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AN APPLICATION OF GENERALIZED DISTRIBUTION INVOLVING MODIFIED MITTAG-LEFFLER FUNCTION

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ABSTRACT. The purpose of present paper is to obtain new class, which was defined by convolution of derivative operator involving generalized Mittag Leffler function and the polynomial whose coefficients are generalized distribution and we obtain the bounds for the new defined class. The results further established geometric properties of the generalized distribution associated with univalent functions.

1. INTRODUCTION

Let us denote by A the class of functions

$$f(z) = z + a_2\, z^2 + \dots \tag{1.1}$$

which is analytic in the punctured disk $U = \{z \in C \ : \ 0 < |z| < 1\}$. For $g \in A$, given by

$$g(z) = z + b_2\, z^2 + \dots . \tag{1.2}$$

The convolution product of the functions f and g is given by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n\, z^n. \tag{1.3}$$

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We say that f is subordinate to g in U and $f \prec g$, if there exists a Schwarz function $\omega \in A$ with $|\omega(z)| < 1$ and $\omega(0) = 0$ such that $f(z) = g(\omega(z))$ in U. Recently, Porwal in [15] investigated generalized distribution and its geometric properties associated with univalent functions.

Let S denote the sum of the convergent series of the form

$$S = \sum_{k=0}^{\infty} a_n, \text{where } a_n \geq 0 \text{ for all } n \in N.$$

The generalized discrete probability distribution , whose probability mass function is given as

$$p(n) = \frac{a_n}{S}, n = 0, 1, 2, \ldots,$$

where $p(n)$ is the probability mass function, because $p(n) \geq 0$ and $\sum_n p(n) = 0$. Furthermore, let

$$\psi(x) = \sum_{n=0}^{\infty} a_n b^n.$$

Then from $S = \sum\limits_{k=0}^{\infty} a_n$, ψ is convergent for both $|x| < 1$ and $x = 1$.

Definition 1.1. *[1], For special values of an various well known discrete probability distribution such as Yule-Simon distribution, binomial distribution, Poisson distribution, logarithmic distribution, Bernoulli distribution can be obtained of particular interest is the polynomial whose coefficients are probabilities of the generalized distribution investigated in [11, 14] and it is defined by*

$$k_{\psi}(z) = z + \sum_{n=2}^{\infty} \frac{a_{n-1}}{s} z^n \tag{1.4}$$

where $S = \sum\limits_{k=0}^{\infty} a_n$.

Definition 1.2. *[12], For* $f \in A$, *the generalized Mittag-Leffler function interm of the extensively investigated Fox-Wright* ${}_p\psi_q$ *function* $\mathcal{M}^m_{\alpha,\beta,\lambda} f(z)$ *and generalized polylogarithm functions* $D^m_\lambda f(z)$ *is defined by* $\mathcal{M}^m_{\alpha,\beta,\lambda} f(z)\colon A \to A$

$$\mathcal{M}^m_{\alpha,\beta,\lambda} f(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\beta)\, n^m\, (n+\lambda-1)}{\Gamma(\alpha(n-1)+\beta)\lambda\,!(n-1)\,!} z^n \tag{1.5}$$

where $m, \lambda \in \mathbb{N} \cup \{0\}$ *and* $min\ Re\{(\alpha), Re(\beta)\} > 0$ $z \in U$.

In the present paper we introduced a new generalized derivative operator $\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ f(z)\ *\mathcal{K}_{\psi}(z)$ obtained by convolution between the generalized distribution given in (1.4), and the linear operator of the generalized Mittag-Leffler function given in (1.5), defined a follows:

$$\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ *\mathcal{K}_{\psi}(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\beta)\ n^{m}\ (n+\lambda-1)}{\Gamma(\alpha(\ n-1)+\beta)\lambda\ !(n-1)\ !} \frac{a_{n-1}}{s}\ z^{n},$$

where $m, \lambda \in \mathbb{N} \cup \{0\}$ and $min\ Re\{(\alpha), Re(\beta)\} > 0$ and $S = \sum\limits_{k=0}^{\infty} a_n\ z \in U, b \neq 0$. Let P denote the analytic functions and $p \in P$ with $Re\quad p(z) > 0$ and $p\ _{k,\gamma} = 1 + p_1\ z + \cdots \in U$ where the function $p\ _{k,\gamma}$ maps the unit disk conformally onto the region $\Omega\ _{k,\gamma}$, which was studied extensively by Kharasani in [2], then we have:

$$\partial\Omega\ _{k,\gamma} = \{u +\ iv : (u-\gamma)^2 >\ k^2(u-1)^2 +\ k^2v^2\}.$$

The details of the functions in conic region can be found in the literatures [3,5–9].

Definition 1.3. *Let* $k \in [0,\infty), \gamma \in [0,1),\ m, \lambda \in \mathbb{N} \cup \{0\}, min\ Re\{(\alpha), Re(\beta)\} > 0$ *, the function* $f \in A$ *is in the class* $\psi S^{m}_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *if*

$$1 + \frac{1}{b}\left(\frac{z\left(\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ *\mathcal{K}_{\psi}(z)\right)'}{\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ *\mathcal{K}_{\psi}(z)} - 1\right) \prec p\ _{k,\gamma},\ (z \in U).$$

Let $\phi(z) = 1 + c_1 z + c_2 z^2 + \ldots\ (c_1 > 0)$ be an analytic function with positive real part on U which maps the open unit disk U onto a region starlike with respect to 1 and symmetric with respect to the real axis. Let $\varphi(z) = d_0 + d_1 z + d_2 z^2 + \ldots$ and $|d_n| \leq 1$ and $f \in A$ is in the class $\tilde{\psi} S^{m}_{\alpha,\beta,\lambda}(b, \phi)$

$$\frac{1}{b}\left(\frac{z\left(\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ *\mathcal{K}_{\psi}(z)\right)'}{\mathcal{M}^{m}_{\alpha,\beta,\lambda}\ *\mathcal{K}_{\psi}(z)} - 1\right) \prec_q (\phi(z)-1),\ (z \in U).$$

Lemma 1.1. *Let* $\omega(z) = \omega_1 z + \omega_1 z^2 + \cdots \in \Omega$ *so that* $|\omega(z)| < 1$ *in* U*. If* ρ *is a complex number, then*

$$|\omega_2 + t\omega_1^2| \leq max(1, |\rho|)$$

The result is sharp for $\omega(z) = z$ *or* $\omega(z) = z^2$*.*

Motivated by the works in [4, 10, 12, 13, 16], we investigate bounds for the class defined in conic domain. The sharp bounds of the coefficients is the information about geometric properties of the functions. For instance, the sharp bounds of the second coefficient of normalized univalent functions readily yields the growth and distortion bounds. Also, sharp bounds of the coefficients functional $|a_2 - \mu a_1^2|$ helps in the investigation of univalence of analytic functions.

2. Main Results

In this section we obtained the first few coefficients bounds for the class defined in definition 1.3 is considered.

Theorem 2.1. *Let* $k \in [0, \infty), \gamma \in [0, 1),\ m, \lambda \in \mathbb{N} \cup \{0\}, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *, the function* f *is given by* (1.1) *belongs to* $\psi S^m_{\alpha,\beta,\lambda}(b, p_{\ k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(\alpha+\beta)}{2^m\Gamma(\beta)(\lambda+1)}$$

and

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)} \max\left\{1, \left|\frac{p_2}{p_1} + bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1) - \mu 3^m[\Gamma(\alpha+\beta)]^2(\lambda+2)}{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1)}\ bp_1\right|\right\}$$

Proof. $f \in \psi S^m_{\alpha,\beta,\lambda}(b, p_{\ k,\gamma})$ and $\omega \in \Omega$ such that

$$1 + \frac{1}{b}\left(\frac{z\left(\mathcal{M}^m_{\alpha,\beta,\lambda} * \mathcal{K}_\psi(z)\right)'}{\mathcal{M}^m_{\alpha,\beta,\lambda} * \mathcal{K}_\psi(z)} - 1\right) \prec p_{\ k,\gamma}\left(\omega(z)\right). \tag{2.1}$$

It is observed that

$$\frac{2^m\Gamma(\beta)(\lambda+1)}{\Gamma(\alpha+\beta)}\frac{a_1}{S}\ z^2 + \frac{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)}{\Gamma(2\alpha+\beta)}\frac{a_2}{S}\ z^3 + \dots$$
$$= b\left[p_1\omega_1 z^2 + p_1\omega_2 Z^3 + p_2\omega_1^2 z^3 + \dots + \frac{2^m\Gamma(\beta)(\lambda+1)}{\Gamma(\alpha+\beta)}p_1\omega_1\frac{a_1}{S}z^3 + \dots\right].$$

Hence we get

$$\frac{a_1}{S} \leq \frac{bp_1\omega_1\Gamma(\alpha+\beta)}{2^m\Gamma(\beta)(\lambda+1)} \tag{2.2}$$

$$\frac{a_2}{S} \leq \frac{bp_1\omega_1\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)}\left(\omega_2+\left(\frac{p_2}{p_1}+bp_1\right)\omega_1^2\right) \tag{2.3}$$

by (2.2) and (2.3), we have

$$\frac{a_2}{S} - \mu\frac{a_1^2}{S^2} \leq \frac{bp_1\omega_1 S\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)}\left(\omega_2+t\omega_1^2\right)$$

where

$$\rho = \frac{p_2}{p_1} + \frac{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1) - \mu 3^m[\Gamma(\alpha+\beta)]^2(\lambda+2)}{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1)}\, bp_1$$

The desired result is obtained by applying Lemma 1.1. □

Corollary 2.1. *Let* $k \in [0,\infty), \gamma \in [0,1),\ m = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.1 the function* f *is belongs to* $\psi S^m_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(\alpha+\beta)}{\Gamma(\beta)(\lambda+1)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{\Gamma(\beta)(\lambda+1)(\lambda+2)} \max\left\{1, \left|\frac{p_2}{p_1}+bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{\Gamma(\beta)(\lambda+1)(\lambda+2)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1) - \mu[\Gamma(\alpha+\beta)]^2(\lambda+2)}{\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1)}\, bp_1\right|\right\}.$$

Corollary 2.2. *Let* $k \in [0,\infty), \gamma \in [0,1), \lambda = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.1 the function* f *is belongs to* $\psi S^m_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(\alpha+\beta)}{2^m\Gamma(\beta)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2\times 3^m\Gamma(\beta)} \max\left\{1, \left|\frac{p_2}{p_1}+bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2\times 3^m\Gamma(\beta)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta) - 2\times\mu 3^m[\Gamma(\alpha+\beta)]^2}{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)}\, bp_1\right|\right\}.$$

Corollary 2.3. *Let* $k \in [0,\infty), \gamma \in [0,1), m = 0, \lambda = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.1 the function* f *is belongs to* $\psi S^{m}_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(\alpha+\beta)}{\Gamma(\beta)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2\Gamma(\beta)} \max\left\{1, \left|\frac{p_2}{p_1} + bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2\Gamma(\beta)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{\Gamma(2\alpha+\beta)\Gamma(\beta) - 2\mu[\Gamma(\alpha+\beta)]^2}{\Gamma(2\alpha+\beta)\Gamma(\beta)}\ bp_1\right|\right\}.$$

Corollary 2.4. *Let* $k \in [0,\infty), \gamma \in [0,1), m = 1, \lambda = 1, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.1 the function* f *is belongs to* $\psi S^{m}_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(\alpha+\beta)}{4 \times \Gamma(\beta)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2 \times 3^2\Gamma(\beta)} \max\left\{1, \left|\frac{p_2}{p_1} + bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1\Gamma(2\alpha+\beta)}{2 \times 3^2\Gamma(\beta)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{2^3 \times \Gamma(2\alpha+\beta)\Gamma(\beta) - 3^2 \times \mu[\Gamma(\alpha+\beta)]^2}{2^3 \times \Gamma(2\alpha+\beta)\Gamma(\beta)}\ bp_1\right|\right\}.$$

Corollary 2.5. *Let* $k \in [0,\infty), \gamma \in [0,1),\ m, \lambda \in \mathbb{N} \cup \{0\}, \alpha = 1/2, \beta = 1$ *in Theorem 2.1, the function* f *is belongs to* $\psi S^{m}_{\alpha,\beta,\lambda}(b, p\ _{k,\gamma})$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|p_1\Gamma(3/2)}{2^m(\lambda+1)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|p_1}{3^m(\lambda+1)(\lambda+2)} \max\left\{1, \left|\frac{p_2}{p_1} + bp_1\right|\right\}$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|p_1}{3^m(\lambda+1)(\lambda+2)} \times \max\left\{1, \left|\frac{p_2}{p_1} + \frac{2^{2m}(\lambda+1) - \mu 3^m[\Gamma(3/2)]^2(\lambda+2)}{2^{2m}(\lambda+1)}\ bp_1\right|\right\}.$$

Theorem 2.2. *Let* $k \in [0,\infty), \gamma \in [0,1),\ m, \lambda \in \mathbb{N} \cup \{0\}, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *, the function* f *is given by* (1.1) *belongs to* $\tilde{\psi} S^m_{\alpha,\beta,\lambda}(b,\phi)$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|c_1\Gamma(\alpha+\beta)}{2^m\Gamma(\beta)(\lambda+1)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)}\left(c_1 + max\left\{c_1, |bc_1^2| + |c_2|\right\}\right)$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{3^m\Gamma(\beta)(\lambda+1)(\lambda+2)}$$
$$\cdot\left(c_1, +max\left\{c_1, \left|\frac{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1) - \mu 3^m[\Gamma(\alpha+\beta)]^2(\lambda+2)}{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1)}\right| |bc_1^2| + |c_2|\right\}\right).$$

Proof. The method of proof is similar to that Theorem 2.1 except that instead using (2.1) we make use of

$$\frac{1}{b}\left(\frac{z\left(\mathcal{M}^m_{\alpha,\beta,\lambda} * \mathcal{K}_\psi(z)\right)'}{\mathcal{M}^m_{\alpha,\beta,\lambda} * \mathcal{K}_\psi(z)} - 1\right) = \varphi(z)\left(\phi(\omega(z)) - 1\right),\ (z \in U).$$

□

Corollary 2.6. *Let* $k \in [0,\infty), \gamma \in [0,1), m = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.2 the function* f *is belongs to* $\tilde{\psi} S^m_{\alpha,\beta,\lambda}(b,\phi)$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|c_1\Gamma(\alpha+\beta)}{\Gamma(\beta)(\lambda+1)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{\Gamma(\beta)(\lambda+1)(\lambda+2)}\left(c_1 + max\left\{c_1, |bc_1^2| + |c_2|\right\}\right)$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{\Gamma(\beta)(\lambda+1)(\lambda+2)}$$
$$\cdot\left(c_1, +max\left\{c_1, \left|\frac{\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1) - \mu[\Gamma(\alpha+\beta)]^2(\lambda+2)}{\Gamma(2\alpha+\beta)\Gamma(\beta)(\lambda+1)}\right| |bc_1^2| + |c_2|\right\}\right).$$

Corollary 2.7. *Let* $k \in [0,\infty), \gamma \in [0,1), \lambda = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.2 the function* f *is belongs to* $\tilde{\psi} S^m_{\alpha,\beta,\lambda}(b,\phi)$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|c_1\Gamma(\alpha+\beta)}{2^m\Gamma(\beta)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{2 \times 3^m\Gamma(\beta)}\left(c_1 + max\left\{c_1, |bc_1^2| + |c_2|\right\}\right)$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{2\times 3^m\Gamma(\beta)}$$
$$\cdot\left(c_1, +max\left\{c_1, \left|\frac{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta) - 2\times\mu 3^m[\Gamma(\alpha+\beta)]^2}{2^{2m}\Gamma(2\alpha+\beta)\Gamma(\beta)}\right| |bc_1^2| + |c_2|\right\}\right).$$

Corollary 2.8. *Let* $k \in [0,\infty), \gamma \in [0,1), m = 0, \lambda = 0, min\ Re\{(\alpha), Re(\beta)\} > 0, b \neq 0$ *in Theorem 2.2 the function* f *is belongs to* $\tilde{\psi}S^m_{\alpha,\beta,\lambda}(b,\phi)$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|c_1\Gamma(\alpha+\beta)}{\Gamma(\beta)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{2\times\Gamma(\beta)}\left(c_1 + max\left\{c_1, |bc_1^2| + |c_2|\right\}\right)$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|\Gamma(2\alpha+\beta)}{2\times\Gamma(\beta)}$$
$$\cdot\left(c_1, +max\left\{c_1, \left|\frac{\Gamma(2\alpha+\beta)\Gamma(\beta) - 2\times\mu[\Gamma(\alpha+\beta)]^2}{\Gamma(2\alpha+\beta)\Gamma(\beta)}\right| |bc_1^2| + |c_2|\right\}\right).$$

Corollary 2.9. *Let* $k \in [0,\infty), \gamma \in [0,1),\ m, \lambda \in \mathbb{N}\cup\{0\}, \alpha = 1/2, \beta = 1, b \neq 0$*in Theorem 2.2 the function* f *is belongs to* $\tilde{\psi}S^m_{\alpha,\beta,\lambda}(b,\phi)$ *then*

$$|\frac{a_1}{S}| \leq \frac{|b|c_1\Gamma(3/2)}{2^m(\lambda+1)}$$

$$\left|\frac{a_2}{S}\right| \leq \frac{|b|}{3^m(\lambda+1)(\lambda+2)}\left(c_1 + max\left\{c_1, |bc_1^2| + |c_2|\right\}\right)$$

$$\left|\frac{a_2}{S} - \mu\frac{a_1^2}{S^2}\right| \leq \frac{|b|}{3^m(\lambda+1)(\lambda+2)}$$
$$\cdot\left(c_1, +max\left\{c_1, \left|\frac{2^{2m}(\lambda+1) - \mu 3^m[\Gamma(3/2)]^2(\lambda+2)}{2^{2m}(\lambda+1)}\right| |bc_1^2| + |c_2|\right\}\right).$$

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