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ON SOFT FUZZY SOFT TOPOLOGICAL VECTOR SPACES AND DIFFERENTIATIONS

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ABSTRACT. Basic properties of soft fuzzy soft topological vector spaces are introduced and discussed. Few results of soft fuzzy soft tangent to 0 and SFS differentiations have been established.

1. INTRODUCTION

Zadeh [**?**] initiated the work on fuzzy set in the year 1965. Fuzzy sets and logic have wide application in the field of Information [8] and control [9]. Chang [1] incorporated the fuzzy language in the topological structures. Ismail U. Tiryaki [10] analysed the hardness of fuzzy set. Applications of soft set was first investigated by Russian researcher Molodstov [6]. Fuzzy soft set provides more accurate solution to the decision making problems which is established by Maji, Biswas and Roy [4]. Serge Lang [7] defines a derivative of a continuous function between any two vector spaces. Katsaras and Liu [2] investigated the topological structre on vector spaces via fuzzy sets. In this connection Mario Ferraro and Foster [5] introduced the differentiability on continuous functions.

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2. Preliminaries

Definition 2.1. *[6] Let $P(U)$ be the power set of U and A be the subset of parameter E. Then (F, A) is known as soft set over U where $F : A \to P(U)$.*

Definition 2.2. *[4] A fuzzy soft set over the universe U is a pair (F, A) where $F : A \to F(U)$ and A is the subset of a set of parameter E.*

Definition 2.3. *[3] $\delta \subset I^E$ is a fuzzy topology on E iff*

(i) *For all α constant, $\alpha \in \delta$,*
(ii) *For all μ, $\eta \in \delta \Rightarrow \mu \wedge \eta \in \delta$.*
(iii) *For all $(\mu_j)_{j\in J} \subset \delta \Rightarrow sup_{j\in J}\mu_j \in \delta$.*

The member in δ is known as open fuzzy set. $\mu \in I^E$ is said to be closed iff μ^c is open.

Definition 2.4. *[10] Let X be a set, $\eta \in I^X$ and M is any subset of X. Then (η, M) is said to be a soft fuzzy set in X.*

3. On Soft Fuzzy Soft Sets

Throughout the paper $X \neq \phi$, E denote the collection of parameters and $I = [0, 1]$. Also soft fuzzy soft is denoted by SFS.

Definition 3.1. *A SFS $\delta_E : X \to I \times P(E)$ with membership $\delta_E(p) = (\delta(p), A)$ where $\delta : X \to I$, A is the member of the collection $P(E)$ of all subsets of E. Moreover its family is represented by $SFS(X, E)$.*

Definition 3.2. *SFS characteristic function $\chi_A : X \to \{(1, E), (0, \phi)\}$ is defined as*

$$\chi_A(p) = \begin{cases} (1, E), & \text{if } p \in A \subset X; \\ (0, \phi), & \text{otherwise.} \end{cases}$$

Definition 3.3. *Let δ_E be a SFS set. Define*

$$p_{\delta_E}(q) = \begin{cases} (\delta(p), A)(\delta(p) \in [0, 1]), & \text{if } p = q; \\ (0, \phi), & \text{otherwise.} \end{cases}$$

p_{δ_E} is a sfs point (in short, SFSP) in $SFS(X, E)$.

Definition 3.4. *Let $\lambda_E \in SFS(X,\ E)$ such that the universal SFS set is $\lambda_E(p) = (1, E)$, $\forall p \in X$ and it is represented by $(1, E)^{\sim}$. The null SFS set is defined as follows $\lambda_E(p) = (0, \phi)$, $\forall p \in X$ and it is represented as $(0, \phi)^{\sim}$.*

Definition 3.5. *Let $\delta_E \in SFS(X,\ E)$ such that $\delta_E(p) = (\delta(p), \mathscr{A})$, then the complement of δ_E is denoted by δ_E^c where $\delta_E^c(p) = (1, E)^{\sim}(p) - \delta_E(p) = (1 - \delta(p), E \backslash \mathscr{A})$, $\forall p \in X$.*

Definition 3.6. *Let δ_E and μ_E be any two SFS sets such that $\delta_E(p) = (\delta(p), \mathscr{A})$ and $\mu_E(p) = (\mu(p), \mathscr{B})$. Then*

(i) $\delta_E(p) \sqsubseteq \mu_E(p) \Leftrightarrow \delta(p) \leq \mu(p)$, $\forall p \in X$, $\mathscr{A} \subseteq \mathscr{B}$.

(ii) $\delta_E(p) \sqsupseteq \mu_E(p) \Leftrightarrow \delta(p) \geq \mu(p)$, $\forall p \in X$, $\mathscr{A} \supseteq \mathscr{B}$.

(iii) $\delta_E(p) \sqcap \mu_E(p) = (min\{\delta(p), \mu(p)\}, \mathscr{A} \cap \mathscr{B})$, $\forall p \in X$.

(iv) $\delta_E(p) \sqcup \mu_E(p) = (max\{\delta(p), \mu(p)\}, \mathscr{A} \cup \mathscr{B})$, $\forall p \in X$.

Definition 3.7. *Let δ_E and μ_E be any two SFS sets. Then*

(i) $\delta_E \Subset \mu_E \Leftrightarrow \delta_E(p) \sqsubseteq \mu_E(p), \forall\, p \in X.$

(ii) $\delta_E \Supset \mu_E \Leftrightarrow \delta_E(p) \sqsupseteq \mu_E(p), \forall\, p \in X.$

(iii) $\delta_E = \mu_E \Leftrightarrow \delta_E(p) = \mu_E(p), \forall\, p \in X.$

Definition 3.8. *Let $f : X \to Y$. If $\delta_E \in SFS(Y,\ E)$, then $f^{-1}(\delta_E)(p) = \delta_E \circ f(p) = \delta_E(f(p)),\ \forall p \in X.$*

Definition 3.9. *Let $f : X \to Y$. If $\mu_E \in SFS(X,\ E)$, then*

$$f(\mu_E)(q) = \begin{cases} \sqcup_{p \in f^{-1}(q)} \mu_E(p), & \text{if } f^{-1}(q) \neq \phi; \\ (0, \phi), & \text{otherwise.} \end{cases}$$

Definition 3.10. *A constant membership function is represented by $\mathfrak{K}_c$ and if $\mathfrak{K}_b(p) = b$ for all $p \in X$, where $0 < b \leq 1$.*

Definition 3.11. *Let δ_{j_E} be any SFS sets and J be an indexed set. A SFS topology on X is a collection $\mathscr{T}$ of SFS sets satisfying:*

(i) $\forall\ \mathfrak{K}_b \in X$ *and* $\forall\ A \in P(E)$, $\mathfrak{K}_{b_A} \in T$ *where* $\mathfrak{K}_{b_A} = (\mathfrak{K}_b(p), A)$ *for all* $p \in X$.

(ii) $\delta_{j_E} \in \mathscr{T}$, $j \in J \Rightarrow \Cup_{j \in J} \delta_{j_E} \in \mathscr{T}$.

(iii) *For any finite* J, $\delta_{j_E} \in \mathscr{T}$, $\Rightarrow \Cap_{j \in J} \delta_{j_E} \in \mathscr{T}$.

Then $(X,\ \mathscr{T})$ is said to be SFS topological space, $SFSTS$. Any member of $\mathscr{T}$ is SFS open set, $SFSOS$. SFS closed set is the complement of SFS open set and whose collection is denoted by $SFSCS$.

Remark 3.1. *$\mathscr{T} = \{\chi_M : \forall M \in \tau\} \cup \{\mathfrak{K}_{b_A} : \forall \mathfrak{K}_b \in X$ and $\forall A \in P(E)\}$ forms an usual SFS topology.*

Proposition 3.1. *Let $f : X \to Y$. If δ_{1_E}, δ_{2_E} be any two SFS sets in X and μ_{1_E}, μ_{2_E} be any two SFS sets in Y. Then*

(i) $\delta_{1_E} \Subset \delta_{2_E} \Rightarrow f(\delta_{1_E}) \Subset f(\delta_{2_E})$.

(ii) $\mu_{1_E} \Subset \mu_{2_E} \Rightarrow f^{-1}(\mu_{1_E}) \Subset f^{-1}(\mu_{2_E})$.

Definition 3.12. *Let $\delta_E \in SFS(X,\ E)$ is said to be a SFS neighbourhood(nghbd) of a SFS point p_{δ_E} in X iff $\exists$ a $\mu_E \in \mathscr{T} \ni p_{\delta_E} \Subset \mu_E \Subset \delta_E$.*

Definition 3.13. *A system of SFS nghbds of a SFS point p_{δ_E} is a set $\mathbb{B}(p_{\delta_E})$ of SFS nghbds of p_{δ_E} such that for each SFS nghbd δ_E of p_{δ_E} there is a $\mu_E \in \mathbb{B}(p_{\delta_E})$ such that $\mu_E \Subset \delta_E$.*

Proposition 3.2. *Let $f : (X,\ \mathscr{T}) \to (Y,\ \mathscr{S})$, then we have the equivalence.*

(i) *f is SFS continuous.*

(ii) *$\delta_E \in SFS(X,\ E)$ and each SFS nghbd δ_E of $f(\delta_E)$, $\exists$ a SFS nghbd μ_E of δ_E such that $f(\mu_E) \Subset \delta_E$.*

4. On SFS Topological Vector Spaces

Throughout the section $\mathscr{V}$ is a vectorspace over the field K.

Definition 4.1. *Let $\{\delta_{j_E}\} \in SFS(\mathscr{V},\ E)$, $j = 1, 2, 3, \ldots, n$. The sum $\delta_E = \delta_{1_E} + \delta_{2_E} + \delta_{3_E} + \ldots + \delta_{n_E}$ of $\{\delta_{j_E}\}$, is the SFS set having membership, $\delta_E(p) = \sqcup_{p_1+\ldots+p_n=p}(\delta_{1_E}(p_1) \sqcap \delta_{2_E}(p_2), \ldots, \delta_{n_E}(p_n)),\ p \in \mathscr{V}$. The scalar product $\alpha\delta_E$, of $\alpha \in K$ and δ_E is a SFS set in $\mathscr{V}$ that has $\alpha\delta_E(p)$, $p \in \mathscr{V}$, given by $\alpha\delta_E(p) = \delta_E(p/\alpha)$ $\forall\ \alpha \neq 0$, $p \in \mathscr{V}$. For $\alpha = 0$, $\alpha\delta_E(p) = (0, \phi),\ p \neq 0$ or $= \sqcup_{q\in\mathscr{V}}\delta_E(q),\ p = 0$.*

Proposition 4.1. *If $f : \mathscr{V}_1 \to \mathscr{V}_2$. Then for all SFS sets δ_E, μ_E in $\mathscr{V}_1$ and all scalars α, $f(\delta_E + \mu_E) = f(\delta_E) + f(\mu_E)$ and $f(\alpha\delta_E) = \alpha f(\delta_E)$.*

Proposition 4.2. *If δ_E, $\mu_E \in SFS(\mathscr{V},\ E)$ and $\alpha \in K$, $\alpha \neq 0$, then $\alpha\delta_E \Subset \mu_E \Rightarrow \delta_E \Subset (1/\alpha)\mu_E$.*

Proposition 4.3. *Let $\delta_E, \delta_{1_E}, \delta_{2_E}, \ldots, \delta_{n_E} \in SFS(\mathscr{V},\ E)$ and $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ scalars. Then the following are equivalent.*

(i) $\alpha_1\delta_{1_E} + \alpha_2\delta_{2_E} + \ldots + \alpha_n\delta_{n_E} \Subset \delta_E$.

(ii) *For all* $p_1, p_2, \ldots, p_n$ *in* $\mathscr{V}$,

Proposition 4.4. *Let* $(\mathscr{V},\ \mathscr{T})$ *be a SFS topological vector space. For every SFS point* 0_{δ_E} *such that* $0_{\delta_E} = (\delta(0), \mathscr{A})$, $(0, \phi) \sqsubset (\delta(0), \mathscr{A}) \sqsubseteq (1, E)$, $\exists$ *a system of SFS nghbds* $\mathbb{B}(0_{\delta_E})$ *in* $\mathscr{V}$ *holds.*

(i) $\forall\ \delta_E \in \mathbb{B}(0_{\delta_E})$ $\exists$ *a* $\mu_E \in \mathbb{B}(0_{\delta_E})$ *with* $\mu_E + \mu_E \Subset \delta_E$.

(ii) $\forall\ \delta_E \in \mathbb{B}(0_{\delta_E})$ $\exists$ *a* $\mu_E \in \mathbb{B}(0_{\delta_E})$ *for which* $k\mu_E \Subset \delta_E\ \forall\ k \in K$, $\mid k \mid \leq 1$.

(iii) *Every* $\delta_E \in \mathbb{B}(0_{\delta_E})$ *is SFS balanced.*

5. On SFS Differentiations

Definition 5.1. $\sigma : (\mathscr{V}_1,\ \mathscr{T}_1) \to (\mathscr{V}_2,\ \mathscr{T}_2)$ *is called SFS tangent to* 0 *if for each SFS nghbd* μ_E *of* 0_{δ_E} *where* $0_{\delta_E}(0) = (\delta(0), \mathscr{A})$, $(0, \phi) \sqsubseteq (\delta(0), \mathscr{A}) \sqsubseteq (1, E)$, *in* $\mathscr{V}_2$ *there exists a SFS nghbd* δ_E *of* 0_{η_E} *where* $0_{\eta_E}(0) = (\eta(0), F)$, $(0, \phi) \sqsubset (\eta(0), F) \sqsubseteq (\delta(0), A)$ *in* $\mathscr{V}_1$ *such that* $\sigma(t\delta_E) \Subset \rho(t)\mu_E$ *for some function* $\rho(t)$.

Proposition 5.1. *If the function* σ *is SFS tangent to* 0, *then* σ *is SFS continuous at* $0 \in \mathscr{V}_1$.

Proposition 5.2. *If* σ *and* η *are two functions SFS tangent to* 0 *then* $\sigma + \eta$ *is a function SFS tangent to* 0.

Proposition 5.3. *Let* $(\mathscr{V}_1,\ \mathscr{T}_1)$, $(\mathscr{V}_2,\ \mathscr{T}_2)$, $(\mathscr{V}_3,\ \mathscr{T}_3)$ *be any three SFS topological vector spaces over* K *with* E *as the set of all parameters. Composition of* SFS *continuous linear map and* SFS *tangent to zero is* SFS *tangent to zero.*

Definition 5.2. *Let* $(\mathscr{V}_1,\ \mathscr{T}_1)$ *and* $(\mathscr{V}_2,\ \mathscr{T}_2)$ *be any two SFS topological vector spaces, each of them is a SFS* $\mathscr{T}_1$ *space. A* SFS *continuous function* $f : \mathscr{V}_1 \to \mathscr{V}_2$ *is called a* SFS *differentiable at* $p \in \mathscr{V}_1$ *if there is a linear SFS continuous function* u *on* $\mathscr{V}_1$ *satisfies* $f(p+q) = f(p) + u(q) + \sigma(q),\ q \in \mathscr{V}_1$ *where* σ *is* SFS *tangent to* 0 *and* u *is* SFS *derivative of* f *at* p.

Proposition 5.4. *Let* $(\mathscr{V}_1,\ \mathscr{T}_1)$, $(\mathscr{V}_2,\ \mathscr{T}_2)$, $(\mathscr{V}_3,\ \mathscr{T}_3)$ *be any three SFS topological vector spaces and also SFS* $\mathscr{T}_1$ *space. Let* f *and* g *be a SFS continuous function on* $\mathscr{V}_1$ *and* $\mathscr{V}_2$ *respectively. Composition of two* SFS *differentiable function is* SFS *differentiable.*

Proof. Assume that f and g are SFS differentiable. Hence $f(p+r)-f(p) = f^{'}(p)(r) + \sigma(r),\ r \in V_1$, $g(q+s)-g(q){=}g^{'}(q)(s)+\eta(s),\ s \in \mathscr{V}_2$, where σ and η are each SFS tangent to 0. Defining $h = f \circ g$, after substitution we get, $h(p+r)-h(p) = g^{'}(q)(f^{'}(p)(r)) + g^{'}(q)(\sigma(r)) + \eta(f^{'}(p)(r)+\sigma(r)),\ r \in \mathscr{V}_1$. By Proposition 5.3, $g^{'}(q) \circ \sigma$ is SFS tangent to 0. Consider the function $\eta \circ (f^{'}(p)+\sigma)$. For every SFS nghbd μ_E of 0_{ν_E} where $0_{\nu_E}(0) = (\nu(0), F)$, $(0,\phi) \sqsubset (\nu(0),F) \sqsubseteq (1,E)$ in $\mathscr{V}_3$ there is a SFS nghbd δ_E of 0_{δ_E} where $0_{\delta_E}(0) = (\delta(0), A)$, $(0,\phi) \sqsubset (\delta(0),A) \sqsubseteq (\nu(0),F)$ in $\mathscr{V}_2$ such that $\eta(t\delta_E)(z) \sqsubseteq \rho(t)\mu_E(z),\ z \in \mathscr{V}_3$. Given δ_E there epists a SFS nghbd $\delta^{'}_E$ of 0_{δ_E} such that $\delta^{'}_E + \delta^{'}_E \Subset \delta_E$. Suppose that both δ_E and $\delta^{'}_E$ belongs to a system of SFS nghbds $\mathbb{B}(0_{\delta_E})$. By the SFS continuity of $f^{'}(p)$ there is a SFS nghbd γ_E of 0_{β_E} where $0_{\beta_E}(0) = (\beta(0), G)$, $(0,\phi) \sqsubset (\beta(0),G) \sqsubseteq (\delta(0),A)$ in $\mathscr{V}_1$ such that $f^{'}(p)(\gamma_E)(q) \sqsubseteq \delta^{'}_E(q)$, which implies that $tf^{'}(p)(\gamma_E)(q) \sqsubseteq t\delta^{'}_E(q)$, that is $f^{'}(p)(t(\gamma_E))(q) \sqsubseteq t\delta^{'}_E(q)$, $q \in \mathscr{V}_2$. For everq $\delta^{'}_E$ there exists a SFS nghbd $\gamma^{'}_E$ of 0_{β_E} in $\mathscr{V}_1$ for which $\sigma(t\gamma^{'}_E)(q) \sqsubseteq \rho(t)\delta^{'}_E(q)$ and, for $\mid \rho(t)/t \mid \leq 1$, $\rho(t)\delta^{'}_E(q) \sqsubseteq t\delta^{'}_E(q)$, $q \in \mathscr{V}_2$. Let $\gamma_{1_E} = \gamma_E \Cap \gamma^{'}_E$ and using Proposition 3.1, we obtain $[\sigma(t\gamma_{1_E}) + f^{'}(p)(t\gamma_{1_E})](q) \sqsubseteq t\delta_E(q)$, which implies that $\eta(\sigma(t\gamma_{1_E}) + f^{'}(p)(t\gamma_{1_E})) \Subset \eta(t\delta_E) \Subset \rho(t)\mu_E$, that is the function $\eta \circ (f^{'}(p)+\sigma)$ from $\mathscr{V}_1$ to $\mathscr{V}_3$ is SFS tangent to 0. Thus $h(p+r)-h(p) = g^{'}(q) \circ f^{'}(p)(r) + \zeta(r)$, $r \in V_1$, where $g^{'}(q) \circ f^{'}(p)$ is linear and SFS continuous, and ζ, is SFS tangent to 0. □

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