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AN APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD FOR UNSTEADY MHD LAMINAR MOMENTUM BOUNDARY LAYER ABOVE A FLAT PLATE WITH LEADING EDGE ACCRETION

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ABSTRACT. The unsteady MHD boundary layer over a flat plate is discussed in this paper. The flow involves the unsteady flow over a flat plate with leading edge accretion. The momentum boundary layer is analysed with the free stream velocities using Blasius - Rayleigh - Stokes variable. The differential transformation method (DTM) is applied to get the better solution of the laminar flow of the flat plate with leading edge accretion. Result shown is efficient and accurate for solving the nonlinear system of equations.

1. INTRODUCTION

To solve the mathematical problem of differential equations, there are different types of methods used for the linear and non-linear differential equations. Among many, Taylor series is one of the method to find higher order derivative of the function. In 1986, Zhou [16] has introduced a new form of Taylor series called Differential Transformation Method (DTM) and successfully implemented this method to solve the ordinary differential equations by the induced recursive equation to find the coefficient of the Taylor series of a function. In the present days, many researchers have adapted this differential transform method to solve the initial and boundary value problems for the partial and ordinary differential

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equations with the restricted and non-restricted limits of boundary conditions. DTM provides the freedom to express solutions of a given nonlinear problems by means of Pade approximant, Fractional order, HAM and Ms-DTM, one can refer [2,3,5–9,13,15] and references therein for history and properties of DTM.

Almost a century ago, boundary layer plays a vital role in determining accurately the flow of certain fluids. Boundary layer concept has developed in 1904 by Lading Prandtl. By Prandtl's hypothesis the fluid friction at high Reynolds number are restricted to a thin layer near the boundary known as boundary layer term. The solution to boundary layer flows is obtained form the reduced "Navier - Stokes" equation for which boundary layer assumptions and approximations have been applied. The solutions to these equations was obtained in 1908 by Blasius, a student of Prandtl's. He has begin with the exact solution method by performing a transformation technique to change the set of two coupled pde into a single ode's [4, 11]. Magnetohydrodynamics (MHD) is the study of magnetic properties and behaviour of electrically conducting fluids. The fundamental concept behind magnetohydrodynamics is that magnetic fields can induce currents in the moving conductive fluid which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The differential equations must be solved either analytically or numerically [1]. In this work Differential Transform Method (DTM) is applied to solve the unsteady Magneto-hydrodynamic (MHD) boundary layer for the flow analysis [14]. Another important advantage is that, this method helps in reducing the size of computational work and provides the accuracy of the series solution with good convergence rate. To be sure, flow characteristics of such an unsteady, MHD boundary layer plays an important role in many engineering and technological problems like initial process and fluid motion [12]. The DTM is considered another technique used for solving BVPs. The power and strength of the DTM is to solve the non-linear differential equation for $\frac{-\pi}{4} \leq \beta \leq \frac{3\pi}{4}$ and is discussed in calculative portion [10].

2. Mathematical formulation

The two dimensional laminar boundary layer flow over a flat plate has been considered. The steady of accretion at leading edge is caused by the unsteadiness of the fluid. The x-axis is taken vertically upwards to the direction of the free stream where y-axis would be perpendicular the free stream, U_∞ as the uniform free stream velocity, a magnetic field B_0. The governing equation of momentum of this problem [4] is given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}B_0^2 u \tag{2.2}$$

with boundary condition

$$u(x,0)=0, \qquad v(x,0)=0, \qquad u(x,\infty)=U_\infty, \tag{2.3}$$

where u and v are velocity components in the x and y directions respectively, t is the time variable, ν is the kinematic viscosity, σ is the thermal diffusivity and ρ is the density.

The stream function $\psi(x,y)$ is defined as

$$\psi(x,y,t)=U_\infty f(\eta)\sqrt{(cos\beta)\gamma t+(sin\beta)\frac{\nu_x}{U_\infty}}$$

such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, with a similarity variable

$$\eta=\frac{y}{\sqrt{(cos\beta)\gamma t+(sin\beta)\frac{\nu x}{U_\infty}}}$$

that satisfies the continuity equation (2.1) which is also called Blasius - Rayleigh Stokes variable by Todd.

$$\psi(x,y,t)=U_\infty\left(\nu t\,cos(\beta)+\frac{\nu x}{U_\infty}sin(\beta)\right)^{\frac{1}{2}}f(\eta)$$

$$u=\frac{\partial\psi}{\partial y}=U_\infty\, f'(\eta)$$

$$v=\frac{-\partial\psi}{\partial x}=\frac{\nu}{2}\,sin(\beta)\,\left(\nu t\,cos(\beta)+\frac{\nu x}{U_\infty}sin(\beta)\right)^{\frac{1}{2}}\,(\eta f'(\eta)-f(\eta))$$

Using the values of u and v by solving we get the Equation of Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Governing equations transformation:

$$(2.2) \Rightarrow \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}$$

$$u = U_\infty f'(\eta), \quad v = \frac{\left(\frac{\nu}{2}\right)(\nu f' - f)(sin\beta)}{\sqrt{(cos\beta)\gamma t + (sin\beta)\left(\frac{\nu_x}{U_\infty}\right)}}$$

Differentiating the velocity components and substituting in (2.2), we get

$$0 = f'''(\eta) + \frac{1}{2}\, cos(\alpha)\, \eta\, f''(\eta) + \frac{1}{2}\, sin(\alpha)\, f(\eta)\, f''(\eta) - Mf'$$

where $M = \dfrac{\sigma B_0^2}{\rho}$ and β is the unsteady parameter of leading edge. The momentum equation (2.2) can be transformed into a similarity equation as

$$f''' + \frac{1}{2}(cos\beta)\eta f'' + \frac{1}{2}(sin\beta)ff'' - Mf' = 0 \tag{2.4}$$

The boundary conditions (2.3) becomes

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1.$$

3. Differential transformation method

Let $u(x)$ is a continuously differentiable function in domain D. Suppose that $u(x)$ is differentiable with respect to $x_0 \in D$, then the Taylor series expansion of $u(x)$ is of the form:

$$u(x) = \sum_{k=0}^{\infty} \frac{1}{k!}\frac{d^k u(x_0)}{dx^k}(x - x_0)^k. \tag{3.1}$$

The differential operator of order k is denoted by

$$DT = \frac{1}{k!}\frac{d^k}{dx^k}$$

The differential transformation of $u(x_0)$ of order k is denoted by

$$U(k) = DT\,(u(x_0))$$

$$U(k) = \frac{1}{k!}\frac{d^k u(x_0)}{dx^k} \tag{3.2}$$

from (3.1) and (3.2)

$$u(x) = \sum_{k=0}^{\infty} U(k)(x - x_0)^k$$

which is called inverse differential transformation of $U(k)$.

(1) If $z(x) = u(x) \pm v(x)$, then $Z(k) = V(k) \pm U(k)$.
(2) If $z(x) = \alpha\, u(x)$, then $Z(k) = \alpha\, U(k)$.
(3) If $z(x) = u'(x)$, then $Z(k) = (k+1)U(k+1)$.
(4) If $z(x) = u''(x)$, then $Z(k) = (k+1)(k+2)U(k+2)$.
(5) If $z(x) = u^m(x)$, then $Z(k) = (k+1)(k+2)\dots(k+m)U(k+m)$.
(6) If $z(x) = u(x)\, v(x)$, then $Z(k) = \sum_{l=0}^{k} U(l)\, V(k-l)$.
(7) If $z(x) = \alpha\, x^m$, then $Z(k) = \alpha\, \delta(k-m)$.
(8) If $z(x) = cos(\alpha x + \beta)$, then $Z(k) = \dfrac{\alpha^k}{k!} cos(\frac{k\pi}{2} + \beta)$.
(9) If $z(x) = sin(\alpha x + \beta)$, then $Z(k) = \dfrac{\alpha^k}{k!} sin(\frac{k\pi}{2} + \beta)$.

4. RESULTS AND DISCUSSION

Applying the fundamental properties of Differential Transformation Method for (2.4), discussed in (3) we have:

$$F(k+3) = \frac{-1}{(k+1)(k+2)(k+3)} \left(\frac{1}{2} \sum_{m=0}^{k} (cos\,\beta)\, \delta(k-m-1)\, (m+1)\, (m+2)\, F(m+2) \right.$$
$$\left. + \frac{1}{2}\, (sin\,\beta) \sum_{m=0}^{k} F(k-m)\, (m+1)\, (m+2)\, F(m+2) - M\, (k+1)\, F(k+1) \right)$$

The boundary condition at $f'(\infty) = 1$ is replaced with a free boundary condition as Asaithambi did in [1]:

$$f'(\eta) = 1,\ f''(\eta) = 0$$

at $\eta = \eta_\infty$ in which η_∞ is the unknown free boundary condition (truncated boundary). Then the original problem [1] becomes the free boundary problem [10] defined on a finite interval, where η_∞ is to be determined as a part of the solution. Then, a shooting algorithm is used to find the solution of the boundary value problem. We add another initial value condition $f''(0) = a$ and

$$
\begin{aligned}
F(0) &= 0\\
F(1) &= 0\\
F(2) &= a\\
F(3) &= 0\\
F(4) &= \frac{1}{12}Ma - \frac{1}{24}a\cos\beta\\
F(5) &= -\frac{1}{60}a^2\sin\beta\\
F(6) &= -\frac{1}{180}Ma\cos\beta + \frac{1}{480}a(\cos\beta)^2 + \frac{1}{360}M^2a
\end{aligned}
$$

$$
\begin{aligned}
F(7) &= \frac{11}{5040}a^2\cos\beta\sin\beta - \frac{1}{315}Ma^2\sin\beta\\
F(8) &= \frac{23}{80640}Ma(\cos\beta)^2 - \frac{1}{10752}a(\cos\beta)^3 - \frac{1}{4480}M^2a\cos\beta + \frac{11}{20160}a^3(\sin\beta)^2\\
&\quad + \frac{1}{20160}M^3a\\
F(9) &= -\frac{43}{241920}a^2(\cos\beta)^2\sin\beta + \frac{17}{40320}a^2\cos\beta\sin\beta - \frac{13}{60480}M^2a^2\sin\beta\\
F(10) &= -\frac{11}{907200}Ma(\cos\beta)^3 + \frac{1}{276480}a(\cos\beta)^4 + \frac{43}{3628800}M^2a(\cos\beta)\\
&\quad - \frac{5}{48384}a^3\cos\beta\sin\beta - \frac{1}{226800}M^3a\cos\beta + \frac{3}{22400}Ma^3(\sin\beta)^2\\
&\quad + \frac{1}{1814400}M^4a
\end{aligned}
$$

The above process is continuous, and by substituting the values in Taylor's series

$$f(\eta) = \sum_{i=0}^{\infty} F(i)\cdot\eta^i$$

there forms a series like

(4.1)

$$
\begin{aligned}
f(\eta) &= a\eta^2 + \left(\frac{1}{12}Ma - \frac{1}{24}a\cos\beta\right)\eta^4 - \frac{1}{60}a^2\sin\beta\eta^5 + \left(-\frac{1}{180}Ma\cos\beta + \frac{1}{480}a(\cos\beta)^2\right.\\
&\quad \left.+\frac{1}{360}M^2a\right)\eta^6 + \left(\frac{11}{5040}a^2\cos\beta\sin\beta - \frac{1}{315}Ma^2\sin\beta\right)\eta^7 + \left(\frac{23}{80640}Ma(\cos\beta)^2\right.
\end{aligned}
$$

$$-\frac{1}{10752}a(\cos\beta)^3-\frac{1}{4480}M^2a\cos\beta+\frac{11}{20160}a^3(\sin\beta)^2+\frac{1}{20160}M^3a\Big)\,\eta^8$$
$$+\left(\frac{43}{241920}a^2(\cos\beta)^2\sin\beta+\frac{17}{40320}a^2\cos\beta\sin\beta-\frac{13}{60480}M^2a^2\sin\beta\right)\eta^9$$
$$+\Big(\frac{11}{907200}Ma(\cos\beta)^3+\frac{1}{276480}a(\cos\beta)^4+\frac{43}{3628800}M^2a(\cos\beta)$$
$$-\frac{5}{48384}a^3\cos\beta\sin\beta-\frac{1}{226800}M^3a\cos\beta+\frac{3}{22400}Ma^3(\sin\beta)^2+\frac{1}{1814400}M^4a\Big)$$

and by substituting the boundary condition into $f(\eta)$ equation (4.1) in point $\eta=5$, the values of a can be found. Then substituting the value of a in equation (4.1), the expression of $f(\eta)$ can be obtained. As seen in terms of above equation, although for increasing the accuracy, the number of statements has been increased and convergence of DTM is completely evident.

The objective of the present study was to apply the differential transformation method to obtain an explicit analytic solution of heat transfer equation of a non-Newtonian fluid flow. The effect of transverse magnetic field $(M\neq 0)$, is easily noticeable in permanent basis the velocity profiles to push over to far away asymptotic conditions. The result that obtained by the differential transform method was well matched with the result carried out by the numerical solutions obtained by an implicit finite difference scheme for different values of β for $M=0$ and 0.25.

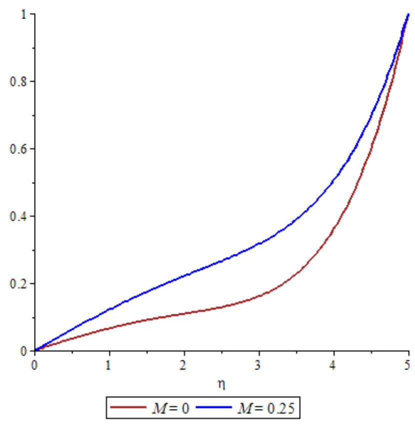

FIGURE 1. Velocity profile for β =0 when $M=0$ and $M=0.25$

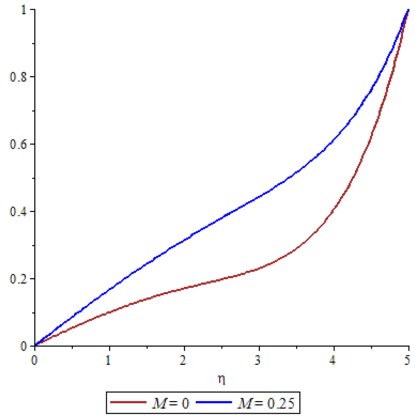

FIGURE 2. Velocity profile for β =0 when $M = 0$ and $M = 0.25$

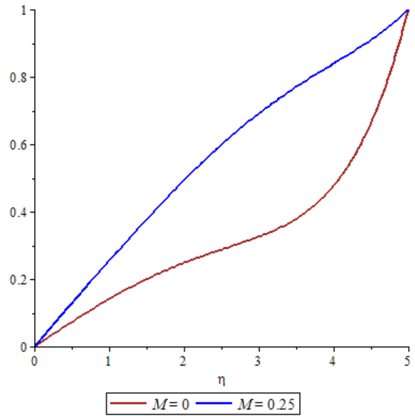

FIGURE 3. Velocity profile for β =0 when $M = 0$ and $M = 0.25$

5. CONCLUSION

The objective of the present study was to apply the differential transformation method to obtain an explicit analytic solution of heat transfer equation of a non-Newtonian fluid flow. The effect of transverse magnetic field $(M \neq 0)$, is easily noticeable in permanent basis the velocity profiles to push over to far away asymptotic conditions. In this paper, it was shown that the leading edge accretion has greatly affected the heat transfer characteristics with the boundary conditions when applied the differential transformation method, which brings the good convergent rate and accuracy of the exact solution through the Taylor series method.

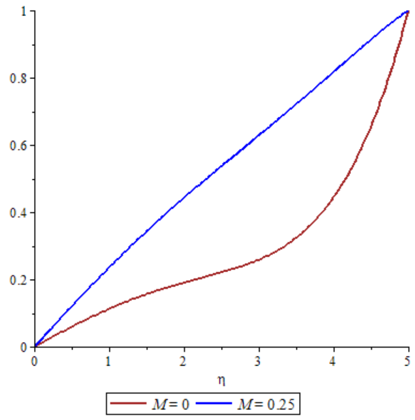

FIGURE 4. Velocity profile for β =0 when $M = 0$ and $M = 0.25$

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