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OPTIMAL SOLUTION OF FIXED COST TRANSPORTATION PROBLEMS BY APPROXIMATING STAIRCASE METHOD

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ABSTRACT. The fixed charge is the main issue facing by many of the production and transportation systems. It is posing real difficulties in the solving process since it does not depend on the number of units transported in that particular route. Even though the mathematical model has been framed and many of the researchers were done their research, very few were succeed to get the optimal solution. Those methods are effective when the sources and destinations are less in numbers and it is still difficult to get the optimal solution when the number of sources and destinations were increasing.

We proposed a method in this paper to solve fixed cost transportation problems based on the linear approximation. It is very easy to understand and not posing any difficulties in the solving process. Using this method we can get a solution which is optimal for the problem in the finite number of steps. Proposed method is effective even though the sources and destinations are large in numbers.

1. INTRODUCTION

Generally, in classical transportation problem, cost for the transportation is directly proportional to the number of units which is transported. But, in many of the real-world cases there will be a fixed cost which will be charged to each transportation route. Hirsch and Dantzig in [11] in 1954, were the first ones

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who formulated the fixed charge problem. Balinski in 1961 in [2] showed a special case which is a fixed cost transportation problem (FCTP) of the fixed charge problem. He proposed an approximation method to get approximate solution of it. Gray in [10], tried to find the optimal (exact) solution to the FCTP by decomposition. He decomposed the problem into an integer program and transportation subprograms. Steinberg in [19], and Palekar et al., in [16] provided algorithms to get the exact solution based on the method of branch and bound. The heuristic approaches were developed and analyzed by many of the researchers like Cooper and Dredes, [4], Drenzler [6], Cooper in [3], Walker in [23], Veena Adlakha et al. in [21], Jesus Saez Aguado in [12], Xie et al. in [7] and Sun et al., [20] in the process of solving a FCTP. Those methods are more like simplex iterations. Among the heuristic methods, Diaby, [5] and Kuhn et al., [13], are the well-known methods. Sandrock in [18] presented a very simple algorithm for small fixed charge problems. In his problem the fixed cost is not associated with the destination routes instead of that, it is associated with supply points. Francisco Ortega in [9] proposed a branch and cut algorithm for uncapacitated, single commodity fixed charge network workflow problems. Andreas Klose in [1] proposed a method based on dynamic programming to solve a single sink FCTP which is a sub problem of FCTPs. The authors in [25] have studied the structure of the projection polyhedron of FCTP.

Veena Adlakha et al., in [22] proposed a non-linear approximation of a cost function to get the better-approximated solution of the FCTP. Roberto Roberti et al.in [16] gave an algorithm to solve the FCTP based on a new integer programming formulation. Farhad Ghassemi Tari in [8] proposed hybrid dynamic programming for solving discounted mechanism of FCTP. Yixin Zhao et al. in [24] proposed a well-built formulation based on Lagrangian decomposition and column generation in the FCTP.

After many methods were developed over the decays, only two methods were guaranteed to get the optimal solution. One is the stage-ranking method, [14, 17] and the other one is branch and bound method [15, 20]. The difficult thing about the branch-and-bound method is, it grows exponentially depends on the number of supply points and destination points; whereas, the method of ranking extreme points, needs to investigate a big allocation field. Analyzing extreme points will be difficult and time taking process if the number of extreme points is large.

A method was proposed to find the optimal solution of the FCTP which is very easy to handle and understand. This method will give the optimal solution (Exact solution) in finite number of steps. The proposed method is based on Balanski's approximation method. In his method the cost was approximated only one time before solving the problem. Whereas, in this proposed method the FCTP were approximated and reframed wherever it requires.

2. Fixed Cost Transportation Problem

The mathematical model of the FCTP is given below,

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + f_{ij}y_{ij},\ where\ y_{ij} = \begin{cases} 1, & \text{if } x_{ij} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Subject to the constraints, $\sum_{j=1}^{n} x_{ij} = a_i$, for $i = 1, 2, \ldots, m$

$$\sum_{i=1}^{m} x_{ij} = b_j, \text{ for } j = 1, 2, \ldots, n,\ x_{ij} \geq 0,$$

$y_{ij}\ is\ 0\ or\ 1.$

m, n - number of supply points and demand points respectively,

x_{ij} - number of units shipped from i^{th} supply point to j^{th} demand point,

c_{ij} - unit shipping cost for shipping the units from i^{th} supply point to j^{th} demand point,

f_{ij} - fixed cost for shipping the units from i^{th} supply point to j^{th} demand point,

a_i - supply at i^{th} supply point,

b_j -demand at j^{th} demand point.

3. Linear Cost Approximation

Balanski proposed the linear approximation in FCTP using the derivative function $D_{ij} = \frac{f_{ij}}{m_{ij}} + c_{ij}$, where m_{ij} is maximum possible allocation (i.e. minimum among a_i and b_j) and D_{ij} is the approximated cost (Figure 1).

4. Approximating Staircase Method

The solving process of "Approximating Staircase Method" has been given below, and it is a step by step procedure which contains ten steps.

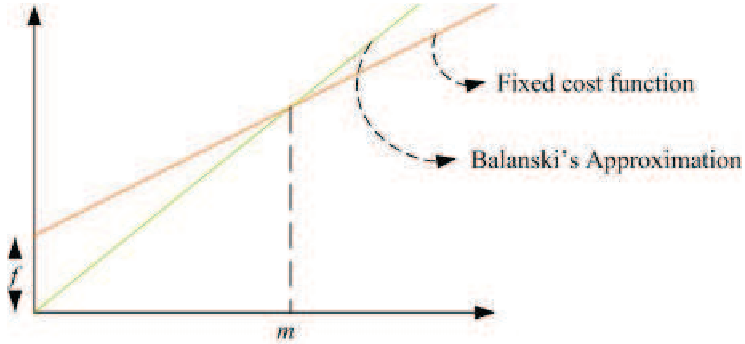

FIGURE 1. Linear approximation

Step 1: Approximate the fixed cost matrix using the derivative function $D_{ij} = \frac{f_{ij}}{m_{ij}} + c_{ij}$ and reframe the problem to the normal transportation problem.

Step 2: Find the minimum of each row and subtract it from the respective row entries. From the resulting cost matrix, find the column minimum of each column and subtract it from the respective column entries.

Step 3: Find the reduced allotment table by checking each row supply is less than or equal to the total of the demands whose reduced costs in that particular row are zero. In the same way each column demand is less than or equal to the total of the supplies whose reduced costs in the column are zero. Go to Step 6 if it satisfies the Step 3. If it not satisfies then, go to Step 4.

Step 4: From the reduced transportation table, to cover all the zeros draw the minimum number of vertical and horizontal lines, such a way that some entries of column(s) or/and row(s) which were not satisfying the condition in the Step 3.

Step 5: Find the smallest among the uncovered entries and add it to the elements which are covered by twice, subtract from the elements which are uncovered and do nothing to the elements which are covered once. Then go to Step 3.

Step 6: In the reduced table, select a cell which has the maximum reduced cost (say λ_{ij}). Select any one if there are multiple maximum.

Step 7: Select a cell in the i^{th}row or/ and j^{th} column of the cell λ_{ij} such a way that selected row or column has only one zero in it. Allocate the maximum possible to that cell. Find the next maximum if such cell does not occur for that λ_{ij} and take that as a newλ_{ij}. (Go to Step 10 if the maximum itself a zero).

Step 8: Reframe the fixed cost table by deleting the supply point which is fully used and destination point which is fully received.

Step 9: Continue the process again from Step 4 to Step 8 until all the demand points and all the supply points are fully vanished.

Step 10: If the maximum cost is zero then reframe the problem according to the new supply and new demand then proceed from Step 1 to Step 9 (Note: Pick the last maximum asλ_{ij} if new maximum is again a zero).

5. Numerical Examples

Example 1. *Consider the fixed charge transportation problem from Katta Murty, [14]. This numerical example has seven destinations and five sources. The available supply from each source, the requirement for each destination and the fixed charge and the unit cost from each route has been given in the table 1.*

TABLE 1. Problem data

	D1	D2	D3	D4	D5	D6	D7	Supply
S1	(6,16)	(8,13)	(0,12)	(3,6)	(7,24)	(4,19)	(19,20)	23
S2	(35,17)	(4,40)	(5,15)	(1,8)	(26,13)	(10,11)	(2,5)	26
S3	(9,19)	(11,109)	(24,8)	(16,29)	(2,26)	(5,5)	(4,25)	38
S4	(12,92)	(36,29)	(6,2)	(31,20)	(19,42)	(8,6)	(5,17)	75
S5	(6,23)	(9,27)	(10,14)	(5,17)	(43,114)	(12,38)	(18,26)	56
Demand	22	9	35	54	8	55	35	

Solution for the above numerical example has been solved by proposed method. Approximate the fixed cost matrix using the derivative function which is in step 1 in the procedure. The resulted approximated cost transportation problem is given in the table 2. Find the

TABLE 2. Approximated cost transportation problem

	D1	D2	D3	D4	D5	D6	D7	Supply
S1	16.2727	13.8889	12.0000	6.1304	24.8750	19.1739	20.8261	23
S2	18.5909	40.4444	15.1923	8.0385	16.2500	11.3846	5.0769	26
S3	19.4091	110.2222	8.6857	29.4211	26.2500	5.1316	25.1143	38
S4	92.5455	33.0000	2.1714	20.5741	44.3750	6.1455	17.1429	75
S5	23.2727	28.0000	14.2857	17.0926	119.3750	38.2182	26.5143	56
Demand	22	9	35	54	8	55	35	

row minimum of each row and subtract it from the respective row entries. From the resulting cost matrix, find the column minimum of each column and subtract it from the respective column entries. The resulted cost matrix is given in table 3.

TABLE 3. Row and column reduced cost transportation problem

	D1	D2	D3	D4	D5	D6	D7	Supply
S1	1.553	0	5.8696	0	7.5715	13.0435	14.6957	23
S2	4.5270	27.6091	10.1154	2.9615	0	6.3077	0	26
S3	5.2905	97.3322	3.5541	24.2895	9.9453	0	19.9827	38
S4	81.3870	23.0701	0	18.4026	31.0305	3.9740	14.9714	75
S-5	0	5.9558	0	2.8069	93.9162	23.9325	12.2286	56
Demand	22	9	35	54	8	55	35	

By Step 3, the row 4 and the columns 4, 6, 7 were not satisfying required condition. Now apply the Step 4 and 5 which is given in the procedure and repeat till it's satisfying the condition in Step 3. After several reputations the reduced cost matrix which is satisfying the condition is given below in the table 4 (we can say this as an allotment table). Now

TABLE 4. Allotment table

	D1	D2	D3	D4	D5	D6	D7	Supply
S1	3.9622	0	17.9410	0	5.7230	21.1409	11.7957	23
S2	10.2339	30.5091	25.0868	5.8615	1.0521	17.3051	0	26
S3	0	89.2348	7.5282	16.1921	0	0	8.9853	38
S4	72.1225	10.9987	0	6.3312	17.1112	0	0	75
S5	0	3.1490	9.2645	0	89.2614	29.2230	6.5217	56
Demand	22	9	35	54	8	55	35	

the above table is ready to do the allotment. After applying the Step 6 to Step 10 we get the allocation as in table 5.

The allocation which is in table 5, will give the optimal solution of the actual problem. Hence the minimized total cost for transportation is 2289.

Followed by, the reputed numerical examples were taken and shown that this method is giving the optimal solution.

Example 2. *Consider the numerical example which is used from Balanski's, [2] approximation. The problem data has been given in Table 6.*

TABLE 5. Allocation table

	D1	*D2*	*D3*	*D4*	*D5*	*D6*	*D7*	*Supply*
S1	*0*	*9*	*0*	*14*	*0*	*0*	*0*	*23*
S2	*0*	*0*	*0*	*0*	*0*	*0*	*26*	*26*
S3	*6*	*0*	*0*	*0*	*8*	*24*	*0*	*38*
S4	*0*	*0*	*35*	*0*	*0*	*31*	*9*	*75*
S5	*16*	*0*	*0*	*40*	*0*	*0*	*0*	*56*
Demand	*22*	*9*	*35*	*54*	*8*	*55*	*35*	

TABLE 6. Problem data of example 2

	D1	*D2*	*D3*	*Supply*
S1	*(10, 2)*	*(30, 3)*	*(20, 4)*	*10*
S2	*(10, 3)*	*(30, 2)*	*(20, 1)*	*30*
S3	*(10, 1)*	*(30, 4)*	*(20, 3)*	*40*
S4	*(10, 4)*	*(30, 5)*	*(20, 2)*	*20*
Demand	*20*	*50*	*30*	

Here, in this problem the fixed charges are depends on the destination points not depend on the routes. Using the proposed method the allocation for this problem is given in table 7.

TABLE 7. Allocation table for example 2

	D1	*D2*	*D3*	*Supply*
S1	*0*	*0*	*10*	*10*
S2	*0*	*30*	*0*	*30*
S3	*20*	*20*	*0*	*40*
S4	*0*	*0*	*20*	*20*
Demand	*20*	*50*	*30*	

The allocation in table 7 is the optimal allocation and it will give the optimal solution to the Balanski's numerical example. Hence the minimized total cost for transportation is 350.

Example 3. *Consider the numerical example which is performed in the research work of Sadagopan, S. and A. Ravindran in [17]. The problem data has been given in table 8.*

TABLE 8. Problem data of example 3

	D1	*D2*	*D3*	*D4*	*Supply*
S1	*(900,760)*	*(1000,71)*	*(700,283)*	*(800,594)*	*50*
S2	*(900,594)*	*(300,64)*	*(700,170)*	*(600,564)*	*15*
S3	*(600,594)*	*(200,69)*	*(400,79)*	*(0,202)*	*5*
Demand	*25*	*20*	*15*	*10*	

Here, in this problem the fixed charges were larger in values and its range is also large (Since there is a fixed charge 0 as the smallest and 1000 as a largest). Using the proposed method the allocation is given in the table 9.

TABLE 9. Allocation table for example 3

	D1	*D2*	*D3*	*D4*	*Supply*
S1	*25*	*20*	*0*	*5*	*50*
S2	*0*	*0*	*15*	*0*	*15*
S3	*0*	*0*	*0*	*5*	*5*
Demand	*25*	*20*	*15*	*10*	

The allocation in table 9 is the optimal allocation and it will give the optimal solution to the numerical example of Sadagopan, S. and A. Ravindran. Hence the minimized total cost for transportation is 30350.

Clearly, all the three examples are differs in nature and they do behave different in the solving process. The numerical Example 1 has large number of supply and destination points. The numerical Example 2 has fixed charge which is depending on the destination points rather than the routes.The numerical Example 3 has large range of fixed charges. From this, It is clear that the proposed method is giving the optimal allocation to all kind of FCTPs.

6. CONCLUSION

Fixed cost transportation problem (FCTP) has been analyzed by many of the researchers and given many methods to solve it. Among them only few were succeed to get the solution which is optimal. Even though the methods were there to get the optimal solution, this method is very easy to find the optimal solution and not posing any difficulties to understand as well as solving process. Using the proposed method, few numerical examples from various research works were examined and the optimal solution has been obtained.

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