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AN M/G/1 FEEDBACK RETRIAL QUEUE WITH NON PREEMPTIVE CUSTOMERS AND NON-PRIORITY CUSTOMERS

L. FRANCIS RAJ AND K. GEETHAPRIYA[1]

ABSTRACT. We develop the retrial queuing system with non-preemptive priority customer.A batch of customer arriving in the system in which the non-preemptive customer do not form any queue. A customer in first class find the server free and he completes his service leaves the system. A customer in second class waits a period of time until the customer in-service and obeys FCFS discipline. The joint probability generating functions are derived using the supplementary variable technique. In this model we obtain the steady state behaviour and average waiting time, finally some examples are presented to the effect of the system.

1. INTRODUCTION

In recent years there have been significant contributions to the retrial queuing system Choi and Park investigated an M_1N_{11} retrial queue with two types of customers in which the service time distribution for both type of customers are the same .For example , in telecommunication transfer protocol , for transaction different layer service for different customers. Priority classes control may appears in header of IP package, (or) in ATM cell , priority control is also widely used in production practice, transportations, management , etc.

[1]*corresponding author*

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In this paper we have studied single server retrial queuing system with non-preemptive priority queuing system with a single server serving two queues. Here we derive time dependent probability generating function for both priority units in terms of Laplace transforms. We also derive the average queue size and average waiting time in the queue for both priority and non priority under the non preemptive priority.

Definition:

When the priority discipline is non preemptive in nature,a job in service is allowed to complete its service normally even if a job of higher priority enters the queue while its service is going on. In the preemptive case, the service is to the ongoing job will be preempted by the new arrival of higher priority.

Mathematical description of our model:

We consider a single server retrial queuing system with types of customers is considered in this paper .Type I non preemptive priority customers arrive in batches of size 'x' with probability and type II non priority arrive in batches of size 'y' with probability z where $x, y, z = 1, 2 \ldots$ according to two independent Poisson process with rate λ_1, λ_2 can be identified as priority and non priority customers in the system.

The server must serve all the priority units present in the system before taking up non priority for service .If no priority present in the system at the time of starting service of a non priority unit. We assume that the server follows a non preemptive priority rule.If one (or) more priority arrive during the service time of a non priority rule, the current service of a non priority is not stopped and a priority will be taken up for service only after the current service of a non priority unit is complete.

The supplementary variable technique is used for the analysis and the variable considered being the residual service time for both non preemptive priority and non priority follows arbitrary distributions.

The Joint Distribution Of Queue Sizes:

We define the following random variables.

$M_1(t)$ = number of customers in the non preemptive priority type at time t.

$M_2(t)$ = number of customers in the non priority type at time t.

Y(t) = residual service time of the customer in the service at time t.

ξ (t) = 0, when the server is idle at time t; 1, when server services the non preemptive priority customer at time t; 2, when server services the non priority customer at time t.

Then, the stochastic process Y(t) = (ξ(t), M_1(t),M_2(t),X(t); t= 0) is the Markovian process with state space {0,1,2} denote by (ξ,M_1,M_2,Y) the limiting random variables of (ξ(t), M_1(t),M_2(t),Y(t)).

We define the related probabilities:

$$Q_Y = P\{\xi = 0, M_1 = i\} \text{ where } i = 0, 1, 2 \ldots\ldots$$

$$P_{zxy}(x)dx = p\{\xi = z, M_1 = x, M_2 = y, X \in (x, x + dx\}$$

and their Laplace –Stieltjes transform of the service time distribution function.

Let $\beta_z(s) = \int_0^8 e^{-sx} d\beta_z(x)dx$ where β_z(x) = z = 1,2 $\beta_{x,z}$ = (-1)$^z\beta_x{}^{(z)}$(0).
The service time β_z has a general distribution with probability of degree of freedom β_z(x) and mean β_z.
$z = 1, 2,$ where z = 1 is related to the non preemptive priority customers &
$z = 2$ is related to the non priority customers

The Laplace Stieltjes transform is

$$p^*{}_{zxy}(s) = \int_0^8 e^{-sx} p_{zxy}(x)dx, z = 1, 2, x, y = 0, 1, 2 \ldots.$$

Also,

$$p^*{}_{zxy}(0) = \int_0^8 .p_{zxy}(x)dx = p\{\xi = z, M_1 = x, M_2 = y\}$$

is the steady –state probability that there are x customers in the non- preemptive priority , y customers in the non priority customers in the retrial group of z- type customer.

A non-preemptive priority customer has received service departs the system with probability $1 - \delta_1$ (or)
Return to the group for add service with probabilityδ_1.

A non priority customer has received service leaved the system with probability $1 - \delta_2$ (or)
Rejoins the retrial type with probability $\delta_{2.}$

The following system of difference equation :

$$(\lambda_1+\lambda_2+YV)Q_y=(1-\delta_1)p_{10y}(0)+\delta_1 p_{10y}(0)+(1-\delta_2)p_{20y}(0)+\delta_2 p_{20y-1}(0), \tag{1.1}$$

$$\begin{aligned} p_{10y}(x) &= -(\lambda_1+\lambda_2)p_{10y}(x)+\lambda_1\beta_1(x)Q_y+\lambda_2 p_{10y-1}(x)+\delta_1\beta_1(x)p_{10y}(0) \\ &+(1-\delta_1)\beta_1(x)p_{11y}(0)+\delta_2\beta 1(x)p_{21y-1}(0)+(1-\delta_2)\beta_1(x)p_{21y}(0) \end{aligned} \tag{1.2}$$

$$\begin{aligned} &-p'_{1xy}(x) \\ &=-(\lambda_1+\lambda_2)p_{1xy}(x)+\lambda_{11}p_{x-1y}(x)+\lambda_2 p_{1xy-1}(x)+\delta_1 b_1(x)p_{1xy}(0) \\ &+(1-\delta_1)\beta_1(x)p_{1x+1y}(0)+\delta_2\beta_2(x)p_{2x+1y-1}(0)+(1-\delta_2)\beta_2(x)p_{1x+1y}(0) \end{aligned} \tag{1.3}$$

$$\begin{aligned} -p'_{20y}(x) &= -(\lambda_1+\lambda_2)p_{20y}(x)+\lambda_2\beta_{2(x)}q_y \\ &+(y+1)v\beta_{2(x)}q_{y+1+}\lambda_2 p_{20y-1}(x) \end{aligned} \tag{1.4}$$

$$-p'_{2xy}(x)=-(\lambda_1+\lambda_2)p_{2xy}(x)+\lambda_1 p_{2x-1y}(x)+\lambda_2 p_{2xy-1}(x) \tag{1.5}$$

$$-p'_{2xyz}(x)=-(\lambda_1+\lambda_2)p_{2xyz}(x)+\lambda_1 p_{2z-1xy}(x)+\lambda_2 p_{2xyz-2}(x) \tag{1.6}$$

Here $x=1,2,\dots,j=0,1,2\dots p_{xyz}$=0 for $xy<0, z=1,2\dots$ and $x=0$. By taking Laplace transform (1.2)-(1.5) we obtain

$$\begin{aligned} &\{S-(\lambda_1+\lambda_2)\}p*_{10y}(s)+\lambda_2 p*_{10y-1}(s) \\ &=p_{10y}(0)-\lambda_1\beta*(s)qy-\delta_1\beta*_1(s)p_{10y}(0)-(1-\delta_1)\beta*_1\rho_{11y}(0) \\ &-\delta_2\beta*_1(s)p_{21y-1}(0)-(1-\delta_2)\beta*_1(s)p_{21y}(0) \end{aligned} \tag{1.7}$$

$$\begin{aligned} &\{(s-(\lambda_1+\lambda_2)\}p*_{1xy}(s)+\lambda_1 p*_{1x-1y}(s)+\lambda_2 p*_{1xy-1}(s) \\ &=p_{1xy}(0)-\delta_1\beta*_1(s)p_{1xy}(0)-(1-\delta_1)\beta*_1(s)p_{1x+1y}(0) \\ &-\delta_2\beta*_1(s)p*_{2x+1y-1}(0)-(1-\delta_2)\beta*_1(s)p_{2x+1y}(0) \end{aligned} \tag{1.8}$$

$$\begin{aligned} &\{s-\ (\lambda_1+\lambda_2)\}yp*_{20y}(s)+\lambda_2 p*_{20y-1}(s) \\ &=p_{20y}(0)-\lambda_2\beta_2*(s)q_y-(y+1)v\beta_2*(s)q_{y+1} \end{aligned} \tag{1.9}$$

$$\{s-(\lambda_1+\lambda_2)\}p*_{2xy}(s)+\lambda_1 p*_{2x-1y}(s)+\lambda_2 p*_{2xy-1}(s)=p_{2xy}(0) \tag{1.10}$$

$$\{s-(\lambda_1+\lambda_2)\}p*_{2xyz}(s)+\lambda^1 p^*2xyz-{}_1(s)+\lambda_2 p*_{2xyz}(s)=p_{2xy}(0) \tag{1.11}$$

We introduce the following generating function for complete z with $|z| < 1$,

$$R(z_2) = \sum_{y=0}^{8} qyz^y 2;$$

$$P^* zx(s, z_2) = \sum_{y=0}^{8} p^*{}_{zxy}(s) z_2{}^y, z = 1, 2;$$

$$P^* zx(0, z_2) = \sum_{y=0}^{8} p_{zxy}(s) z_2{}^y, z = 1, 2.$$

Multiplying equations (1.1) & (1.10) by $z_2{}^y$ and summing over all y,

$$\begin{aligned} &(\lambda_1 + \lambda_2)R(z_2) + vz_2 R'(Z_2) \\ &= (1 - \delta_1)P_{10}(0, Z_2) + (1 - \delta_1 + \delta_2)PZ_0(0, Z_2) \end{aligned} \tag{1.12}$$

$$\begin{aligned} &\{S - (\lambda_1 + \lambda_2) + \lambda_2 Z_2\}P^*{}_{10}(S, Z_2) \\ &= (1 - \delta_1 \beta * {}_1(s)p_{10}(0, Z_2) - \lambda_1 \beta * {}_1(s)R(z_2) \\ &- (1 - \delta_1)\beta * {}_1(s)p_{11}(0, Z_2) - (1 - \delta_1 + \delta_2 z_2)\beta * {}_1(s)p_{21}(0, Z_2) \end{aligned} \tag{1.13}$$

$$\begin{aligned} &\{S - (\lambda_1 + \lambda_2) + \lambda_2 Z_2\}P^*{}_{1x}(S, Z_2) + \lambda_1 P^*{}_{1x}(S, Z_2) + \lambda_1 P^*{}_{1x-1}(S, Z_2) \\ &= (1 - \delta_1 \beta * {}_1(s)p_{1x}(0, Z_2)\delta_1 \beta * {}_1(s)p_{1x+1}(0, Z_2) \\ &- (1 - \delta_1 + \delta_2 z_2)\beta * {}_1(s)p_{2x+1}(0, Z_2) \end{aligned} \tag{1.14}$$

$$\begin{aligned} &\{S - (\lambda_1 + \lambda_2) + \lambda_2 Z_2\}P^*{}_{20}(S, Z_2) \\ &= P_{20}(0, Z_2) - \lambda_2 \beta * {}_2(s)R(z_2) - v\lambda_1 \beta * {}_2(s)R'(z_2) \end{aligned} \tag{1.15}$$

$$\{S - (\lambda_1 + \lambda_2) + \lambda_2 Z_2\}P^*{}_{21}(S, Z_2) + \lambda_1 P_{2x-1}(S, Z_2) = p_{2x}(0, z_2) \tag{1.16}$$

Define the generating functions:

$P^*{}_z(s, z_1, z_2)$ and $p_z(0, z_1, z_2)$ $for\ z = 1, 2 \ldots$

$$P_{0xy} = p\{Y(t) = 0, M_1(t) = x, M_2(t) = y\}$$

$$P_{1xy}(x) = \frac{d}{dx} p\{Y(t) = 1, \xi(t) < x, M_1(t) = x, M_2(t) = y\}$$

$$P_{2xy}(x) = \frac{d}{dx} p\{Y(t) = 2, \xi(t) < x, M_1(t) = x, M_2(t) = y\}$$

And corresponding partial generating functions:

$$P_0(z_1,z_2)=\sum_{x=0}^{8}\sum_{y=0}^{8} {z_1}^x {z_2}^y p_{0xy},$$

$$P_1(s,z_1,z_2)=\sum_{x=0}^{8}\sum_{y=0}^{8} {z_1}^x {z_2}^y p_{1xy(x)},$$

$$P_2(s,z_1,z_2)=\sum_{x=0}^{8}\sum_{y=0}^{8} {z_1}^x {z_2}^y p_{2xy(x)}.$$

Since, $M_1(t)=0$ if $Y(t)=0$ then $p_{0,x,y}=0$ for $x=1$.

The generating function, $p_0(z_1,z_2)=p_0(0,z_2)$. That is, does not depend on variable z_1. Also, ${P_2}^*(0,z_1,z_2)=E(Z^{M1}{}_{1,}Z^{M2}{}_2;\xi=Z)$, which is the joint generating function of (M_1,M_2) when the server services the z-type customer.

Multiplying equations (1.13)-(1.16) by ${z_1}^x$ and summing over all x, we obtain

$$\begin{aligned}&\{S-\lambda_1(1-z_1)+\lambda_2(1-z_2)\}{p_1}^*(s,z_1,z_2)\\&=\{1-\delta_1\beta*{}_1(s)-(1-\delta_1)\beta*{}_1(s)/z_1p_1(0,z_1,z_2)\\&-(1-\delta_2+\delta_2z_2)\beta*{}_1(s)/z_1p_2(0,z_1,z_2)\\&+(1-\delta_1)\beta*{}_1(s)/z_1p_{10}(0,z_2)+(1-\delta_2+\delta_2z_2)\beta\\&*{}_1(s)/z_1z_1p_{20}(0,z_2)-\lambda_1\beta*{}_1(s)R(z_1)\end{aligned}\tag{1.17}$$

$$\begin{aligned}&\{S-\lambda_1(1-z_1)+\lambda_2(1-z_2)\}{p_2}^*(s,z_1,z_2)+{p_1}^*(s,z_1,z_2)\\&=p_2(0,z_1,z_2)-\lambda_2\beta*{}_2(s)R(z_2)+p_1(0,z_1,z_2)\\&-\lambda_1\beta*{}_1(s)R(z_2)-v\beta*{}_1(s)R'(z_2)\end{aligned}\tag{1.18}$$

By choosing $s=\lambda_1(1-z_1)--\lambda_2(1-z_2)$ into (1.17) and (1.18), we eliminate $P^*{}_1(s,z_1,z_2)$ & $P^*{}_2(s,z_1,z_2)$ from (1.17), (1.18)

$$\begin{aligned}&\{Z_1-(1-\delta_1+\delta_1z_2)\gamma_1(z_1,z_2)\}P_1(0,z_1,z_2)\\&=\gamma_1(z_1,z_2)(1-\delta_2+\delta_2z_2)P_2(0,z_1,z_2)\\&+\lambda_1z_1R(z_2)-1-\delta_1P_{10}(0,Z_2)-(1-\delta_2+\delta_2z_2)P_{20}(0,Z_2)\end{aligned}\tag{1.19}$$

$$P_2(0,Z_{1,}Z_2)=\gamma_2(z_1,z_2)\{\lambda_2R(z_2)+VR'(z_2)\}\tag{1.20}$$

Hwere, $\gamma_2(z_1,z_2)=\beta*{}_Z(\lambda_1(1-z_1)+\lambda_2(1-z_2)),Z=1,2.$

Now, consider the functions

$$h(z_1, z_2) = Z_1 - (1 - \delta_1 + \delta_1 z_2)\gamma_1(z_1, z_2).$$

It follows that for each z_2 with $|z_2| < 1$, there is a unique solution $Z_1 = \varphi(z_2)$ of the equation $h(z_1, z_2) = 0$ in the unit circle

$$h(\varphi(z_2), z_2) = \varphi(z_2) - (1 - \delta_1 + \delta_1\varphi(z_2))\gamma_1(\varphi(z_2), z_2) = 0.$$

We conclude that $z_1 = \varphi(z_2)$ is analytic on $|z_2| < 1$ and is continuous at $z_2 = 1$ and $\varphi(1) = 1$. By substituting $z_1 = \varphi(z_2)$ into (1.19), (1.20) $P_1(0, z_1, z_2)$ is eliminated and we get

$$\begin{aligned}(1.21)\qquad &(1 - \delta_1 + \delta_1 z_2)P_2(0, z_1, z_2) + \lambda_1\varphi(z_2)R(z_2)\\ &= (1 - \delta_1)p_{10}(0, z_2) + (1 - \delta_1 + \delta_1 z_2)p_{20}(0, z_2)\end{aligned}$$

$$(1.22)\qquad P_2(0, z_1, z_2) = \gamma_1(\varphi(z_2), z_2)\{\lambda_2 Q(z_2) + vR'(z_2)\}.$$

From (1.21) and (1.22), we obtain

$$\begin{aligned}(1.23)\qquad &p_{10}(0, z_2) + (1 - \delta_1 + \delta_1 z_2)p_{20}(0, z_2)\\ &= \{\lambda_1\varphi(z_2) + (1 - \delta_2 + \delta_2 z_2)\lambda_2\gamma_2(\varphi(z_2), z_2)\}R(z_2)\\ &+ (1 - \delta_2 + \delta_2 z_2)v\gamma_2(\varphi(z_2), z_2)R'(z_2).\end{aligned}$$

By equating (1.12) and (1.13), we obtain the difference equation

$$\begin{aligned}(1.24)\qquad R'(z_2) = &\frac{1}{v\{((1{-}\delta 2{+}\delta 2\mathrm{z}2)\,\gamma 2\,(\varphi\,(\mathrm{z}2)\,,\mathrm{z}2)\,{-}\mathrm{z}2\}}\\ &* [\lambda_1(1 - Q(z_2)) + \lambda_2\{1 - (1 - \delta_2 + \delta_2 z_2)\gamma_2(\varphi(z_2), z_2)]R(z_2),\end{aligned}$$

$$\begin{aligned}(1.25)\qquad R(z_2) = c.exp[-\frac{1}{v}\int_1^{\mathrm{z}1} &\frac{1}{((1{-}\delta_2{+}\delta_2\mathrm{z}_2)\,\gamma_2\,(\varphi\,(\mathrm{x})\,,\mathrm{x})\,{-}\mathrm{x}}\\ &* \{\lambda_1(1 - Q(x)) + \lambda_2\{1 - (1 - \delta_2 + \delta_2 z_2)\gamma_2(\varphi(x), x)\}\}dx],\end{aligned}$$

where $c = (1 - \rho_1 - \rho_2), \varphi(x) = h(x)\gamma_2 = k_2$ in equation (1.25).

An M/G/1 retrial queue with non-preemptive priority subscribers is

$$P_0(z_2) = (1 - \rho_1 - \rho_2) * exp\frac{1}{v}\int_1^{\mathrm{z}2} \frac{\lambda 1{-}\lambda 1 h(x)\ +\ \lambda 2\ -\lambda 2 k2(h(x), x)}{k2\,(h\,(x)\,, x) - x}$$

$$
\begin{aligned}
P_1(z_1,z_2,x) = \{&\lambda_1 - \lambda_1 h(z_2) + \lambda_2 - \lambda_2 k_2(h(z_2),z_2)(k_2(z_1,z_2) - z_2) \\
&- (\lambda_1 - \lambda_1 z_{1+}\lambda_2 - \lambda_2 k_2(z_1,z_2) k_2(h(z_2),z_2) - z_2\} \\
&\cdot \{k_2(h(z_2),z_2) - z_2)(z_1 - -k_1(z_1,z_2))\}^{-1} \\
&\cdot `p_0(z_2)[1-\beta_1(x)] e^{-(\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2)x}
\end{aligned} \tag{1.26}
$$

$$
\begin{aligned}
P_2(z_1,z_2,x) = \frac{\lambda_1 - \lambda_1 \mathrm{h}\,(\mathrm{z}_2) + \lambda_2 - \lambda_2\ \mathrm{z}_2}{\mathrm{k}_2(\mathrm{h}(\mathrm{z}_2),\ \mathrm{z}_2) - \mathrm{z}_2)} \cdot p_0(z_2)[1-\beta_1(x)] \\
\cdot e^{-(\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2)x}
\end{aligned} \tag{1.27}
$$

We find that $P_1(z_1,z_2,x)$ and $P_2(z_1,z_2,x)$ depend upon x as follows :

$$P_1(z_1,z_2,x) = P_1(z_1,z_2,0) * [1-\beta_1(x)] e^{-(}\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2)x \tag{1.28}$$

$$P_2(z_1,z_2,x) = P_2(z_1,z_2,0) * [1-\beta_2(x)] e^{-(}\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2)x. \tag{1.29}$$

For the generating functions $p_0(z_2), P_1(z_1,z_2,x), P_2(z_1,z_2,x)$ these equations give

$$
\begin{aligned}
t\gamma z_2 \frac{d\mathrm{p}_0(\mathrm{z}_2)}{d\mathrm{z}_2} = -(\lambda_1+\lambda_2) p_0(z_2) + \int_0^8 p_1(0,z_2,x)\beta_1(x)dx \\
+ \int_0^8 p_2(0,z_2,x)\beta_2(x)dx
\end{aligned} \tag{1.30}
$$

$$\frac{\partial \mathrm{p}_{1(}\mathrm{z}_1,\mathrm{z}_2,\mathrm{x})}{\partial x} = (\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2 + \beta_1(x)) * P(z_1,z_2,x) \tag{1.31}$$

$$\frac{\partial \mathrm{p}_{2(}\mathrm{z}_1,\mathrm{z}_2,\mathrm{x})}{\partial x} = (\lambda_1 - \lambda_1 z_1 + \lambda_2 - \lambda_2 z_2 + \beta_2(x)) * P(z_1,z_2,x) \tag{1.32}$$

$$
\begin{aligned}
z_1 P_2(z_1,z_2,0) = \int_0^8 p_1(z_1,z_2,x) - p_1(0,z_2,x)\beta_1(x)dx \\
+ \int_0^8 p_2(z_1,z_2,x) - p_2(0,z_2,x)\beta_2(x)dx + e_1 z_1 p_0(z_2)
\end{aligned} \tag{1.33}
$$

$$P_2(z_1,z_2,0) = e_2 p_0(z_2) + v\frac{d\mathrm{p}_0(\mathrm{z}_2)}{d\mathrm{z}_2}. \tag{1.34}$$

From (1.28), (1.33), (1.28) and(1.29) we have

$$K_2(z_1,z_2)P_2(z_1,z_2,0) = [z_1 - K_1(z_1,z_2)].P_1(z_1,z_2,x) + (e_1 - e_1z_1 + {}_2)p_0(z_2) + vz_2\frac{d\mathrm{p}_0(\mathrm{z}_2)}{d\mathrm{z}_2}. \tag{1.35}$$

Eliminating $P_2(z_1,z_2,0)$ from (1.35) and (1.34) we get

$$v[K_2(z_1,z_2) - z_2]\frac{d\mathrm{p}_0(\mathrm{z}_2)}{d\mathrm{z}_2} = [e_1 - e_1z_1 + e_2 - e_2K_2(z_1,z_2)]p_0(z_2) + [z_1 - K_1(z_1,z_2)]P_1(z_1,z_2,0). \tag{1.36}$$

Consider equation $z_1 - K_1(z_1,z_2) = 0$, rewritten as

$$z_1 - (s + e_1 - e_1z_1) = 0, \tag{1.37}$$

where $s = e_2 - e_2z_1$ if $\rho_1 < 1$ then has a unique root $z_1(s)$, in the unit disk $|z_1| = 1\&|z_2| = 1$ is continuous in the closed disk $|z| = 1$. The normalizing condition is $p_0(1) + p_1(1,1) + p_2(1,1) = 1$. We obtain

$$C = \frac{(1-\delta_2)\{(1-(\delta_1 + \lambda_1\ \beta_1)\}-(1-\delta_2)\{(1-(\delta_1 + 2\lambda_1\ \beta_1)\}}{(1-\delta_1)\{\ (1-\ (\delta_1 + \lambda_1\ \beta_1)\} + \lambda_1\ \beta_1(1-\delta_2)}.$$

The probability that the server is idle $\&P_1{}^*(0,1,1) + P_2{}^*(0,1,1)$

$$\frac{({}_{1-}\ \delta_1)\{1-\ (\delta_1 + \lambda_1\beta_1) + \lambda_2\beta_2)\}-(1-\delta_2)\{1-\ (\delta_1 + 2\lambda_1\beta_1)}{(1-\delta_1)\{1-\ (\delta_1 + \lambda_1\beta_1) + \lambda_2\beta_2)\} + \lambda_1\ \beta_1(1-\delta_2)}.$$

The probability that the server is busy.

Numerical Example: (Non Preemptive Priority)

Assume that, we have two classes, $k = 1,2$, exponential service with rates $\mu_1 = \mu_2 = 10$ customers/min, $\lambda_1 = 4$, $\lambda_2 = 3$. When no priorities are applied we have that

$$E(W_q{}^1) = E(W_q{}^2) = E(W) = \frac{\rho}{\mu(1-\rho)} = 14\ sec.$$

When non preemptive priorites are applied we have

$$E(W_q{}^1) = \frac{\rho}{\mu(1-\rho)} = 7\ sec,$$

$$E(W_q{}^2) = \frac{\rho}{\mu\,(1-\rho1\ )\,(1-\rho1-\rho2)} = 23.32\ sec.$$

Conclusion:

In this paper we studied the server provides two types of service, namely priority and non priority under non-preemptive priority groups are found by using the supplementary variable technique, the average waiting time for the priority and non-priority are obtained.The above model finds different layers service for different customers.

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PG & RESEARCH DEPARTMENT OF MATHEMATICS,
VOORHEES COLLEGE, VELLORE
Email address: francisraj73@yahoo.co.in

DEPARTMENT OF MATHEMATICS,
D.K.M COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE – 632 001.
Email address: geethasaipriya@gmail.com