LST $G_i^*(y)$, i = 1, 2 respectively.

In addition let $A^0(t), S_1^0(t), S_2^0(t), V^0(t), G_1^0(t), G_2^0(t)$ be the elapsed retrial time, normal service time, optional re-service time, vacation time, repair on normal service time and repair on optional re-service time respectively. In the steady state, we assume that $A(0) = 0, A(\infty) = 1, S_1(0) = 0, S_1(\infty) = 1, S_2(0) = 0, S_2(\infty) = 1, V(0) = 0, V(\infty) = 1$ are continuous at $x = 0$ and $G_i(0) = 0, G_i(\infty) = 1$ are continuous at $y = 0$.

The state of system at time t can be described by the Markov process $\{C(t), X(t), t \geq 0\}$ where C(t) denotes the server state 0, 1, 2, 3, 4, 5 according to the server is idle, busy, re-service, repair on normal service, repair on optional re-service, vacation respectively.

The functions are $\theta(x)dx, \mu_1(x)dx, \mu_2(x)dx, v(x)dx, r_1(y)dy, r_2(y)dy$ the conditional probability of completion of repeated attempts, normal service, re-service, vacation, repair time x, i.e., $\theta(x)dx = \frac{dA(x)}{1-A(x)}, \mu_1(x)dx = \frac{dS_1(x)}{1-S_1(x)}, \mu_2(x)dx = \frac{dS_2(x)}{1-S_2(x)}$, $v(x) = \frac{dv(x)}{1-v(x)}, r_i(y)dy = \frac{dG_i(y)}{1-G_i(y)}, i = 1, 2$.

We analyze the ergodicity of the embedded Markov chain at departure completion epochs. Let $\{t_n/n \in N\}$ be the sequence of epochs at which either a service period completion time occurs or a vacation time ends or a repair period ends. The sequence of random vectors $Z_n = \{C(t_n+), X(t_n+)\}$ form a Markov chain, which is embedded Markov chain for the queueing system.

Theorem 2.1. *The embedded Markov chain $\{z_n/n \in N\}$ is ergodic if and only if $\rho < 1$ where $\rho = (1 - A^*(\lambda)) + (\lambda[E(S_1)(1 + \alpha_1 E(G_1)) + r(E(s_2)(1 + \alpha_1 E(G_2))) + aE(V)])$.*

3. STEADY STATE DISTRIBUTION OF THE SERVER STATE

For the process $\{X(t), t \geq 0\}$, the probabilities are defined as

$P_0(t) = P\{C(t) = 0, X(t) = 0\}$
$P_n(x,t)dx = P\{C(t) = 0, X(t) = n, x \leq A^0(t) < x + dx\}, n \geq 1$
$\pi_{(1,n)}(x,t)dx = P\{C(t) = 1, X(t) = n, x \leq S_1^0(t) < x + dx\}$ for $t, x, n \geq 0$
$\pi_{(2,n)}(x,t)dx = P\{C(t) = 2, X(t) = n, x \leq S_2^0(t) < x + dx\}$ for $t, x, n \geq 0$
$R_{(1,n)}(x,y,t)dy =$
$P\{C(t) = 3, X(t) = n, y \leq G_1^0(t) < y + dy/S_1^0(t) = x\}$ for $t, (x,y), n \geq 0$
$R_{(2,n)}(x,y,t)dy$
$= P\{C(t) = 4, X(t) = n, y \leq G_2^0(t) < y + dy/S_2^0(t) = x\}$ for $t, (x,y), n \geq 0$
$V_n(x,t)dx = P\{C(t) = 5, X(t) = n, x \leq V^0(t) < x + dx\}, n \geq 0$

We assume that the steady state condition is fulfilled. Hence we can set

$P_0 = \lim_{n\to\infty} P_0(t) \geq 0$, $P_n(x) = \lim_{n\to\infty} P_n(x,t)$ for $t \geq 0, x \geq 0, n \geq 1$,
$\pi_{(1,n)}(x) = \lim_{n\to\infty} \pi_{(1,n)}(x,t)$ for $t \geq 0, x \geq 0, n \geq 0$,
$\pi_{(2,n)}(x) = \lim_{n\to\infty} \pi_{(2,n)}(x,t)$ for $t \geq 0, x \geq 0, n \geq 0$,
$R_{(1,n)}(x,y) = \lim_{n\to\infty} R_{(1,n)}(x,y,t)$ for $t \geq 0$,
$R_{(2,n)}(x,y) = \lim_{n\to\infty} R_{(2,n)}(x,y,t)$ for $t \geq 0$,
$V_n(x) = \lim_{n\to\infty} V_n(x,t)$ for $t \geq 0$

We obtain the following steady state balance equations

$$\lambda P_0 = (1-a)\left[\bar{r}\int_0^\infty \pi_{(1,0)}(x)\mu_1(x)dx + \int_0^\infty \pi_{(2,0)}(x)\mu_2(x)dx\right] + \int_0^\infty V_0(x)v(x)dx \tag{3.1}$$

$$\frac{dP_n(x)}{dx} + (\lambda+\theta(x))P_n(x) = 0; n \geq 1 \tag{3.2}$$

$$\frac{d\pi_{1,0}(x)}{dx} + [\lambda+\alpha_1+\mu_1(x)]\pi_{1,0}(x) = \int_0^\infty r_1(y)R_{1,0}(x,y)dy, n=0 \tag{3.3}$$

$$\frac{d\pi_{1,n}(x)}{dx} + [\lambda+\alpha_1+\mu_1(x)]\pi_{1,n}(x) = \lambda\pi_{1,n-1}(x) + \int_0^\infty r_1(y)R_{1,n}(x,y)dy, n \geq 1 \tag{3.4}$$

$$\frac{d\pi_{2,0}(x)}{dx} + [\lambda+\alpha_2+\mu_2(x)]\pi_{2,0}(x) = \int_0^\infty r_2(y)R_{2,0}(x,y)dy, n=0 \tag{3.5}$$

$$\frac{d\pi_{2,n}(x)}{dx} + [\lambda+\alpha_2+\mu_2(x)]\pi_{2,n}(x) = \lambda\pi_{2,n-1}(x) + \int_0^\infty r_2(y)R_{2,n}(x,y)dy, n \geq 1 \tag{3.6}$$

$$\frac{dR_{1,0}(x,y)}{dy} + (\lambda+r_1(y))R_{1,0}(x,y) = 0, n=0 \tag{3.7}$$

$$\frac{dR_{1,n}(x,y)}{dy} + (\lambda+r_1(y))R_{1,n}(x,y) = \lambda R_{1,n-1}(x,y), n \geq 1 \tag{3.8}$$

$$\frac{dR_{2,0}(x,y)}{dy} + (\lambda+r_2(y))R_{2,0}(x,y) = 0, n=0 \tag{3.9}$$

$$\frac{dR_{2,n}(x,y)}{dy} + (\lambda+r_2(y))R_{2,n}(x,y) = \lambda R_{2,n-1}(x,y), n \geq 1 \tag{3.10}$$

$$\frac{dV_0(x)}{dx} + (\lambda+v(x))V_0(x) = 0, n=0 \tag{3.11}$$

$$\frac{dV_n(x)}{dx} + (\lambda+v(x))V_n(x) = \lambda V_{n-1}(x), n \geq 1 \tag{3.12}$$

The set of equations (3.1) to (3.12) are solved under the following boundary conditions at $x=0, y=0$:

$$P_n(0) = (1-a)[\bar{r}\int_0^\infty \pi_{(1,n)}(x)\mu_1(x)dx + \int_0^\infty \pi_{(2,n)}(x)\mu_2(x)dx] + \int_0^\infty V_n(x)v(x)dx, n \geq 1 \tag{3.13}$$

$$\pi_{1,n}(0) = \int_0^\infty P_1(x)\theta(x)dx + \lambda P_0 \tag{3.14}$$

$$\pi_{1,n}(0) = \int_0^\infty P_{n+1}(x)\theta(x)dx + \lambda\int_0^\infty P_n(x)dx, n \geq 1 \tag{3.15}$$

$$\pi_{2,n}(0) = r\int_0^\infty \pi_{1,n}(x)\mu_1(x)dx, n \geq 1 \tag{3.16}$$

$$R_{1,n}(x,0) = \alpha_1\pi_{1,n}(x), n \geq 0 \tag{3.17}$$

$$R_{2,n}(x,0) = \alpha_2\pi_{2,n}(x), n \geq 0 \tag{3.18}$$

$$V_n(0) = \bar{r}\int_0^\infty \pi_{1,0}(x)\mu_1(x)dx + \int_0^\infty \pi_{2,0}(x)\mu_2(x)dx, n = 0 \tag{3.19}$$

$$V_n(0) = a\bar{r}\int_0^\infty \pi_{1,n}(x)\mu_1(x)dx + a\int_0^\infty \pi_{2,n}(x)\mu_2(x)dx, n \geq 1 \tag{3.20}$$

The normalizing condition is given by:

$$P_0 + \sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx + \sum_{n=0}^{\infty}\Bigg[\int_0^\infty \pi_{1,n}(x)dx + \int_0^\infty \pi_{2,n}(x)dx + \int_0^\infty V_n(x)dx + \int_0^\infty\int_0^\infty R_{1,n}(x,y)dxdy + \int_0^\infty\int_0^\infty R_{2,n}(x,y)dxdy\Bigg] = 1 \tag{3.21}$$

Multiplying the equations (3.2) to (3.20) by suitable powers of z and summing over n, we get

$$\frac{dP(x,z)}{dx} + (\lambda + \theta(x))P(x,z) = 0 \tag{3.22}$$

$$\frac{d\pi_1(x,z)}{dx} + [\lambda(1-z) + \alpha_1 + \mu_1]\pi_1(x,z) = \int_0^\infty r_1(y)R_1(x,y,z)dy \tag{3.23}$$

$$\frac{d\pi_2(x,z)}{dx} + [\lambda(1-z) + \alpha_2 + \mu_2]\pi_2(x,z) = \int_0^\infty r_2(y)R_2(x,y,z)dy \tag{3.24}$$

$$\frac{dR_1(x,y,z)}{dy} + (\lambda(1-z) + r_1(y))R_1(x,y,z) = 0 \tag{3.25}$$

$$\frac{dR_2(x,y,z)}{dy} + (\lambda(1-z) + r_2(y))R_2(x,y,z) = 0 \tag{3.26}$$

$$\frac{dV(x,z)}{dx} + (\lambda(1-z) + v(x))V(x,z) = 0 \tag{3.27}$$

$$\begin{aligned} P(0,z) &= (1-a)\Big[\bar{r}\textstyle\int_0^\infty \pi_1(x,z)\mu_1(x)dx \\ &+ \displaystyle\int_0^\infty \pi_2(x,z)\mu_2(x)dx\Big] + \int_0^\infty V(x,z)v(x)dx - \lambda P_0 \end{aligned} \tag{3.28}$$

$$\pi_1(0,z) = \frac{1}{z}\int_0^\infty P(x,z)\theta(x)dx + \lambda\int_0^\infty P(x,z)dx + \lambda P_0 \tag{3.29}$$

$$\pi_2(0,z) = r\int_0^\infty \pi_1(x,z)\mu_1(x)dx \tag{3.30}$$

$$R_1(x,0,z) = \alpha_1\pi_1(x,z), n \geq 0 \tag{3.31}$$

$$R_2(x,0,z) = \alpha_2\pi_2(x,z), n \geq 0 \tag{3.32}$$

$$V(0,z) = a\bar{r}\int_0^\infty \pi_1(x,z)\mu_1(x)dx + a\int_0^\infty \pi_2(x,z)\mu_2(x)dx, n \geq 0 \tag{3.33}$$

Solving the above PDE (3.22) to (3.27), it follows that

$$P(x,z) = P(0,z)[1 - A(x)]e^{-\lambda x} \tag{3.34}$$

$$\pi_1(x,z) = \pi_1(0,z)[1 - S_1(x)]e^{-A_1(z)x} \tag{3.35}$$

$$\pi_2(x,z) = \pi_2(0,z)[1 - S_2(x)]e^{-A_2(z)x} \tag{3.36}$$

$$R_1(x,y,z) = R_1(x,0,z)[1 - G_1(y)]e^{-\lambda_0(z)y} \tag{3.37}$$

$$R_2(x,y,z) = R_2(x,0,z)[1 - G_2(y)]e^{-\lambda_0(z)y} \tag{3.38}$$

$$V(x,z) = V(0,z)[1 - V(x)]e^{-\lambda_0(z)x} \tag{3.39}$$

where $A_1(z) = \lambda_0(z) + \alpha_1[1 - G_1^*(\lambda_0(z))]$, $A_2(z) = \lambda_0(z) + \alpha_2[1 - G_2^*(\lambda_0(z))]$ and $\lambda_0(z) = \lambda(1 - z)$.

Substituting the equations (3.28) to (3.33) in (3.34) to (3.39), we get

$$P(x,z) = \frac{Nr(z)}{Dr(z)}(1 - A(x))e^{\lambda x} \tag{3.40}$$

$$Nr(z) = \lambda P_0 z\Big\{\big[arV^*(\lambda_0(z)) + (1-a)r)S_1^*(A_1(z))S_2^*(A_2(z))$$

$$+(a\bar{r}V^*(\lambda_0(z)) + (1-a)\bar{r})S_1^*(A_1(z))\big] - 1\Big\}$$

$$Dr(z) = \Big\{z - \Big[\{z + (1-z)A^*(\lambda)\}(arV^*(\lambda_0(z))$$

$$+(1-a)r)S_1^*(A_1(z))S_2^*(A_2(z)) + (a\bar{r}V^*(\lambda_0(z)) + (1-a)\bar{r})S_1^*(A_1(z))\Big]\Big\}$$

$$\pi_1(x,z) = \frac{\lambda P_0[(z-1)A^*(\lambda)](1 - S_1(x))e^{-A_1(z)x)}}{Dr(z)} \tag{3.41}$$

$$\pi_2(x,z) = \frac{r\lambda P_0[(z-1)A^*(\lambda)]S_1^*(A_1(z))(1 - S_2(x))e^{-A_2(z)x)}}{Dr(z)} \tag{3.42}$$

$$R_1(x,y,z) = \frac{\lambda P_0\alpha_1\{(z-1)A^*(\lambda)(1 - G_1(y))e^{-\lambda_0(z)y}(1 - S_1(x))e^{-A_1(z)x}\}}{Dr(z)} \tag{3.43}$$

$$R_2(x,y,z) = $$

$$\frac{\lambda P_0\alpha_2\{(z-1)A^*(\lambda)(1 - G_2(y))e^{-\lambda_0(z)y}(1 - S_2(x))e^{-A_2(z)x}S_1^*(A_1(z)\}}{Dr(z)} \tag{3.44}$$

$$V(x,z)$$

$$= \frac{\lambda P_0[((z-1)A^*(\lambda))(a\bar{r}S_1^*(A_1(z)) + aS_1^*(A_1(z))S_2^*(A_2(z))(1 - V(x))e^{-\lambda_0(z)x}]}{Dr(z)} \tag{3.45}$$

Integrating the above equations with respect to x from 0 to ∞, we get

$$P(z) = \frac{Nr(z)}{Dr(z)} \tag{3.46}$$

$$Nr(z) = zP_0(1 - A^*(\lambda))\Big[(arV^*(\lambda_0(z))$$

$$+(1-a)r)S_1^*(A_1(z))S_2^*(A_2(z)) + (a\bar{r}V^*(\lambda_0(z)) + (1-a)\bar{r})S_1^*(A_1(z))\Big] - 1$$

$$Dr(z) = \Big\{z - [\{(z + (1-z)A^*(\lambda))\}(arV^*(\lambda_0(z)))$$

$$+(1-a)r)S_1^*(A_1(z))S_2^*(A_2(z)) + (a\bar{r}V^*(\lambda_0(z)) + (1-a)\bar{r}))S_1^*(A_1(z))]\Big\}$$

$$\pi_1(z) = \frac{\lambda P_0\left[\frac{1-S_1^*(A_1(z))}{A_1(z)}\right][(z-1)A^*(\lambda)]}{Dr(z)} \tag{3.47}$$

$$\pi_2(z) = \frac{\lambda P_0\left[\frac{1-S_2^*(A_2(z))}{A_2(z)}\right]S_1^*(A_1(z))[r(z-1)A^*(\lambda)]}{Dr(z)} \tag{3.48}$$

$$R_1(z) = \frac{P_0\left[\frac{1-S_1^*(A_1(z))}{A_1(z)}\right][\alpha_1\lambda_0(z)A^*(\lambda)]}{Dr(z)} \tag{3.49}$$

$$R_2(z) = \frac{P_0\left[\frac{1-S_2^*(A_2(z))}{A_2(z)}\right][G_2^*(\lambda_0(z)-1)][S_1^*(A_1(z)][\alpha_2 r\lambda_0(z)A^*(\lambda)]}{Dr(z)} \tag{3.50}$$

$$V(z) = \frac{P_0[V^*(\lambda_0(z)-1][\lambda_0(z)A^*(\lambda)(a\bar{r}S_1^*(A_1(z)) + arS_1^*(A_1(z))S_2^*(A_2(z))]}{Dr(z)} \tag{3.51}$$

From the above equations, the only unknown is P_0 can be obtained by normalizing condition

$$P_0 + P(1) + \pi_1(1) + \pi_2(1) + Q_1(1) + Q_2(1) + R_1(1) + R_2(1) + V(1) = 1$$

$$P_0 = \frac{A^*(\lambda) - \lambda[E(S_1)[1+\alpha_1E(G_1)] + rE(S_2)[1+\alpha_2E(G_2)] + aE(V)]}{A^*(\lambda)} \tag{3.52}$$

We define the probability generating functions of the number of customer in the system is

$$K(z) = P_0 + P(z) + V(1) + z[\pi_1(z) + \pi_2(z) + Q_1(z) + Q_2(z) + R_1(z) + R_2(z)],$$

(3.53)

$$K(z) = \frac{P_0 A^*(\lambda) S_1^*(A_1(z))(z-1)(\bar{r} + rS_2^*(A_2(z)))}{z - (z + (1-z)A^*(\lambda))(S_1^*(A_1(z))(aV^*(\lambda_0(z)) + 1 - a)(\bar{r} + rS_2^*(A_2(z))))}.$$

We define the probability generating functions of the number of customers in the orbit is

$$H(z) = P_0 + P(z) + V(z) + \pi_1(z) + \pi_2(z) + Q_1(z) + Q_2(z) + R_1(z) + R_2(z),$$

(3.54)

$$H(z) = \frac{P_0 A^*(\lambda)\{1-z\}}{z - (z + (1-z)A^*(\lambda))(S_1^*(A_1(z))(aV^*(\lambda_0(z)) + 1 - a)(\bar{r} + rS_2^*(A_2(z)))}.$$

4. Performance measures

We derive the system performance measures of M/G/1 retrial queue with optional re-service and repair which is subject to modified Bernoulli vacation under a steady state. Differentiating (3.53) with respect to z and evaluating at z = 1, the mean number of customer in the system as

$$\begin{aligned} L_S = \lambda^2 \Big[& E(s_1^2) + (1 + \alpha_1 E(G_1))^2 + rE(S_2^2)(1 + \alpha_2 E(G_2))^2 \\ & + aE(V^2) + 2rE(S_1)(1 + \alpha_1 E(G_1))E(S_2)(1 + \alpha_2 E(G_2)) \\ & + 2aE(S_1)(1 + \alpha_1 E(G_1))E(V) + 2arE(S_2)(1 + \alpha_2 E(G_2)) \Big] \\ & + 2\lambda\{[E(s_1)(1 + \alpha_1 E(G_1)) + rE(S_2)(1 + \alpha_2 E(G_2)) + aE(V)] \\ & \cdot[\lambda E(S_1)(1 + \alpha_1 E(G_1)) + rE(S_2)(1 + \alpha_2 E(G_2))] + A^*(\lambda)\}\big)/2A^*(\lambda) \\ & - \{rE(S_2)(1 + \alpha_2 E(G_2)) + aE(V)\}\ \lambda(E(S_1)(1 + \alpha_1 E(G_1)) \\ & + rE(S_2)(1 + \alpha_2 E(G_2)) + aE(V)) \end{aligned}$$

Differentiating (3.54) with respect to z and evaluating at $z = 1$, the mean number of customer in the orbit as

$$\begin{aligned} L_q = (1 - A^*(\lambda))\{[& E(S_1)(1 + \alpha_1 E(G_1)) + rE(S_2)(1 + \alpha_2 E(G_2)) \\ & + aE(V)]\} - \lambda^2 \Big[E(S_1^2)(1 + \alpha_1 E(G_1))^2 + rE(S_2^2)(1 + \alpha_2 E(G_2))^2 \\ & + aE(V^2) + 2rE(S_1)(1 + \alpha_1 E(G_1))E(S_2)(1 + \alpha_2 E(G_2)) \end{aligned}$$

$$+2aE(S_1)(1+\alpha_1 E(G_1))E(V)+2arE(S_2)(1+\alpha_2 E(G_2))E(V)-1\Big]$$
$$\Big/2\Big\{\lambda(E(S_1)(1+\alpha_1 E(G_1))+rE(S_2)(1+\alpha_2 E(G_2))+aE(V))-A^*(\lambda)\Big\}$$

Let U be the steady state probability that the server is busy on normal service period and optional re-service period, I be the steady state probability that the server is idle during the retrial time.

$$\begin{aligned} U &= \pi_1(z)+\pi_2(z) \\ &= \frac{P_0 A^*(\lambda)\lambda[E(S_1)[1+\alpha_1 E(G_1)]+rE(S_2)[1+\alpha_2 E(G_2)]]}{A^*(\lambda)-\lambda[E(S_1)[1+\alpha_1 E(G_1)]+rE(S_2)[1+\alpha_2 E(G_2)]+aE(V)]} \end{aligned}$$

4.1. **SPECIAL CASES. Case (i):** Let us consider No breakdown, No optional re-service and No retrial then this model takes the form

$$K(z)=\frac{P_0 S_1^*(\lambda(1-z))(z-1)}{(z-(S_1^*(\lambda(1-z))(aV^*(\lambda_0(z))+1-a)))},$$

where $P_0 = 1-\lambda[E(S_1)+aE(V)]$.

Case (ii): No vacation, No breakdown, No retrial this model can be reduced to the following form.

Let a = 1,$\alpha_1=\alpha_2=0, A^*(\lambda)\to 1$

$$H(z)=\frac{P_0 S_1^*(\lambda(1-z))(z-1)(\bar{r}+rS_2^*(\lambda(1-z)))}{(z-(S_1^*(\lambda(1-z)))(\bar{r}+rS_2^*(\lambda(1-z)))V^*(\lambda_0(z))},$$

where $P_0 = 1-\lambda[E(S_1)+rE(S_2)+E(V)]$.

5. CONCLUSION

We have introduced a single server retrial queue with regular and extra service. The retrial time, service time, repair time and vacation time each have a general distribution. Further, some performance measures, such as the number of jobs in the system, orbit size, server utilization and probability that the orbit is empty are obtained. The result of this work finds application in the telephone systems, call centre, and in the area of computer processing systems. The proposed model can be extended, the concepts of working vacation policies, randomized policy, setup time, catastrophes in future.

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