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ON PAIRWISE FUZZY GENERALIZED VOLTERRA SPACES

G. THANGARAJ, V. CHANDIRAN[1], AND P. ASHOKKUMAR

ABSTRACT. In this paper, the notion of generalized Volterra spaces in fuzzy bitopological spaces is introduced. Some examples of pairwise fuzzy generalized Volterra spaces are illustrated and several characterizations of pairwise fuzzy generalized Volterra spaces are also studied in this paper. The relations between pairwise fuzzy generalized Volterra spaces and other fuzzy bitopological spaces are also investigated in this paper.

1. INTRODUCTION

L. A. Zadeh in [15], introduced the concept of fuzzy sets in 1965. The fundamental objects are closed sets in topological spaces. The concept of generalized closed sets as a generalization of closed sets in topological spaces was first initiated by N.Levine in [10], in 1970. C. L. Chang in [3] initiated the nation of fuzzy topological spaces as a generalization of topological spaces in 1968. In 1997, G. Balasubramanian and P. Sundaram in [2], gave the notion of fuzzy generalized closed set in fuzzy topology. The meaning of bitopological spaces was initiated by J .C. Kelly in [9] in 1963. The notion of fuzzy bitopological spaces (FBTS, in short) was introduced by A. Kandil in [11] in 1989. The concepts of Volterra spaces have been studied extensively in classical topology in [4–8]. In this paper, the notion of generalized Volterra spaces in FBTS's is introduced. Several

[1]*corresponding author:*

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characterizations of pairwise fuzzy generalized Volterra spaces are also studied in this paper. The relations between pairwise fuzzy generalized Volterra spaces and other FBTS's are also investigated in this paper.

2. Preliminaries

In this section, some important definitions and theorems are given for the subsequent sections.

Definition 2.1. *[3], A fuzzy topology is a family 'T'of fuzzy sets in X which satisfies the following conditions:*

(1) $\Phi, X \in T$,
(2) *If* $A, B \in T$, *then* $A \cap B \in T$,
(3) *If* $A_i \in T$, *for each* $i \in I$, *then* $\cup_{i \in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space or fts, in short. Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open. When no confusion is likely to arise, we shall call a T-open (T-closed) fuzzy set simply an open (closed) fuzzy set.

Definition 2.2. *[3], Let λ and μ be fuzzy sets in X. Then for all $x \in X$,*

1. $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
2. $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
3. $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$,
4. $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$,
5. $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i \mid i \in I\}$ of fuzzy sets in X, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by $\psi(x) = sup_i\{\lambda_i(x) \mid x \in X\}$, and $\delta(x) = inf_i\{\lambda_i(x) \mid x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.3. *[3], Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). The closure and interior of a fuzzy set λ in a fuzzy topological space (X, T) are respectively denoted as $cl(\lambda)$ and $int(\lambda)$ are defined as*

(1) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$ *and*
(2) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$.

Lemma 2.1. *[1], For a fuzzy set λ of a fuzzy space X,*

(a) $1 - cl(\lambda) = int(1 - \lambda)$ *and*
(b) $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.4. *A fuzzy set λ in a FBTS (X, T_1, T_2) is called a*

(1) *pairwise fuzzy open set (pfo set, for short) if $\lambda \in T_i,\ (i = 1, 2)$, [13];*
(2) *pairwise fuzzy closed set (pfc set, for short) if $1 - \lambda \in T_i$, [13];*
(3) *pairwise fuzzy G_δ-set (pfG_δ-set, for short) if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfo sets, [13];*
(4) *pairwise fuzzy F_σ-set (pfF_σ-set, for short) if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfc sets, [13];*
(5) *pairwise fuzzy dense set (pfd set, for short) if $cl_{T_1}cl_{T_2}(\lambda) = 1$ and $cl_{T_2}cl_{T_1}(\lambda) = 1$, [12];*
(6) *pairwise fuzzy nowhere dense set (pfnd set, for short) if $int_{T_1}cl_{T_2}(\lambda) = 0$ and $int_{T_2}cl_{T_1}(\lambda) = 0$, [14];*
(7) *pairwise fuzzy first category set (pffc set, for short) if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfnd sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set and $1 - \lambda$ is a pairwise fuzzy residual set (pfr set, for short), [14];*
(8) *pairwise fuzzy σ-nowhere dense set (pfσ-nd set, for short) if λ is a pfF_σ-set in (X, T_1, T_2) such that $int_{T_1}int_{T_2}(\lambda) = int_{T_2}int_{T_1}(\lambda) = 0$, [13].*

Definition 2.5. *A FBTS (X, T_1, T_2) is said to be a*

(1) *pairwise fuzzy Volterra space (pfVs, for short) if $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, $(i = 1, 2)$, where (λ_k)'s are pfd and pfG_δ-sets in (X, T_1, T_2), [13];*
(2) *pairwise fuzzy σ-Baire space (pfσ-Bs, for short) if $int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 0,\ (i = 1, 2)$ where (λ_k)'s are pfσ-nd sets in (X, T_1, T_2), [13];*
(3) *pairwise fuzzy P-space (pf$P$-s, for short) if every non-zero pfG_δ-set in (X, T_1, T_2), is a pfo set in (X, T_1, T_2). That is, if (X, T_1, T_2) is a pf$P$-s if $\lambda \in T_i,\ (i = 1, 2)$ for $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfo sets in (X, T_1, T_2), [13];*
(4) *pairwise fuzzy hyperconnected space (pfhs, for short) if λ is a pfo set in (X, T_1, T_2), then $cl_{T_i}(\lambda) = 1,\ (i = 1, 2)$, [13];*
(5) *pairwise fuzzy Baire space (pfBs, for short) if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0,\ (i = 1, 2)$, where (λ_k)'s are pfnd sets in (X, T_1, T_2), [14].*

Theorem 2.1. *[13], If a fuzzy set λ_k is a T_i $(i = 1, 2)$-fuzzy dense set in a FBTS (X, T_1, T_2), then λ_k is a pfd set in (X, T_1, T_2).*

Theorem 2.2. *[13], In a FBTS (X, T_1, T_2), a fuzzy set λ is a pfσ-nd set in (X, T_1, T_2) if and only if $1 - \lambda$ is a pfd and pfG_δ-set in (X, T_1, T_2).*

Theorem 2.3. *[14], If λ is a pfnd set in a FBTS (X, T_1, T_2), then $1 - \lambda$ is a pfd set in (X, T_1, T_2).*

Theorem 2.4. *[13], If each pfnd set λ in a FBTS (X, T_1, T_2) is a pfF_σ-set, then (X, T_1, T_2) is a pfBs.*

Theorem 2.5. *[13], If the pfnd sets are pfF_σ-sets in a pfBs (X, T_1, T_2), then (X, T_1, T_2) is a pfσ-Bs.*

Theorem 2.6. *[13], If a pfo set λ in a FBTS (X, T_1, T_2) such that $cl_{T_i}(\lambda) = 1,\ (i = 1, 2)$, then $1 - \lambda$ is a pfnd set in (X, T_1, T_2).*

Theorem 2.7. *[13], If a pfG_δ-set λ in a FBTS (X, T_1, T_2) such that $cl_{T_i}(\lambda) = 1,\ (i = 1, 2)$, then $1 - \lambda$ is a pffc set in (X, T_1, T_2).*

3. Pairwise fuzzy generalized G_δ-sets

Definition 3.1. *A fuzzy set λ in a FBTS (X, T_1, T_2) is called a pairwise fuzzy generalized closed set (pfgc set, for short) if $cl_{T_i} cl_{T_j}(\lambda) \leq \mu,\ (i \neq j \text{ and } i, j = 1, 2)$ whenever $\lambda \leq \mu$ and $\mu \in T_i$.*

Proposition 3.1. *If λ and μ are pfgc sets in a FBTS (X, T_1, T_2), then $\lambda \vee \mu$ is also a pfgc set in (X, T_1, T_2).*

Proof. Let λ and μ be pfgc sets and let γ be any pfo set such that $\lambda,\ \mu \leq \gamma$. Then $cl_{T_i} cl_{T_j}(\lambda),\ cl_{T_i} cl_{T_j}(\mu) \leq \gamma,\ (i \neq j \text{ and } i, j = 1, 2)$. Suppose $\lambda \vee \mu \leq \gamma$. Then $cl_{T_i} cl_{T_j}(\lambda \vee \mu) = cl_{T_i} cl_{T_j}(\lambda) \vee cl_{T_i} cl_{T_j}(\mu) \leq \gamma$. This implies that $cl_{T_i} cl_{T_j}(\lambda \vee \mu) \leq \gamma$ and hence $\lambda \vee \mu$ is also a pfgc set.

But, the intersection of two pfgc sets is not a pfgc set in a FBTS (X, T_1, T_2). □

Proposition 3.2. *If λ is a pfgc set in a FBTS (X, T_1, T_2) such that $\lambda \leq \mu \leq cl_{T_i}(\lambda),\ (i = 1, 2)$, then μ is a pfgc set in (X, T_1, T_2).*

Proof. Let γ be a pfo set such that $\mu \leq \gamma$. Since $\lambda \leq \mu$, $\lambda \leq \gamma$ and since λ is a pfgc set, $cl_{T_i}cl_{T_j}(\lambda) \leq \gamma,\ (i \neq j \text{ and } i,j=1,2)$. Also, since $\mu \leq cl_{T_i}(\lambda)$, $cl_{T_1}cl_{T_2}(\mu) \leq cl_{T_1}cl_{T_2}[cl_{T_2}(\lambda)] = cl_{T_1}cl_{T_2}(\lambda)$ and $cl_{T_2}cl_{T_1}(\mu) \leq cl_{T_2}cl_{T_1}[cl_{T_1}(\lambda)] = cl_{T_2}cl_{T_1}(\lambda)$. This implies that $cl_{T_i}cl_{T_j}(\mu) \leq cl_{T_i}cl_{T_j}(\lambda)$. Hence $cl_{T_i}cl_{T_j}(\mu) \leq \gamma$. Therefore, μ is a pfgc set. □

Proposition 3.3. *If λ is a pfnd set in a FBTS (X,T_1,T_2), then λ is a pfgc set in (X,T_1,T_2).*

Proof. Let λ be a pfnd set. Then there does not exists any pfo set between λ and $cl_{T_i}cl_{T_j}(\lambda),\ (i \neq j \text{ and } i,j=1,2)$. Suppose that $\lambda \leq \mu$, for any $\mu \in T_i$ and hence $cl_{T_i}cl_{T_j}(\lambda) \leq \mu$. Therefore, λ is a pfgc set. □

Proposition 3.4. *If λ is a pfc set in a FBTS (X,T_1,T_2) with $int_{T_i}(\lambda)=0,\ (i=1,2)$, then λ is a pfgc set in (X,T_1,T_2).*

Proof. Let λ be a pfc set. Then $cl_{T_j}(\lambda) = \lambda$. Now $int_{T_i}cl_{T_j}(\lambda) = int_{T_i}(\lambda)\ (i \neq j \text{ and } i,j=1,2)$ and $int_{T_i}(\lambda)=0$, implies that $int_{T_i}cl_{T_j}(\lambda)=0$. Then λ is a pfnd set. Hence by proposition 3.3, λ is a pfgc set. □

Definition 3.2. *A fuzzy set λ in a FBTS (X,T_1,T_2) is called a pairwise fuzzy generalized open set (pfgo set, for short) if and only if $1-\lambda$ is a pfgo set in (X,T_1,T_2).*

Proposition 3.5. *A fuzzy set λ is a pfgo set in a FBTS (X,T_1,T_2) if and only if $\mu \leq int_{T_i}int_{T_j}(\lambda),\ (i \neq j \text{ and } i,j=1,2)$ whenever μ is a pfc set in (X,T_1,T_2) and $\mu \leq \lambda$.*

Proof. Let λ be a pfgo set and let μ be a pfc set such that $\mu \leq \lambda$. Then $1-\mu$ is a pfo set and $(1-\lambda) \leq (1-\mu)$. Since $1-\lambda$ is a pfgc set, $cl_{T_i}cl_{T_j}(1-\lambda) \leq (1-\mu),\ (i \neq j \text{ and } i,j=1,2)$ and hence $1-int_{T_i}int_{T_j}(\lambda) = cl_{T_i}cl_{T_j}(1-\lambda) \leq (1-\mu)$. That is., $1-int_{T_i}int_{T_j}(\lambda) \leq (1-\mu)$. Therefore, $\mu \leq int_{T_i}int_{T_j}(\lambda)$.

Conversely, assume that λ be a fuzzy set such that $\mu \leq int_{T_i}int_{T_j}(\lambda)$ and let μ be a pfc set and $\mu \leq \lambda$. It has to proved that λ is a pfgo set. It is enough to prove that $1-\lambda$ is a pfgc set. Let γ be a pfo set such that $1-\lambda \leq \gamma$. Then $1-\gamma \leq \lambda$. By assumption, $1-\gamma \leq int_{T_i}int_{T_j}(\lambda)$. That is., $1-int_{T_i}int_{T_j}(\lambda) \leq \gamma$ and hence $cl_{T_i}cl_{T_j}(1-\lambda) \leq \gamma$. Therefore, $1-\lambda$ is a pfgc set. □

Remark 3.1. (1). *The union of two pfgo sets is not a pfgo set.*

(2). *The intersection of two pfgo sets is a pfgo set.*

Proposition 3.6. *If λ is a pfgo set in a FBTS (X, T_1, T_2) such that $int_{T_i}(\lambda) \leq \mu \leq \lambda,\ (1 = 1, 2)$, then μ is a pfgo set in (X, T_1, T_2).*

Proof. Let $int_{T_i}(\lambda) \leq \mu \leq \lambda,\ (1 = 1, 2)$. Then $(1 - \lambda) \leq (1 - \mu) \leq (1 - int_{T_i}(\lambda)) = cl_{T_i}(1 - \lambda)$. Since λ is a pfgo set, $1 - \lambda$ is a pfgc set. Hence by proposition 3.2, $1 - \mu$ is a pfgc set, implies that μ is a pfgo set. □

Definition 3.3. *A fuzzy set λ in a FBTS (X, T_1, T_2) is called a pairwise fuzzy generalized G_δ-set (pfgG_δ-set, for short) if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgo sets in (X, T_1, T_2).*

Definition 3.4. *A fuzzy set λ in a FBTS (X, T_1, T_2) is called a pairwise fuzzy generalized F_σ-set (pfgF_σ-set, for short) if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgc sets in (X, T_1, T_2).*

Proposition 3.7. *A fuzzy set λ is a pfgG_δ-set in a FBTS (X, T_1, T_2) if and only if $1 - \lambda$ is a pfgF_σ-set in (X, T_1, T_2).*

Proof. Let λ be a pfgG_δ-set. Then $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgo sets. Now $1-\lambda = 1-\wedge_{k=1}^{\infty}(\lambda_k) = \vee_{k=1}^{\infty}(1-\lambda_k)$, where $(1-\lambda_k)$'s are pfgc sets. Let $\mu_k = 1-\lambda_k$. Then $1-\lambda = \vee_{k=1}^{\infty}(\mu_k)$, where (μ_k)'s are pfgc sets. Therefore, $1-\lambda$ is a pfgF_σ-set.

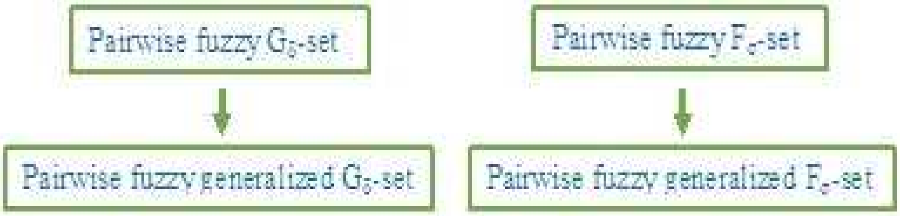
Conversely, let λ be a pfgF_σ-set. Then $\lambda = \vee_{k=1}^{\infty}(\mu_k)$, where (μ_k)'s are pfgc sets. Now $1 - \lambda = 1 - \vee_{k=1}^{\infty}(\mu_k) = \wedge_{k=1}^{\infty}(1 - \mu_k)$, where $(1 - \mu_k)$'s are pfgo sets. Let $\lambda_k = 1 - \mu_k$. Then $1 - \lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgo sets. Therefore, $1 - \lambda$ is a pfgG_δ-set. □

Proposition 3.8. *If λ is a pfG_δ-set in a FBTS (X, T_1, T_2), then λ is a pfgG_δ-set in (X, T_1, T_2).*

Proof. Let λ be a pfG_δ-set. Then $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfo sets. Since every pfo set is a pfgo set, (λ_k)'s are pfgo sets. This implies that $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgo sets. Therefore, λ is a pfgG_δ-set. □

Proposition 3.9. *If λ is a pfF_σ-set in a FBTS (X, T_1, T_2), then λ is a pfgF_σ-set in (X, T_1, T_2).*

Proof. Let λ be a pfF_σ-set. Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfc sets. Since every pfc set is a pfgc set, (λ_k)'s are pfgc sets. This implies that $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgc sets. Therefore, λ is a pfgF_σ-set. □



Remark 3.2. The inter-relations between pfG_δ-set and pfgG_δ-set, pfF_σ-set and pfgF_σ-set can be summarized as follows:

Proposition 3.10. *If λ is a pffc set in a FBTS (X, T_1, T_2), then λ is a pfgF_σ-set in (X, T_1, T_2).*

Proof. Let λ be a pffc set. Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfnd sets. By proposition 3.3, the pfnd sets (λ_k)'s are pfgc sets. Hence $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pfgc sets. Therefore, λ is a pfgF_σ-set. □

Proposition 3.11. *If λ is a pfr set in a FBTS (X, T_1, T_2), then λ is a pfgG_δ-set in (X, T_1, T_2).*

Proof. Let λ be a pfr set. Then $1 - \lambda$ is a pffc set. By proposition 3.10, $1 - \lambda$ is a pfgF_σ-set. Therefore, by proposition 3.7, λ is a pfgG_δ-set. □

Proposition 3.12. *If λ is a pfσ-nd set in a FBTS (X, T_1, T_2), then $1 - \lambda$ is a pfd and pfgG_δ-set in (X, T_1, T_2).*

Proof. Let λ be a pfσ-nd set. Then by theorem 2.2, $1 - \lambda$ is a pfd and pfG_δ-set. By proposition 3.8, the pfG_δ-set $1 - \lambda$ is a pfgG_δ-set. Therefore $1 - \lambda$ is a pfd and pfgG_δ-set. □

4. Pairwise fuzzy generalized Volterra spaces

Definition 4.1. *A FBTS (X, T_1, T_2) is called a pairwise fuzzy generalized Volterra space (pfgVs, for short) if $cl_{T_i}\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1,\ (i = 1, 2)$, where (λ_k)'s are pfd and pfgG_δ-sets in (X, T_1, T_2).*

Example 1. *Let $X = \{a, b, c\}$. The fuzzy sets $\alpha_1, \alpha_2, \alpha_3$ and α_4 are defined on X as follows:*

$\alpha_1 : X \to [0, 1]$ *is defined as* $\alpha_1(a) = 0.6;\ \ \alpha_1(b) = 0.9;\ \ \alpha_1(c) = 0.8,$

$\alpha_2 : X \to [0,1]$ *is defined as* $\alpha_2(a) = 0.7;\ \ \alpha_2(b) = 0.8;\ \ \alpha_2(c) = 0.9$,

$\alpha_3 : X \to [0,1]$ *is defined as* $\alpha_3(a) = 0.8;\ \ \alpha_3(b) = 0.6;\ \ \alpha_3(c) = 0.7$,

$\alpha_4 : X \to [0,1]$ *is defined as* $\alpha_4(a) = 0.7;\ \ \alpha_4(b) = 0.5;\ \ \alpha_4(c) = 0.9$.

Clearly $T_1 = \{0, \alpha_1, \alpha_2, \alpha_3, \alpha_1 \vee \alpha_2, \alpha_1 \vee \alpha_3, \alpha_2 \vee \alpha_3, \alpha_1 \wedge \alpha_2, \alpha_1 \wedge \alpha_3, \alpha_2 \wedge \alpha_3, \alpha_1 \vee (\alpha_2 \wedge \alpha_3), \alpha_3 \vee (\alpha_1 \wedge \alpha_2), \alpha_2 \wedge (\alpha_1 \vee \alpha_3), \alpha_1 \vee \alpha_2 \vee \alpha_3, 1\}$ *and* $T_2 = \{0, \alpha_1, \alpha_4, \alpha_3, \alpha_1 \vee \alpha_4, \alpha_1 \vee \alpha_3, \alpha_4 \vee \alpha_3, \alpha_1 \wedge \alpha_4, \alpha_1 \wedge \alpha_3, \alpha_4 \wedge \alpha_3, \alpha_1 \vee (\alpha_4 \wedge \alpha_3), \alpha_4 \vee (\alpha_1 \wedge \alpha_3), \alpha_3 \vee (\alpha_1 \wedge \alpha_4), \alpha_1 \wedge (\alpha_4 \vee \alpha_3), \alpha_4 \wedge (\alpha_1 \vee \alpha_3), \alpha_3 \wedge (\alpha_1 \vee \alpha_4), \alpha_1 \wedge \alpha_4 \wedge \alpha_3, \alpha_1 \vee \alpha_4 \vee \alpha_3, 1\}$ *are fuzzy topologies on* X. *The fuzzy sets* $\alpha_1, \alpha_3, \alpha_1 \vee \alpha_2, \alpha_1 \vee \alpha_3, \alpha_1 \vee \alpha_4, \alpha_1 \wedge \alpha_3, \alpha_2 \wedge \alpha_3, \alpha_3 \wedge (\alpha_1 \vee \alpha_4), \alpha_1 \vee (\alpha_2 \wedge \alpha_3), \alpha_1 \vee (\alpha_4 \wedge \alpha_3), \alpha_1 \vee \alpha_2 \vee \alpha_3, \alpha_1 \vee \alpha_4 \vee \alpha_3, 1$ *are pfgo sets.*

Now $\lambda = \alpha_1 \wedge (\alpha_1 \vee \alpha_2) \wedge (\alpha_1 \vee \alpha_3) \wedge [\alpha_1 \vee (\alpha_2 \wedge \alpha_3)],\ \mu = \alpha_3 \wedge (\alpha_1 \wedge \alpha_3) \wedge (\alpha_2 \wedge \alpha_3)$ *and* $\nu = (\alpha_1 \vee \alpha_4) \wedge [\alpha_3 \wedge (\alpha_1 \vee \alpha_4)] \wedge [\alpha_1 \vee (\alpha_2 \wedge \alpha_3)] \wedge (\alpha_1 \vee \alpha_4 \vee \alpha_3)$ *are pfg*G_δ*-sets. Also,* $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1;\ cl_{T_1} cl_{T_2}(\mu) = cl_{T_2} cl_{T_1}(\mu) = 1$ *and* $cl_{T_1} cl_{T_2}(\nu) = cl_{T_2} cl_{T_1}(\nu) = 1$. *Then* λ, μ *and* ν *are pfd sets. Hence* λ, μ *and* ν *are pfd and pfg*G_δ*-sets. Now* $cl_{T_i}(\lambda \wedge \mu \wedge \nu) = 1,\ (i = 1, 2)$*, implies that the FBTS* (X, T_1, T_2) *is a pfgVs.*

Example 2. *Let* $X = \{a, b, c\}$. *The fuzzy sets* $\alpha_1, \alpha_2, \alpha_3$ *and* α_4 *are defined on* X *as follows:* $\alpha_1 : X \to [0,1]$ *is defined as* $\alpha_1(a) = 0;\ \ \alpha_1(b) = 0.8;\ \ \alpha_1(c) = 0.7$,

$\alpha_2 : X \to [0,1]$ *is defined as* $\alpha_2(a) = 0.7;\ \ \alpha_2(b) = 0.6;\ \ \alpha_2(c) = 0$,

$\alpha_3 : X \to [0,1]$ *is defined as* $\alpha_3(a) = 0.2;\ \ \alpha_3(b) = 0.9;\ \ \alpha_3(c) = 0.5$,

$\alpha_4 : X \to [0,1]$ *is defined as* $\alpha_4(a) = 0.6;\ \ \alpha_4(b) = 0;\ \ \alpha_4(c) = 0.4$.

Clearly $T_1 = \{0, \alpha_1, \alpha_2, \alpha_3, \alpha_1 \vee \alpha_2, \alpha_1 \vee \alpha_3, \alpha_2 \vee \alpha_3, \alpha_1 \wedge \alpha_2, \alpha_1 \wedge \alpha_3, \alpha_2 \wedge \alpha_3, \alpha_1 \vee (\alpha_2 \wedge \alpha_3), \alpha_2 \vee (\alpha_1 \wedge \alpha_3), \alpha_3 \wedge (\alpha_1 \vee \alpha_2), \alpha_1 \vee \alpha_2 \vee \alpha_3, 1\}$ *and* $T_2 = \{0, \alpha_1, \alpha_4, \alpha_3, \alpha_1 \vee \alpha_4, \alpha_1 \vee \alpha_3, \alpha_4 \vee \alpha_3, \alpha_1 \wedge \alpha_4, \alpha_1 \wedge \alpha_3, \alpha_4 \wedge \alpha_3, \alpha_1 \vee (\alpha_4 \wedge \alpha_3), \alpha_4 \vee (\alpha_1 \wedge \alpha_3), \alpha_3 \wedge (\alpha_1 \vee \alpha_4), \alpha_1 \vee \alpha_4 \vee \alpha_3, 1\}$ *are fuzzy topologies on* X. *The fuzzy sets* $\alpha_1, \alpha_3, \alpha_1 \vee \alpha_3, \alpha_1 \wedge \alpha_3, \alpha_1 \vee (\alpha_2 \wedge \alpha_3), \alpha_1 \vee (\alpha_4 \wedge \alpha_3), \alpha_3 \wedge (\alpha_1 \vee \alpha_2), \alpha_3 \wedge (\alpha_1 \vee \alpha_4), 1$ *are pfgo sets.*

Now $\lambda = \alpha_1 \wedge (\alpha_1 \vee \alpha_3) \wedge [\alpha_1 \vee (\alpha_2 \wedge \alpha_3)],\ \mu = \alpha_3 \wedge (\alpha_1 \vee \alpha_3) \wedge [\alpha_1 \vee (\alpha_2 \wedge \alpha_3)] \wedge [\alpha_3 \wedge (\alpha_1 \vee \alpha_2)]$ *and* $\nu = \alpha_1 \wedge (\alpha_1 \wedge \alpha_3) \wedge [\alpha_1 \vee (\alpha_2 \wedge \alpha_3)] \wedge [\alpha_3 \wedge (\alpha_1 \vee \alpha_2)]$ *are pfg*G_δ*-sets. Also,* $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1;\ cl_{T_1} cl_{T_2}(\mu) = cl_{T_2} cl_{T_1}(\mu) = 1$ *and* $cl_{T_1} cl_{T_2}(\nu) = cl_{T_2} cl_{T_1}(\nu) = 1$. *Then* λ, μ *and* ν *are pfd sets. Hence* λ, μ *and* ν *are pfd and pfg*G_δ*-sets. Now* $cl_{T_1}(\lambda \wedge \mu \wedge \nu) = 1$. *But* $cl_{T_2}(\lambda \wedge \mu \wedge \nu) = 1 - \alpha_4 \neq 1$. *This implies that the FBTS* (X, T_1, T_2) *is not a pfgVs.*

5. Characterization of pairwise fuzzy generalized Volterra spaces

Proposition 5.1. *If a FBTS (X, T_1, T_2) is a pfgVs, then $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) = 0,\ (i = 1, 2)$, where (λ_k)'s are pfσ-nd sets in (X, T_1, T_2).*

Proof. Let (λ_k)'s ($k = 1$ to N) be pfσ-nd sets. Then by proposition 3.12, $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Since (X, T_1, T_2) is a pfgVs, $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right) = 1$. Then, $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) = 0$, where (λ_k)'s are pfσ-nd sets. □

Proposition 5.2. *If $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) = 1,\ (i = 1, 2)$, where (λ_k)'s are pfG_δ-sets in a FBTS (X, T_1, T_2), then (X, T_1, T_2) is a pfgVs.*

Proof. Suppose that $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) = 1$, where (λ_k)'s are pfG_δ-sets. Since (λ_k)'s are pfG_δ-sets and by proposition 3.8, (λ_k)'s are pfgG_δ-sets. Now $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) \leq \wedge_{k=1}^{\infty} cl_{T_i}(\lambda_k)$, implies that $1 \leq \wedge_{k=1}^{\infty} cl_{T_i}(\lambda_k)$. That is., $\wedge_{k=1}^{\infty} cl_{T_i}(\lambda_k) = 1$. Then $cl_{T_i}(\lambda_k) = 1$ and hence by theorem 2.1, (λ_k)'s are pfd sets. Thus, (λ_k)'s are pfd and pfgG_δ-sets. Now $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) \leq cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)$, implies that $1 \leq cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)$. That is., $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, where (λ_k)'s are pfd and pfgG_δ-sets. Hence (X, T_1, T_2) is a pfgVs. □

Proposition 5.3. *If $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) = 0, (i = 1, 2)$, where (λ_k)'s are pfnd and pfF_σ-sets in a FBTS (X, T_1, T_2), then (X, T_1, T_2) is a pfgVs.*

Proof. Suppose that $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) = 0$, where (λ_k)'s are pfnd and pfF_σ-sets. Since (λ_k)'s are pfnd sets and by theorem 2.3, $(1-\lambda_k)$'s are pfd sets. But (λ_k)'s are pfF_σ-sets and by proposition 3.9, (λ_k)'s are pfgF_σ-sets and hence $(1-\lambda_k)$'s are pfgG_δ-sets. Therefore, $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Now $1 - int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) = 1$. Then $cl_{T_i}\left(1 - \vee_{k=1}^{N}(\lambda_k)\right) = 1$. This implies that $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right) = 1$. Thus, $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right) = 1$, where $(1-\lambda_k)$'s are pfd and pfgG_δ-sets, implies that (X, T_1, T_2) is a pfgVs. □

Proposition 5.4. *If $int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 0$, where (λ_k)'s are pfσ-nd sets in a FBTS (X, T_1, T_2), then (X, T_1, T_2) is a pfgVs.*

Proof. Suppose that $int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 0$, where (λ_k)'s are pfσ-nd sets. Now, $1 - int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 1$ and hence $cl_{T_i}\left(\wedge_{k=1}^{\infty}(1-\lambda_k)\right) = 1$. But $cl_{T_i}\left(\wedge_{k=1}^{\infty}(1-\lambda_k)\right) \leq cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right)$ and $cl_{T_i}\left(\wedge_{k=1}^{\infty}(1-\lambda_k)\right) = 1$. By proposition 3.12, $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Hence, $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right) = 1$, where $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Therefore (X, T_1, T_2) is a pfgVs. □

Proposition 5.5. *Let (X, T_1, T_2) be a FBTS. Then the following are equivalent:*

(i) (X, T_1, T_2) *is a pfgVs.*

(ii) $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) = 0,\ (i = 1, 2)$, *where* (λ_k)*'s are pfd and pfg*G_δ*-sets in* (X, T_1, T_2).

Proof. $(i) \implies (ii)$. Let (X, T_1, T_2) be a pfgVs. Then, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, where (λ_k)'s are pfd and pfgG_δ-sets. Now $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) = int_{T_i}\left(1-\wedge_{k=1}^{N}(\lambda_k)\right) = 1 - cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1 - 1 = 0$. Hence $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) = 0$, where (λ_k)'s are pfd and pfgG_δ-sets.

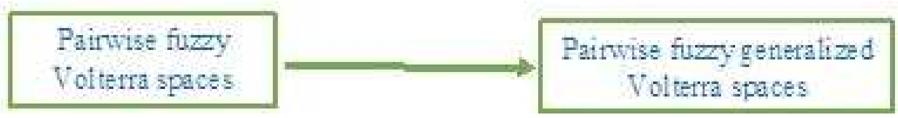
$(ii) \implies (i)$. Let $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) = 0$, where (λ_k)'s are pfd and pfgG_δ-sets. Now $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) = 0$, implies that $int_{T_i}\left(1-\wedge_{k=1}^{N}(\lambda_k)\right) = 0$ and hence $1 - cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 0$. Then $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$. Therefore, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, where (λ_k)'s are pfd and pfgG_δ-sets. Hence (X, T_1, T_2) is a pfgVs. □

6. Relations between pairwise fuzzy generalized Volterra spaces and other fuzzy bitopological spaces

Proposition 6.1. *A FBTS (X, T_1, T_2) is a pfVs, then (X, T_1, T_2) is a pfgVs.*

Proof. Let (X, T_1, T_2) be a pfVs. Then $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, where (λ_k)'s are pfd and pfG_δ-sets. Since (λ_k)'s are pfG_δ-sets and by proposition 3.8, (λ_k)'s are pfgG_δ-sets. Hence (λ_k)'s are pfd and pfgG_δ-sets. Therefore $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, where (λ_k)'s are pfd and pfgG_δ-sets, implies that (X, T_1, T_2) is a pfgVs. □

Remark 6.1. The inter-relations between pfVs and pfgVs can be summarized as follows:



Proposition 6.2. *If each pfnd set λ in a FBTS (X, T_1, T_2) is a pfF_σ-set, then (X, T_1, T_2) is a pfgVs.*

Proof. Suppose that the pfnd set λ is a pfF_σ-set. Then, by theorem 2.4, (X, T_1, T_2) is a pfBs. Hence $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0,\ (i = 1, 2)$, where (λ_k)'s are pfnd sets. Then $1 - int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$. Since (λ_k)'s are pfnd sets and by theorem 2.3, $(1-\lambda_k)$'s are pfd sets. Also, since (λ_k)'s are pfF_σ-sets, $(1-\lambda_k)$'s are pfG_δ-sets. But by proposition 3.8, $(1-\lambda_k)$'s are pfgG_δ-sets. Hence $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Now $cl_{T_i}\left(\wedge_{k=1}^{\infty}(1-\lambda_k)\right) \leq cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right)$, implies that $1 \leq cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right)$. That is., $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right)=1$, where $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Therefore (X,T_1,T_2) is a pfgVs. □

Proposition 6.3. *If the FBTS (X,T_1,T_2) is a pfσ-Bs, then (X,T_1,T_2) is a pfgVs.*

Proof. Let (X,T_1,T_2) be a pfσ-Bs. Then $int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right)=0,\ (i=1,2)$, where (λ_k)'s are pfσ-nd sets. Now $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right) \leq int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right)=0$, implies that $int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right)=0$. Then, $1-int_{T_i}\left(\vee_{k=1}^{N}(\lambda_k)\right)=1$. This implies that $cl_{T_i}\left(\wedge_{k=1}^{N}(1-\lambda_k)\right)=1$. Since (λ_k)'s are pfσ-nd sets and by proposition 3.12, $(1-\lambda_k)$'s are pfd and pfgG_δ-sets. Hence (X,T_1,T_2) is a pfgVs. □

Proposition 6.4. *If the pfnd sets are pfF_σ-sets in a pfBs (X,T_1,T_2), then (X,T_1,T_2) is a pfgVs.*

Proof. Suppose that every pfnd set is a pfF_σ-set in a pfBs. Then by theorem 2.5, (X,T_1,T_2) is a pfσ-Bs. Also, by proposition 6.3, (X,T_1,T_2) is a pfgVs. □

Proposition 6.5. *If a FBTS (X,T_1,T_2) is a pf$P$-s and pfhs, then (X,T_1,T_2) is a pfgVs.*

Proof. Let (λ_k)'s $(k=1 \text{ to } N)$ be pfG_δ-sets. Since (X,T_1,T_2) is a pf$P$-s and (λ_k)'s are pfG_δ-sets, (λ_k)'s are pfo sets. This implies that $\lambda_k \in T_i$. Then $\wedge_{k=1}^{N}(\lambda_k) \in T_i,\ (i=1,2)$. Thus $\wedge_{k=1}^{N}(\lambda_k)$ is a pfo set. Also, since (X,T_1,T_2) is a pfhs and (λ_k)'s are pfo sets, (λ_k)'s are pfd sets. Since (λ_k)'s are pfG_δ-sets and by proposition 3.8, (λ_k)'s are pfgG_δ-sets. Hence (λ_k)'s are pfd and pfgG_δ-sets. Since $\wedge_{k=1}^{N}(\lambda_k)$ is a pfo set in a pfhs, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$. Thus, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$, where (λ_k)'s are pfd and pfgG_δ-sets. Therefore (X,T_1,T_2) is a pfgVs. □

Proposition 6.6. *If a FBTS (X,T_1,T_2) is a pfB and pf$P$-s, then (X,T_1,T_2) is a pfgVs.*

Proof. Let (λ_k)'s $(k=1 \text{ to } N)$ be pfG_δ-sets such that $cl_{T_i}(\lambda_k)=1,\ (i=1,2)$. Since (X,T_1,T_2) is a pf$P$-s, the pfG_δ-sets (λ_k)'s are pfo sets. Then, (λ_k)'s are pfo sets such that $cl_{T_i}(\lambda_k)=1$. By theorem 2.6, $(1-\lambda_k)$'s are pfnd sets. Since (X,T_1,T_2) is a pfBs, $int_{T_i}\left(\vee_{\alpha=1}^{\infty}(\mu_\alpha)\right)=0$, where (μ_α)'s are pfnd sets. Let us take the first N pfnd sets in (μ_α)'s as $(1-\lambda_k)$. Then, $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right) \leq$

$int_{T_i}\left(\vee_{\alpha=1}^{\infty}(\mu_\alpha)\right)$, implies that $int_{T_i}\left(\vee_{k=1}^{N}(1-\lambda_k)\right)=0$. Hence $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$. Since $cl_{T_i}(\lambda_k)=1$, $cl_{T_1}cl_{T_2}(\lambda_k)=cl_{T_1}(1)=1$ and $cl_{T_2}cl_{T_1}(\lambda_k)=cl_{T_2}(1)=1$ and hence (λ_k)'s are pfd sets. Since (λ_k)'s are pfG_δ-sets and by proposition 3.8, (λ_k)'s are pfgG_δ-sets. Hence (λ_k)'s are pfd and pfgG_δ-sets. Thus, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$, where (λ_k)'s are pfd and pfgG_δ-sets. Therefore (X,T_1,T_2) is a pfgVs. □

Proposition 6.7. *If the pffc sets are pfc sets in a pfBs (X,T_1,T_2), then (X,T_1,T_2) is a pfgVs.*

Proof. Let (X,T_1,T_2) be a pfBs and (λ_k)'s ($k=1$ to ∞) be pfG_δ-sets such that $cl_{T_i}(\lambda_k)=1$, $(i=1,2)$. Then by theorem 2.7, $(1-\lambda_k)$'s are pffc sets. Then by hypothesis, $(1-\lambda_k)$'s are pfc sets. Hence (λ_k)'s are pfo sets. Now (λ_k)'s are pfo sets such that $cl_{T_i}(\lambda_k)=1$. Hence by theorem 2.6, $(1-\lambda_k)$'s are pfnd sets. Since (X,T_1,T_2) is a pfBs, $int_{T_i}\left(\vee_{k=1}^{\infty}(1-\lambda_k)\right)=0$. This implies that $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right)=1$. Now $cl_{T_i}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right)\leq cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)$, implies that $1\leq cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)$. That is., $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$. Since $cl_{T_i}(\lambda_k)=1$, $cl_{T_1}cl_{T_2}(\lambda_k)=cl_{T_1}(1)=1$ and $cl_{T_2}cl_{T_1}(\lambda_k)=cl_{T_2}(1)=1$. Hence (λ_k)'s are pfd sets. Since (λ_k)'s are pfG_δ-sets and by proposition 3.8, (λ_k)'s are pfgG_δ-sets. Hence (λ_k)'s are pfd and pfgG_δ-sets. Therefore, $cl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right)=1$, where (λ_k)'s are pfd and pfgG_δ-sets. Hence (X,T_1,T_2) is a pfgVs. □

Proposition 6.8. *If the pfr sets are pfo sets in a pfBs (X,T_1,T_2), then (X,T_1,T_2) is a pfgVs.*

Proof. Let the pfr sets (λ_k)'s ($k=1$ to ∞) be pfo sets in a pfBs. Then $(1-\lambda_k)$'s are pffc sets such that $(1-\lambda_k)$'s are pfc sets. Hence, by proposition 6.7, (X,T_1,T_2) is a pfgVs. □

Proposition 6.9. *If FBTS (X,T_1,T_2) is a pfBs, then $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))=0$, $(i=1,2)$, where (λ_k)'s are pfgc sets in (X,T_1,T_2).*

Proof. Since (X,T_1,T_2) is a pfBs, $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))=0$, where (λ_k)'s are pfnd sets. Also, since (λ_k)'s are pfnd sets and by proposition 3.3, (λ_k)'s are pfgc sets. Therefore, $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))=0$, where (λ_k)'s are pfgc sets. □

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DEPARTMENT OF MATHEMATICS,
THIRUVALLUVAR UNIVERSITY,
VELLORE-632115, TAMIL NADU, INDIA.
Email address: `g.thangaraj@rediffmail.com`

POST GRADUATE DEPARTMENT OF MATHEMATICS,
BESANT THEOSOPHICAL COLLEGE,
MADANAPALLE-517325, ANDHRA PRADESH, INDIA.
Email address: `profvcmaths@gmail.com`

DEPARTMENT OF MATHEMATICS,
KINGS ENGINEERING COLLEGE,
IRUNGATTUKOTTAI, SRIPERUMBUDUR-602117, TAMIL NADU, INDIA.
Email address: `ashokkumar@kingsedu.ac.in`