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CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTIONS DEFINED BY SĂLĂGEAN OPERATOR WITH THE (p, q)-LUCAS POLYNOMIALS

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ABSTRACT. Making use of Sălăgean differential operator, in this paper, we introduce two new subclasses of the function class Σ of bi-univalent functions which are associated with the (p, q)-Lucas polynomials defined in the open unit disc. Furthermore, we find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses. Further, we obtain Fekete-Szegö inequalities for defined class and its specials cases. Also consequences of the results are pointed out.

1. INTRODUCTION AND DEFINITIONS

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C}$ be the set of complex numbers and $\mathbb{N} := \{1, 2, 3, \ldots\} = \mathbb{N}_0 \setminus \{0\}$ be the set of positive integers. Let also $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $\Delta = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in Δ.

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It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \Delta)$$

and $f(f^{-1}(w)) = w(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4})$, where $g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \ldots$.

A function $f \in \mathcal{A}$ is said to be bi-univalent in Δ if both a function f and it's inverse f^{-1} are univalent in Δ. Let Σ denote the class of bi-univalent functions in Δ given by (1.1).

In 2010, Srivastava *et al.* [20] revived the study of bi-univalent functions by their pioneering work on the study of coefficient problems. Various subclasses of the bi-univalent function class Σ were introduced and non-sharp estimates on the first two coefficients $|a_2|$ and $|a_3|$ in the Taylor-Maclaurin series expansion (1.1) were found in the recent investigations (see, for example, [1–9, 13–17, 19]) and including the references therein. The afore-cited all these papers on the subject were actually motivated by the work of Srivastava *et al.* [20]. However, the problem to find the coefficient bounds on $|a_n|$ $(n = 3, 4, \ldots)$ for functions $f \in \Sigma$ is still an open problem.

For analytic functions f and g in $\Delta,$ f is said to be subordinate to g if there exists an analytic function w such that $w(0) = 0$, $|w(z)| < 1$ and $f(z) = g(w(z))$, $z \in \Delta$. This subordination will be denoted here by $f \prec g$, $z \in \Delta$, or, conventionally, by $f(z) \prec g(z)$, $z \in \Delta$.

In particular, when g is univalent in Δ,

$$f \prec g \qquad (z \in \Delta) \;\Leftrightarrow\; f(0) = g(0) \quad \text{and} \quad f(\Delta) \subset g(\Delta).$$

Let $p(x)$ and $q(x)$ be polynomials with real coefficients. The (p, q)-polynomials $L_{p,q,n}(x),$ or briefly $L_n(x)$ are given by the following recurrence relation (see [11, 12]):

$$L_n(x) = p(x)L_{n-1}(x) + q(x)L_{n-2}(x) \qquad (n \in \mathbb{N} \setminus \{1\}),$$

with

$$L_0(x) = 2, \quad L_1(x) = p(x),$$

$$L_2(x) = p^2(x) + 2q(x), \quad L_3(x) = p^3(x) + 3p(x)q(x), \ldots$$

The generating function of the Lucas polynomials $L_n(x)$ is given by:

$$\mathcal{G}_{L_n(x)}(z) := \sum_{n=0}^{\infty} L_n(x)z^n = \frac{2 - p(x)z}{1 - p(x)z - q(x)z^2}\,. \tag{1.2}$$

Note that for particular values of p and q, the $(p,q)-$polynomial $L_n(x)$ leads to various polynomials, among those, we list few cases here (see, for more details, also [5]):

(i) For $p(x)=x$ and $q(x)=1$, we obtain the Lucas polynomials $L_n(x)$.
(ii) For $p(x)=2x$ and $q(x)=1$, we attain the Pell-Lucas polynomials $Q_n(x)$.
(iii) For $p(x)=1$ and $q(x)=2x$, we attain the Jacobsthal-Lucas polynomials $j_n(x)$.
(iv) For $p(x)=3x$ and $q(x)=-2$, we attain the Fermat-Lucas polynomials $f_n(x)$.
(v) For $p(x)=2x$ and $q(x)=-1$, we have the Chebyshev polynomials $T_n(x)$ of the first kind.

We want to remark explicitly that, in [5] Altınkaya and S. Yalçin, first introduced a subclass of bi-univalent functions by using the $(p,q)-$Lucas polynomials. This methodology builds a bridge between the Theory of Geometric Functions and that of Special Functions, which are known as different areas. Thus, we aim to introduce several new classes of bi-univalent functions defined through the $(p,q)-$Lucas polynomials. Furthermore, we derive coefficient estimates and Fekete-Szegö inequalities for functions defined in those classes.

Let $\mathcal{A}_\phi$ denoted the class of functions of the form

$$f_\phi(z)=z+\sum_{n=2}^{\infty}\frac{2}{1+e^{-s}}a_nz^n:=\sum_{n=2}^{\infty}\phi(s)a_nz^n, \tag{1.3}$$

where $\phi(s)=\dfrac{2}{1+e^{-s}}$ is the sigmoid activation function and $s\geq 0$. Also, $\mathcal{A}_1:=\mathcal{A}$ (see [10]).

We consider a differential operator $\mathcal{D}^k$, $k\in\mathbb{N}_0$, (see [18]) for f_ϕ belongs to $\mathcal{A}_\phi$, defined by

$$\mathcal{D}^0f_\phi(z)=f_\phi(z);\qquad \mathcal{D}^1f_\phi(z)=\mathcal{D}f_\phi(z)=zf'_\phi(z);\qquad \mathcal{D}^kf_\phi(z)=\mathcal{D}(\mathcal{D}^{k-1}f_\phi(z)).$$

We note that

$$\mathcal{D}^kf_\phi(z)=z+\sum_{n=2}^{\infty}\frac{2n^k}{1+e^{-s}}a_nz^n:=\sum_{n=2}^{\infty}n^k\phi(s)a_nz^n. \tag{1.4}$$

Definition 1.1. *A function $f\in\Sigma$ of the form* (1.1) *belongs to the class* $\mathcal{G}_\Sigma(\lambda,k,\ \phi(s);x)$, $\lambda\geq 1$ *and* $\phi(s)=\dfrac{2}{1+e^{-s}}$, $s\geq 0$, *if the following conditions are* *satisfied:*

$$\frac{z\left[\left(\mathcal{D}^k f_\phi(z)\right)'\right]^\lambda}{\mathcal{D}^k f_\phi(z)} \prec \mathcal{G}_{L_n(x)}(z) - 1, \quad (z \in \Delta) \tag{1.5}$$

and for $g_\phi(w) = f_\phi^{-1}(w)$*,*

$$\frac{w\left[\left(\mathcal{D}^k g_\phi'(w)\right)'\right]^\lambda}{\mathcal{D}^k g_\phi(w)} \prec \mathcal{G}_{L_n(x)}(w) - 1, \quad (w \in \Delta). \tag{1.6}$$

The geometric properties of the function class $\mathcal{G}_\Sigma(\lambda,\ k,\ \phi(s);x)$ vary according to the values assigned to the parameters involved. For example, $a = 2$; $p(x) = bx$, $q(x) = q$ with $\mathcal{G}_\Sigma(1,\ 0,\ \phi(0);x) \equiv \mathcal{S}^*_\Sigma(x)$. The class $\mathcal{S}^*_\Sigma(x)$ was introduced and studied by Srivastava et al. [19]. Also, $\mathcal{G}_\Sigma(\lambda,\ 0,\ \phi(0);x) := \mathcal{G}_\Sigma(\lambda\ ;x)$ was introduced and studied by authors [1].

In this investigation, we find the estimates for the coefficients $|a_2|$ and $|a_3|$ for functions in the subclass $\mathcal{G}_\Sigma(\lambda,\ \phi(s);x)$. Also, we obtain the Fekete-Szegö inequality.

2. Coefficient Estimates and Fekete-Szegö Inequality

In the following theorem, we obtain coefficient estimates for function f in the class $\mathcal{G}_\Sigma(\lambda,\ \phi(s);x)$.

Theorem 2.1. *Let* $f_\phi(z) = z + \sum_{n=2}^{\infty} \phi(s) a_n z^n$ *be in the class* $\mathcal{G}_\Sigma(\lambda,\ \phi(s);x)$*. Then*

$$|a_2| \leq \frac{|p(x)|\sqrt{|p(x)|}}{2^k\phi(s)\sqrt{|Q|}}, \qquad \textit{and} \qquad |a_3| \leq \frac{|p(x)|}{(3\lambda-1)3^k\phi(s)} + \frac{p^2(x)}{(2\lambda-1)^2 3^k\phi(s)},$$

and for $\mu \in \mathbb{R}$*,*

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} \dfrac{|p(x)|}{(3\lambda-1)3^k\phi(s)} & ; \left|1 - \mu\dfrac{3^k}{2^{2k}\phi(s)}\right| \leq \dfrac{|Q|}{p^2(x)(3\lambda-1)} \\ \dfrac{|p(x)|^3\left|1 - \mu\dfrac{3^k}{2^{2k}\phi(s)}\right|}{3^k\phi(s)\,|Q|} & ; \left|1 - \mu\dfrac{3^k}{2^{2k}\phi(s)}\right| \geq \dfrac{|Q|}{p^2(x)(3\lambda-1)} \end{cases},$$

where

$$Q = (2\lambda-1)(1-\lambda)\,p^2(x) - 2q(x)(2\lambda-1)^2.$$

Proof. Let $f \in \mathcal{G}_{\Sigma}(\lambda,\ \phi(s);x)$ be given by Taylor-Maclaurin expansion (1.1). Then, from the Definition 1.1, for some analytic functions Ψ and Φ such that

$$\Psi(0)=0; \quad \Phi(0)=0, \quad |\Psi(z)|<1 \quad \text{and} \quad |\Phi(z)|<1 \quad (\forall\, z,\ w\in\Delta),$$

we can write

$$\frac{z\left[\left(\mathcal{D}^k f_\phi(z)\right)'\right]^\lambda}{\mathcal{D}^k f_\phi(z)} = \mathcal{G}_{L_n(x)}(\Psi(z))-1 \tag{2.1}$$

and

$$\frac{w\left[\left(\mathcal{D}^k g_\phi(w)\right)'\right]^\lambda}{\mathcal{D}^k g_\phi(w)} = \mathcal{G}_{L_n(x)}(\Phi(z))-1. \tag{2.2}$$

Or, equivalently,

$$\frac{z\left[\left(\mathcal{D}^k f_\phi(z)\right)'\right]^\lambda}{\mathcal{D}^k f_\phi(z)} = -1+L_0(x)+L_1(x)u(z)+L_2(x)[u(z)]^2+\dots \tag{2.3}$$

and

$$\frac{w\left[\left(\mathcal{D}^k g_\phi(w)\right)'\right]^\lambda}{\mathcal{D}^k g_\phi(w)} = -1+L_0(x)+L_1(x)v(w)+L_2(x)[v(w)]^2+\dots. \tag{2.4}$$

From (2.3) and (2.4) and in view of (1.2), we obtain

$$\frac{z\left[\left(\mathcal{D}^k f_\phi(z)'\right)\right]^\lambda}{\mathcal{D}^k f_\phi(z)} = 1+L_1(x)u_1z+[L_1(x)u_2+L_2(x)u_1^2]z^2+\dots \tag{2.5}$$

and

$$\frac{w\left[\left(\mathcal{D}^k g_\phi(w)\right)'\right]^\lambda}{\mathcal{D}^k g_\phi(w)} = 1+L_1(x)v_1w+[L_1(x)v_2+L_2(x)v_1^2]w^2+\dots. \tag{2.6}$$

It is fairly well known that

$$|\Psi(z)| = \left|\psi_1 z+\psi_2 z^2+\dots\right|<1 \qquad \text{and} \qquad |\Phi(z)| = \left|\phi_1 w+\phi_2 w^2+\dots\right|<1,$$

and $|\psi_k|\le 1$ and $|\phi_k|\le 1$ $(k\in\mathbb{N})$.

Thus upon comparing the corresponding coefficients in (2.5) and (2.6), we have

$$(2\lambda-1)2^k\phi(s)a_2 = L_1(x)\psi_1 \tag{2.7}$$

$$(3\lambda-1)3^k\phi(s)a_3+(2\lambda^2-4\lambda+1)2^{2k}\phi^2(s)a_2^2 = L_1(x)\psi_2+L_2(x)\psi_1^2 \tag{2.8}$$

$$-(2\lambda-1)2^k\phi(s)a_2 = L_1(x)\phi_1 \tag{2.9}$$

and

$$(2\lambda^2+2\lambda-1)2^{2k}\phi^2(s)a_2^2-(3\lambda-1)3^k\phi(s)a_3 = L_1(x)\phi_2+L_2(x)\phi_1^2. \tag{2.10}$$

From (2.7) and (2.9), we can easily see that

$$\psi_1 = -\phi_1, \tag{2.11}$$

and

$$\begin{aligned} 2(2\lambda-1)^2 2^{2k}\phi^2(s)a_2^2 &= [L_1(x)]^2(\psi_1^2+\phi_1^2) \\ a_2^2 &= \frac{[L_1(x)]^2(\psi_1^2+\phi_1^2)}{2(2\lambda-1)^2 2^{2k}\phi^2(s)} . \end{aligned} \tag{2.12}$$

If we add (2.8) to (2.10), we get

$$2\lambda\,(2\lambda-1)\,2^{2k}\phi^2(s)a_2^2 = L_1(x)(\psi_2+\phi_2)+L_2(x)(\psi_1^2+\phi_1^2). \tag{2.13}$$

By substituting (2.12) in (2.13), we obtain

$$a_2^2 \;=\; \frac{[L_1(x)]^3\,(\psi_2+\phi_2)}{[2\lambda\,(2\lambda-1)\,[L_1(x)]^2-2L_2(x)(2\lambda-1)^2]2^{2k}\phi^2(s)}, \tag{2.14}$$

which yields

$$|a_2| \;\le\; \frac{|p(x)|\,\sqrt{|p(x)|}}{2^k\phi(s)\sqrt{|(1-\lambda)\,(2\lambda-1)p^2(x)-2q(x)(2\lambda-1)^2|}}. \tag{2.15}$$

By subtracting (2.10) from (2.8) and in view of (2.11) , we obtain

$$\begin{aligned} 2(3\lambda-1)3^k\phi(s)a_3-2(3\lambda-1)2^{2k}\phi^2(s)a_2^2 &= L_1(x)\,(\psi_2-\phi_2)+L_2(x)\left(\psi_1^2-\phi_1^2\right) \\ a_3 &= \frac{L_1(x)\,(\psi_2-\phi_2)}{2(3\lambda-1)3^k\phi(s)}+\frac{2^{2k}\phi(s)a_2^2}{3^k}. \end{aligned} \tag{2.16}$$

Then, in view of (2.12), (2.16) becomes

$$a_3 \;=\; \frac{L_1(x)\,(\psi_2-\phi_2)}{2(3\lambda-1)3^k\phi(s)}+\frac{[L_1(x)]^2(\psi_1^2+\phi_1^2)}{2(2\lambda-1)^2 3^k\phi(s)}.$$

Applying (1.2), we deduce that

$$|a_3| \;\le\; \frac{|p(x)|}{(3\lambda-1)3^k\phi(s)}+\frac{p^2(x)}{(2\lambda-1)^2 3^k\phi(s)}.$$

From (2.16), for $\mu\in\mathbb{R}$, we write

$$a_3-\mu a_2^2 = \frac{L_1(x)\,(\psi_2-\phi_2)}{2(3\lambda-1)3^k\phi(s)}+\left(\frac{2^{2k}\phi(s)}{3^k}-\mu\right)a_2^2. \tag{2.17}$$

By substituting (2.14) in (2.17), we have

$$\begin{aligned} a_3-\mu a_2^2 &= \frac{L_1(x)\,(\psi_2-\phi_2)}{2(3\lambda-1)3^k\phi(s)} \\ &+\left(\frac{2^{2k}\phi(s)}{3^k}-\mu\right)\left(\frac{[L_1(x)]^3\,(\psi_2+\phi_2)}{[2\lambda\,(2\lambda-1)\,[L_1(x)]^2-2L_2(x)(2\lambda-1)^2]2^{2k}\phi^2(s)}\right) \\ &= L_1(x)\left\{\left(\Omega(\mu,\ x)+\frac{1}{2(3\lambda-1)3^k\phi(s)}\right)\psi_2\right. \\ &\left.+\left(\Omega(\mu,\ x)-\frac{1}{2(3\lambda-1)3^k\phi(s)}\right)\phi_2\right\}, \end{aligned} \tag{2.18}$$

where

$$\Omega(\mu,\ x)=\frac{\left(\dfrac{2^{2k}\phi(s)}{3^k}-\mu\right)[L_1(x)]^2}{[2\lambda\,(2\lambda-1)\,[L_1(x)]^2-2L_2(x)(2\lambda-1)^2]2^{2k}\phi^2(s)}.$$

Hence, in view of (1.2), we conclude that

$$\left|a_3-\mu a_2^2\right|\leq\begin{cases}\dfrac{|L_1(x)|}{(3\lambda-1)3^k\phi(s)} & ;0\leq|\Omega(\mu,\ x)|\leq\dfrac{1}{2(3\lambda-1)3^k\phi(s)}\\ \\ 2\,|L_1(x)|\,|\Omega(\mu,\ x)| & ;|\Omega(\mu,\ x)|\geq\dfrac{1}{2(3\lambda-1)3^k\phi(s)}\end{cases}$$

which evidently completes the proof of Theorem 2.1. □

Corollary 2.1. *Let* $f_\phi(z)=z+\sum_{n=2}^{\infty}\phi(s)a_nz^n$ *be in the class* $\mathcal{S}^*_\Sigma(\phi(s);\ x)$*. Then*

$$|a_2|\leq\frac{|p(x)|\sqrt{|p(x)|}}{2^k\phi(s)\sqrt{2\,|q(x)|}},\qquad \textit{and}\qquad |a_3|\leq\frac{|p(x)|}{2\phi(s)3^k}+\frac{p^2(x)}{3^k\phi(s)},$$

and for $\mu\in\mathbb{R}$

$$\left|a_3-\mu a_2^2\right|\leq\begin{cases}\dfrac{|p(x)|}{2\phi(s)3^k} & ;\left|1-\mu\dfrac{3^k}{2^{2k}\phi(s)}\right|\leq\dfrac{|q(x)|}{|p(x)|^2}\\ \dfrac{|p(x)|^3\left|1-\mu\dfrac{3^k}{2^{2k}\phi(s)}\right|}{2\phi(s)3^k\,|q(x)|} & ;\left|1-\mu\dfrac{3^k}{2^{2k}\phi(s)}\right|\geq\dfrac{|q(x)|}{|p(x)|^2}\end{cases}.$$

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