NUMERICAL ANALYSIS USING RK - 4 IN TRANSIENT ANALYSIS OF RLC CIRCUIT

M. MALARVIZHI [1], S. KARUNANITHI, AND N. GAJALAKSHMI

ABSTRACT. In this paper, the transient analysis of an RLC circuit (Resistance, Inductance and Capacitance) is analyzed using the second order Runge-Kutta method of order 4. Kirchhoff's voltage and current laws are used to generate differential equations of second order. Second order differential equation is linearized into the first order differential equation using the convolution method. We derive the voltage values using the RK-4 method. The electric current, the voltage of the RLC circuit and the line graph arrive between the frequency and the following
  (a) Current Magnitude
  (b) Current phase angle (both in degrees and in radians)
  (c) Comparison between real and imaginary values
using MATLAB to identify changes in the circuit. The characterization of the damping factor is also analyzed to determine the accuracy. The simulation result shows that RK-4 provides the best approximation and high precision during steady-state values that change from one state to another.

1. INTRODUCTION

Numerical analysis obviously discovers application in all fields of engineering and the physical sciences, social sciences, medicine, business and all kinds of

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scientific calculations. However, for practical purposes, as in engineering, numerical approximations to the solutions are usually sufficient [5]. Numerical methods for solving second-order IVP's are classified into two major categories: multi-phase method or single-phase method. Further division can be achieved by dividing the methods into the explicit and implicit ones. For second-order ODE's, and all higher-order ODE's can be transformed and reduced into first-order ODE's [4]. Numerical methods are one of the best techniques for solving almost all mathematical equations that cannot be solved analytically [6].

Transient is the sudden explosion of energy in an electrical circuit which can damage some components of the circuit [1]. The transient normally produces changes in the state of the components of an electrical circuit. It is difficult for the voltage of the capacitor and the current of the inductor in an electrical circuit to assume a new steady state value. Therefore, transient analysis is used to establish how the capacitor voltage and the inductor current evolve over the period of time.

Transient analysis can be described through the variable defining the state of a system does not vary over time, the system is said to be in a stable state. Thus, for an electrical current or voltage flowing through an electrical circuit, there can be various forms of the voltage and current. For instance, considering circuits which are time-varying signals with resistive circuits, the resulting Kirchhoff's current law and Kirchhoff's voltage law are normally in the form of differential equations rather than algebraic equations. But these differential equations are not easily solved analytically when the order is high and complex. These equations are converted into ordinary differential equations by differentiation from time. Therefore, the analysis of the transient in an RLC circuit can be addressed numerically. Numerical techniques, i.e., the Runge - Kutta method, are utilized to analyze transient behaviors in an RLC circuit taking into account through damping factor and the variation of voltage with respect to time and determine the precision [2].

This paper is organized as follows: In Section 2, we discuss the current Kirchhoff law and the voltage law for the formation of second-order differential equations. In Section 3, we linearized the second order differential equation. Damping factor values are also obtained. In section 4, the first order linearized differential equations are solved using the RK method of order 4. The simulation results were performed in section 5 and section 6 concludes the paper.

2. Kirchhoff's voltage and current law

In general Kirchhoff's law is used to find current and voltage. There are two generalized laws: (i) Kirchhoff's current law (ii) Kirchhoff's voltage law.

Kirchhoff's current law states that the algebraic sum of the current gathering at any junction in a circuit is zero.

$$I_{l_1} + (-I_{l_2}) + (-I_{l_3}) + I_{l_4} + I_{l_5} = 0 \ \text{ or } \ I_{l_1} + I_{l_4} + I_{l_5} = I_{l_2} + I_{l_3}$$

Clockwise current is considered positive and counterclockwise current is considered negative.

3. RLC circuit

Analyzing the RLC circuit is a simple task; unfortunately the analysis is still difficult. Although we considered a second-order system, most of our attention was directed to the first-order system described by a first-order linear differential equation. Furthermore, it will be necessary to determine the initial conditions for derivatives. Finally, we will see that the presence of inductance and capacitance in the same circuit leads to a response that assumes different functional forms for circuits that have same configuration but values of different elements. So we quickly review the methods and results we found useful for first-order systems, so that we can extend this information as intelligently as possible to the second-order system.

In general RLC circuit consists of a capacitor, an inductor and a resistance. This circuit forms a harmonic oscillator for the current that extinguishes over time with the presence of the resistance. That is, the presence of resistance reduces the resonance frequency. The RLC filter is normally observed as a second order circuit [3] and let

$$I_l(t) = C_1 \frac{dvc(t)}{dt}$$

where C_1 is the capacitance and $V_{C_1}(t)$ is the voltage, which is expressed in equation (3.1) as:

$$(3.1) \qquad\qquad L\frac{dI_l(t)}{dt} + RI_l(t) + V_{C_1}(t) = V_{in}$$

where V_{in} is the input voltage. Substituting $I_l(t)$ in equation (3.1), we get

$$LC_1\frac{d^2v_{C_1}(t)}{dt^2} + RC_1\frac{dV_{C_1}(t)}{dt} + V_{C_1}(t) = V_{in}$$

Again, for a given RLC circuit, parameters such as, α and ω_0 are written in the form of equations as

$$\alpha_i = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC_1}}$$

where ω_0 is the natural frequency. Moreover, there is a useful parameter in an RLC circuit, called the damping factor. This damping factor, ς is estimated as

$$\eta = \frac{\alpha_i}{\omega_0}$$

Suppose the given RLC circuit is in series then the damping factor is estimated by

$$\eta = \frac{R}{2}\sqrt{\frac{C_1}{L}}$$

Transient response or the type exhibited by a circuit depends on the value of the damping factor and the amount with which the oscillation of a system gradually decreases over time [1]. Various characterizations of the damping factor are given below:

    i. If $\eta > 1$, called as over damped.
    ii. If $\eta = 1$, called as critically damped.
    iii. If $\eta < 1$, called as under damped.

Since the second order RLC circuit [2] is given by equation (3.1). But

(3.2)
$$C_1\frac{dv_{C_1}(t)}{dt} = 0$$

Thus, substituting Equation (3.2) into Equation (3.1) yields

$$L\frac{dI_l(t)}{dt} + RI_l(t) + V_{C_1}(t) = V_{in}$$

$$\frac{dI_l(t)}{dt} = \frac{V_{in} - RI_l(t) - V_{C_1}(T)}{I}$$

Now let $I_l(t) = x_1$ and $V_{C_1}(t) = x_2 l$, this leads to equation (3.3).

(3.3)
$$\frac{dI(t)}{dt} = \frac{V_{in} - R(x_1 + x_2)}{L} = g(t, x_1, x_2)$$

$$\frac{x_1}{c} = \frac{dv_c(t)}{dt} = f(t, x_1, x_2)$$

Formulating Runge-Kutta 4th order method for the RLC circuit;

$f_1 = h * f(t, x_1, x_2)$

$g_1 = h * g(t, x_1, x_2)$

$f_2 = hf[(t + h/2), (x_1 + f_1/2), (x_2 + g_1/2)]$

$g_2 = hf[(t + h/2), (x_1 + f_1/2), (x_2 + g_1/2)]$

$f_3 = hf[(t + h/2), (x_1 + f_2/2), (x_2 + g_2/2)]$

$g_3 = hf[(t + h/2), (x_1 + f_2/2), (x_2 + g_2/2)]$

$f_4 = hf[(t + h), (x_1 + f_3), (x_1 + g_3)]$

$g_4 = hg[(t + h), (x_1 + f_3), (x_1 + g_3)]$

h represents the step size.

4. MAGNITUDE, PHASE ANGLE AND REAL & IMAGINARY VALUES

In this section we allow an alternate emf source to be connected to a series combination of a resistor R, an inductor of inductance L and a capacitor of capacitance C_1. Let the current following through the circuit be I_1.

The voltage drop across the resistor is $V_R = I_1 R$ (This is phase with I).

The voltage across the inductor coil is $V_L = I_1 X_L$ (V_L leads 1 by $\pi/2$).

The voltage across the capacitor is $V_{C_1} = I_1 X_{C_1}$ (V_{C_1} lags behind 1 by $\pi/2$).

V_L and V_{C_1} are 180^0. Out of phase with each other and the resultant of V_L and V_{C_1} is $(V_L - V_{C_1})$, assuming the circuit to be predominantly inductive. The applied voltage V equals the vector sum of V_R, V_L and V_{C_1} is: $\qquad OB^2 = OA^2 + AB^2$

$$V^2 = V_R^2 + (V_L - V_{C_1})^2$$
$$V = \sqrt{V_R^2 + (V_L - V_{C_1})^2}$$
$$V = \sqrt{(I_1 R)^2 + (I_1 X_L - I_1 X_{C_1})^2}$$
$$V = I_1 \sqrt{R^2 + (X_L - X_{C_1})^2}$$
$$\frac{V}{I_1} = Z = \sqrt{R^2 + (X_L - X_{C_1})^2}$$

Now calculate the phase angle ϕ as:

$$\tan \phi = \frac{V_L - V_{C_1}}{V_R} = \frac{IX_L - IX_{C_1}}{IR}$$
$$\tan \phi = \frac{X_L - X_{C_1}}{R} = \frac{net\ reactance}{resistance}$$

$$\phi = \tan^{-1}(\frac{X_L - X_{C_1}}{R})$$

$I_0 \sin(\omega t \pm \phi)$ is the instantaneous current that follows from the circuit.

5. Runge - Kutta method

The Runge - Kutta method is the popular method because it is fairly accurate, stable and easy to program. It does not require major derivatives of $y(x)$ as in the Taylor's series method. The fourth order Runge-Kutta (RK4) for the ordinary differential equation is commonly used to solve initial value problems. The general formula for the Runge-Kutta approach is

$$y_{n+1}(x) = y_n(x) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), n = 0, 1, 2, 3, ...$$
$$k_1 = hf(x_n, y_n)$$
$$k_3 = hf(x_n + 1/2h, y_n + 1/2k_2)$$
$$k_2 = hf(x_n + 1/2h, y_n + 1/2k_1)$$
$$k_4 = hf(x_n + h, y_n + k_3)$$

One of the most popular methods for the numerical solution of differential equations is that generated by Runge and developed by Heun, Kutta, Nystrom, and others. Throughout numerical methods, this approach is usually given significant prominence. Simulations results are shown in Figure 1 - Figure 6.

6. Conclusion

The transient analysis of the RLC circuit is analyzed by the exact method and RK-4 method. The process of obtaining the results of the transient analysis is performed through MATLAB. Voltage and current values are arrived using second order RK-4 method. Frequency, current phase angle, current magnitude, Degrees and Real and Imaginary values are simulated through linear graph between current and frequency, Log based linear graph between current and frequency and Graph between Real and Imaginary values. Finally the voltage and Time graph (over-damped) are also discussed in this paper. Analysis reveals that, the RK-4 method would be efficient and the most excellent numerical method for solving higher orders differential equations. Thus, the RK method

of 4th order is recommended for transient analysis of complex electrical circuits since it is more accurate.

7. TABLES AND FIGURES

TABLE 1. Shows the values of Voltage and Current for Over-Damping factor using RK-4 Method

Time	Voltage	Current
0.00000	0.000000	0.000000
0.00100	4.531250	0.027344
0.00200	8.192139	0.025706
0.00300	10.717279	0.018636
0.00400	12.350733	0.012320
0.00500	13.375623	0.007813
0.00600	14.008749	0.004853
0.00700	14.396663	0.002982
0.00800	14.633287	0.001822
0.00900	14.777281	0.001110
0.01000	14.564791	0.000675

TABLE 2. Shows the values of frequency, phase angle & magnitude, degrees, real and imaginary values

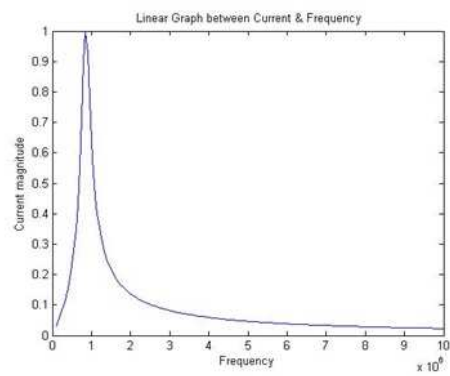| Frequency	Current Phase Angle & Magnitude	Hz Degrees	Real and Imaginary values
100000	(0.0313766 +0.000985465)	88.2011	0.0313921
150000	(0.0478363+0.00229357)	87.255	0.047891
200000	(0.0652742+0.00427903	86.2494	0.0654143
250000	(0.0841107+0.00712538)	85.1578	0.0844119
300000	(0.104861+0.0111194)	83.947	0.1054493
35000	(0.128177+0.0167084)	82.5731	0.129261
400000	(0.154907+0.0246015)	80.976	0.156849
450000	(0.186176+0.035541)	79.0697	0.189616
500000	(0.223469+0.0527174)	76.7263	0.229603
550000	(0.268688+0.0783283)	73.7474	0.279872+

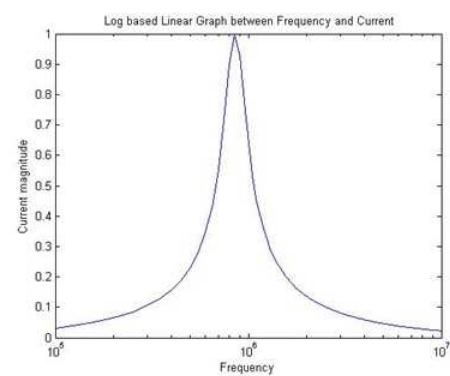FIGURE 1. Linear Graph between current Vs frequency

FIGURE 2. Shows the Log based linear Graph between Frequency and current magnitude
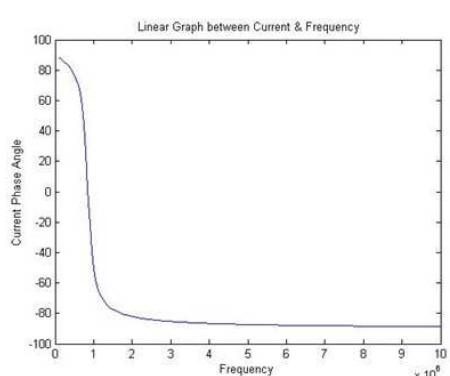

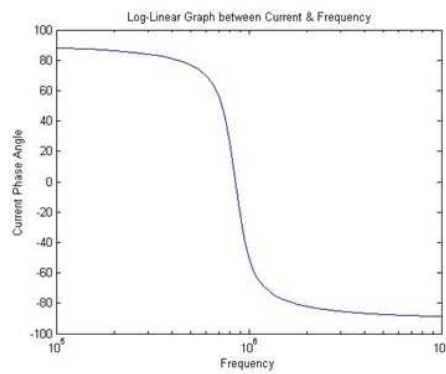FIGURE 3. Frequency Vs Current Phase Angle

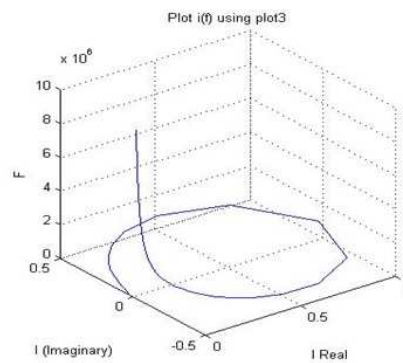FIGURE 4. Log linear graph between current and Frequency

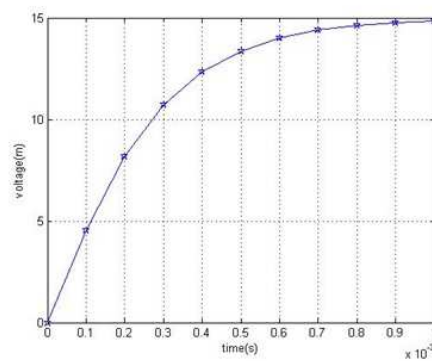FIGURE 5. Shows the difference between the real and Imaginary values

FIGURE 6. Graph between voltage and Time for over-damped

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PG & RESEARCH DEPARTMENT OF MATHEMATICS
GOVERNMENT THIRUMAGAL MILLS COLLEGE
TAMIL NADU, INDIA.
Email address: malar2308@gmail.com

PG & RESEARCH DEPARTMENT OF MATHEMATICS
GOVERNMENT THIRUMAGAL MILLS COLLEGE
TAMIL NADU, INDIA.
Email address: kap232008@gmail.com

PG & RESEARCH DEPARTMENT OF MATHEMATICS
GOVERNMENT THIRUMAGAL MILLS COLLEGE
TAMIL NADU, INDIA.
Email address: n.gajalakshmi@rocketmail.com