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FUZZY TRANSSHIPMENT MODEL USING FUZZY ONE POINT METHOD

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ABSTRACT. Fuzzy transshipment problem is a special case of fuzzy transportation problem. The fuzzy transshipment model recognizes that in real life it can be cheaper to ship through intermediate or transient nodes before reaching the final destination. In this paper, transshipment problem with fuzzy parameters is converted to a regular fuzzy transportation model. Then it is solved using fuzzy one point method in k - stages. The procedure adopted here is independent of the conventional procedure. Suitable numerical examples are included for the proposed approach along with the simulation result using TORA software.

1. INTRODUCTION

In the Fuzzy Transportation Problem (FTP) shipment of commodity takes place directly from sources to destinations. Whereas in the fuzzy transshipment problem shipments are allowed between sources and between destinations, sometimes there may also be intermediate points through which goods can be transshipped on their journey from a source to a destination. This type of shipment can be less expensive than the direct shipment in many cases. Hence fuzzy transshipment problem is very useful to reduce the fuzzy transportation cost. A number of effective solutions are available in the literature for fuzzy transshipment problem. Few among them are mentioned here. Nagoor Gani et al. [6–8] proposed the procedures to solve the fuzzy transshipment problem.

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Amit Kumar et al. [1] proposed fuzzy linear programming approach for solving fuzzy transportation problems with transshipment. Baskaran et al. [4] converted the fuzzy transshipment problem with transit points to fuzzy transportation problem and solved the problem using fuzzy cost deviation algorithm. Baskaran and Dharmalingam [3] proposed a fuzzy programming approach for solving multi objective transshipment problem.

Motivation of the current work: Pandian and Natarajan [9] proposed a new algorithm namely fuzzy zero point method for finding a fuzzy optimal solution for a fuzzy transportation problems, whereas the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Sujatha and Elizabeth [11] proposed fuzzy one point method for finding the fuzzy optimal solution for fuzzy transportation problem and fuzzy unbalanced assignment problem in three stages. Elizabeth et al. [5] developed fuzzy zero point method for finding the fuzzy objective value for unbalanced fuzzy transportation problem in three stages. Srinivasan and Geetharamani [10] applied Robust's ranking technique for solving the fuzzy assignment problem using one's assignment method proposed by Basirzadeh [2]. The concepts of these research articles made us present the procedures which are independent of the conventional procedure to solve the fuzzy transshipment problem.

The paper is structured as follows: In section 2, the basic concepts of fuzzy set theory and the mathematical formulation for fuzzy transportation model is reviewed. In section 3, Fuzzy Transshipment model under fuzzy one point method is presented if (i) Sources and destinations acting as transient nodes (ii) Intermediate nodes acting as transient nodes (iii) Intermediate and destination nodes acting as transient nodes. In section 4, Mathematical formulation (both balanced and unbalanced) and procedure are presented for converted fuzzy transportation problem in the case of fuzzy one point method in k - stages. Suitable numerical examples are provided for the proposed method. Section 5 concludes the paper.

2. PRELIMINARIES

Definition 2.1. ***(Triangular fuzzy number)*** *Let $\widetilde{A}$ be a triangular fuzzy number. It can be represented by $\widetilde{A} = (a_1, a_2, a_3; 1) = (a_1, a_2, a_3); a_1, a_2, a_3 \in R; a_1 < a_2 < a_3$; with membership function $\mu_{\widetilde{A}}(x)$ is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)}, a_1 & \leq x < a_2 \\ 1, & x = a_2 \\ \frac{(a_3-x)}{(a_3-a_2)}, & a_2 < x \leq a_3 \\ 0, & otherwise \end{cases}$$

Definition 2.2. ***(Addition operation on triangular fuzzy numbers)*** *Let* $\widetilde{A} = (a_1, a_2, a_3)$ *and* $\widetilde{B} = (b_1, b_2, b_3)$ *be two triangular fuzzy numbers. Then* $\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

Definition 2.3. ***(Defuzzification formulae for triangular fuzzy number)*** *[11]*
a) Mean measure (MM) of triangular fuzzy number: *Let* $\widetilde{A} = (a_1, a_2, a_3)$ *be a triangular fuzzy number. Mean measure of* $\widetilde{A}$ *is given by* $(a_2 - a_1) = (a_3 - a_2) \Rightarrow a_2 = \frac{a_1+a_3}{2} = MM(\widetilde{A})$. *This is utilized when right spread* (α) *is same as the left spread. Here* $\widetilde{A} = (a_1, a_2, a_3) = (a_2, \alpha, \alpha)_{LR}$*, where* $a_1 = a_2 - \alpha$ *and* $a_3 = a_2 + \alpha$.
b) Left-Right measure (LRM) of triangular fuzzy number: *Let* $\widetilde{A} = (a_1, a_2, a_3)$ *be a triangular fuzzy number. The left-right measure of* $\widetilde{A}$ *is given by* $LRM(\widetilde{A}) = \int_0^1 \{\lambda[a_3 - (a_3 - a_2)\alpha'] + (1 - \lambda)[(a_2 - a_1)\alpha' + a_1]\} \, d\alpha'$*, where* $\lambda \in (0, 1]$*,* $\alpha' \in (0, 1]$*. If* $\lambda = 0.5$*, then* $LRM(\widetilde{A}) = \frac{1}{2}[a_2 + \frac{a_1+a_3}{2}]$*. Here* $\widetilde{A} = (a_1, a_2, a_3) = (a_2, \alpha, \beta)_{LR}$*, where* $a_1 = a_2 - \alpha$ *and* $a_3 = a_2 + \beta$*, then* $LRM(\widetilde{A}) = \frac{1}{2}[2a_2 + \frac{\beta-\alpha}{2}]$*. This is utilized only when right spread* (β) *is not the same as the left spread* (α).

2.1. **Mathematical Formulation for Fuzzy Transportation Model.**

$$Minimize \ \widetilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c_{ij}} \otimes \widetilde{x_{ij}} \tag{2.1}$$

$$Subject \ to \ \sum_{j=1}^{n} \widetilde{x_{ij}} \approx \widetilde{a_i}, i = 1, 2, 3, \ldots, m \tag{2.2}$$

$$\sum_{i=1}^{m} \widetilde{x_{ij}} \approx \widetilde{b_j}, j = 1, 2, 3, \ldots, n \tag{2.3}$$

$$\widetilde{x_{ij}} \geq 0. \tag{2.4}$$

If $\sum_{i=1}^{m} \widetilde{a_i} \approx \sum_{j=1}^{n} \widetilde{b_j}$, then the problem is balanced otherwise unbalanced.

2.2. Mathematical Formulation for Fuzzy Transportation Model in k – stages.

$$Z_k = Minimize\ z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{kij}x_{kij} \tag{2.5}$$

$$Subject\ to\ \sum_{j=1}^{n} x_{kij} = a_{ki}, i = 1, 2, 3, \ldots, m \tag{2.6}$$

$$\sum_{i=1}^{m} x_{kij} = b_{kj}, j = 1, 2, 3, \ldots, , n \tag{2.7}$$

$$x_{kij} \geq 0\ for\ all\ i, j. \tag{2.8}$$

If $\sum_{i=1}^{m} a_{ki} = \sum_{j=1}^{n} b_{kj}$, then the problem is balanced otherwise unbalanced.

3. Fuzzy Transshipment Model under fuzzy one point method

3.1. Fuzzy Transshipment Model if Sources and Destinations acting as the transient nodes.

Guidelines to solve the problem: Here the number of starting nodes as well as the number of ending nodes is the sum of the number of sources and the number of destinations of the original problem. Let $\widetilde{c_{ij}}=(c_{1ij}, c_{2ij}, c_{3ij})$ takes the value (1,1,1) along the transient nodes to itself. Let $\widetilde{T}$ be the buffer which must be maintained at each of the transient sources and transient destinations. The buffer $\widetilde{T}$ can be equal to the sum of the supplies or the sum of the demands or $\widetilde{T} = \sum_{i=1}^{m} \widetilde{a}_i=\sum_{j=1}^{n} \widetilde{b}_j$. So, a constant $\widetilde{T}$ is added to all the starting nodes and all ending nodes.

Supply at a transient node = Original supply + Buffer amount

Demand at a transient node = Original demand + Buffer amount

Now the fuzzy transshipment problem is converted to the fuzzy transportation problem. The problem is solved by applying fuzzy one point method in k - stages.

3.2. **Fuzzy Transshipment Model if Intermediate nodes acting as the transient nodes.**

Guidelines to solve the problem: Add the transient nodes as additional sources as well as additional destinations and form a regular fuzzy transportation table with necessary cost details. Let $\widetilde{c_{ij}}$ takes the value (1,1,1) along the transient nodes to itself. Assume (∞, ∞, ∞), a very large value for the $\widetilde{c_{ij}}$ values between different sources and destinations. Let $\widetilde{T}$ be the buffer which must be maintained at each of the transient intermediate nodes. The buffer $\widetilde{T}$ can be equal to the sum of the supplies or the sum of the demands or $\widetilde{T} = \sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} \widetilde{b}_j$. So, a constant $\widetilde{T}$ is added to all the intermediate nodes.

Supply at a pure supply node = Original supply
Demand at a pure demand node = Original demand
Supply at a transient node = Original supply + Buffer amount
Demand at a transient node = Original demand + Buffer amount

Now the fuzzy transshipment problem is converted to the fuzzy transportation problem. The problem is solved by applying fuzzy one point method in k - stages.

3.3. **Fuzzy Transshipment Model if Intermediate and Destination nodes acting as transient nodes.**

Guidelines to solve the problem: The guideline given in subsection 3.2 is utilized here. The buffer amount is added to the transient intermediate nodes and transient destination nodes.

4. Mathematical Formulation and Procedure for Converted Fuzzy Transportation Problem using Fuzzy One Point Method

4.1. **Mathematical Formulation.** The objective function of the converted fuzzy transportation model is:

$$Minimize\ \widetilde{z} = \sum_{i} \sum_{j} \widetilde{c_{ij}} \otimes \widetilde{x}_{ij} - \sum \widetilde{x}_{ij} = {I_1}^{*} - {I_2}^{*} \tag{4.1}$$

$\widetilde{c_{ij}}$ may take the value (1,1,1) in other cells also, but $\widetilde{c_{ij}}$ takes the value (1,1,1) for the transient nodes to itself. Summation is taken over the transient nodes to itself, ${I_2}^{*}$ is obtained. Equation (4.1) is subject to the constraints given in the equations (2.2) to (2.4).

The objective function of the converted fuzzy transportation model in k – stages is

$$Z_k = \ Minimize\ z_k = \sum_i \sum_j c_{kij} x_{kij} - \sum x_{kij} = I_1 - I_2 \tag{4.2}$$

Here $c_{kij} \geq 1$. c_{kij} may take the value 1 in other cells also, but c_{kij} takes the value 1 for the transient nodes to itself. Summation is taken over the transient nodes to itself, I_2 is obtained. Equation (4.2) is subject to the constraints given in the equations (2.6) to (2.8).

4.2. **Mathematical formulation for unbalanced form of the converted fuzzy transportation model in the case of fuzzy one point method in k-stages.**

If

$$\sum Supply \neq \sum Demand$$

in k-stages then

(4.3)

$$Z_k = \ Minimize\ z_k = \sum_i \sum_j c_{kij} x_{kij} - \sum x_{kij} - \sum_i x_{kij} = I_1 - I_2 - I_3, c_{kij} \geq 1$$

if dummy column is introduced. Here c_{kij} may take the value 1 in other cells (columns) also, but c_{kij} takes the value 1 for the transient nodes to itself. Summation is taken over the transient nodes to itself, I_2 is obtained. Summation is taken over the dummy column whose cost is "1", I_3 is obtained.

(4.4)

$$Z_k = \ Minimize\ z_k = \sum_i \sum_j c_{kij} x_{kij} - \sum x_{kij} - \sum_j x_{kij} = I_1 - I_2 - I_4, c_{kij} \geq 1$$

if dummy row is introduced. Here c_{kij} may take the value 1 in other cells (rows) also, but c_{kij} takes the value 1 for the transient nodes to itself. Summation is taken over the transient nodes to itself, I_2 is obtained. Summation is taken over the dummy row whose cost is "1", I_4 is obtained.

Both equations (4.3) and (4.4) are subject to the constraints given in the equations (2.6) to (2.8).

4.3. **Procedure for Converted Fuzzy Transportation Problem in k – stages using fuzzy one point method.**

Let k=1,2,3 be the 3-stage of FTP.

Step 4.1 to 4.6 is adopted from [11].

Step 4.1: FTP is now divided into 3 stages. In the 1^{st} stage, the transportation cost c_{1ij} of transportation table TT $[c_{1ij}]$, supply a_{1i} and demand $b_{1j}$are considered. In the 2^{nd} stage, c_{2ij} of TT $[c_{2ij}]$, supply a_{2i} and demand b_{2j} are considered. In the 3^{rd} stage, c_{3ij} of TT $[c_{3ij}]$, supply a_{3i} and demand b_{3j} are considered. This is for all $i = 1$ to m and $j = 1$ to n.
Procedure for the 1^{st} stage of FTP (It is in the form of crisp transportation problem).

Step 4.2: Check whether the given transportation problem is a balanced one. If not convert it into a balanced one by introducing a dummy column / dummy row with cost entry as "1".

Step 4.3: Divide each row entries of the transportation table by row minimum that is if u_{1i} is the minimum of the i^{th} row of the table $[c_{1ij}]$. Then divide the i^{th} row entries by u_{1i}, so that the resulting table is $[c_{1ij}/u_{1i}]$.

Step 4.4: Divide each column entries of the resulting transportation table after applying the step 4.3 by the column minimum that is if v_{1j} is the minimum of j^{th} column of the resulting table $[c_{1ij}/u_{1i}]$ then divide j^{th} column entries by v_{1j} so that the resulting table is $[(c_{1ij}/u_{1i})/v_{1j}]$. It may be noted that $[(c_{1ij}/u_{1i})/v_{1j}] \geq 1$ for all i, j. Each row and each column of the resulting table $[(c_{1ij}/u_{1i})/v_{1j}]$ has atleast one "1" entry.

Step 4.5: Choose the row or column with only one "1" and allot the minimum of source and demand corresponding to that cell. Check whether the supply points are fully used and all the demand points are fully received. If so go to step 4.7 if not, go to step 4.6.

Step 4.6: Draw minimum number of lines horizontally and vertically to cover all the 1's . Then choose the least uncovered element and divide all the uncovered elements using it and multiply it at the intersection of lines, leaving other elements unchanged. Now, check whether each row and each column has at least one "1" entry. If so, go to step 4.5, else go to step 4.3, step 4.4 and then to step 4.5.

Step 4.7: This allotment yields an optimal solution x^0_{1ij} to the given 1^{st} stage of converted fuzzy transportation problem:

(1) with the objective function given in the equation (4.2), if the problem is a balanced one.
(2) with the objective function given in the equation (4.3), if the problem is balanced by introducing a dummy column.
(3) with the objective function given in the equation (4.4), if the problem is balanced by introducing a dummy row.

All the three equations (4.2), (4.3) and (4.4) are subject to the constraints given in the eqn.(2.6) to eqn.(2.8).

Now repeat Step 4.2 to Step 4.7 for the 2^{nd} stage and the 3^{rd} stage of converted FTP. Finally, the combination of the optimal objective value obtained in the 3-stages gives the optimal fuzzy objective value $\widetilde{Z} = (Z_1, Z_2, Z_3)$ to the given converted fuzzy transportation problem / fuzzy transshipment problem.

Example 1. ***Numerical Example (Sources and Destinations acting as transient nodes)***
Consider the following transshipment problem involving 4 sources and 2 destinations. The supply values of the sources S_1, S_2, S_3 and S_4 are (95,100,105) units, (198,200,202) units, (145,150,155) units and (349,350,351) units respectively. The demand values of destinations D_1 and D_2 are (320,350,380) units and (430,450,470) units respectively. The transportation cost per unit between different sources and destinations are summarized as in Table 1.

TABLE 1. Fuzzy Transshipment Problem

Sources	*Destinations*					
	S_1	S_2	S_3	S_4	D_1	D_2
S_1	(1,1,1)	(2,4,6)	(15,20,25)	(4,5,6)	(20,25,30)	(10,12,14)
S_2	(5,10,15)	(1,1,1)	(3,6,9)	(5,10,15)	(4,5,6)	(15,20,25)
S_3	(10,15,20)	(15,20,25)	(1,1,1)	(6,8,10)	(40,45,50)	(6,7,8)
S_4	(15,20,25)	(20,25,30)	(5,10,15)	(1,1,1)	(25,30,35)	(3,6,8)
D_1	(15,20,25)	(16,18,20)	(55,60,65)	(10,15,20)	(1,1,1)	(5,10,15)
D_2	(5,10,15)	(20,25,30)	(25,30,35)	(22,23,24)	(2,4,6)	(1,1,1)

Solution. *Here the number of sources is 4 and the number of destinations is 2. Therefore the total number of starting nodes as well as the total number of ending nodes of the fuzzy transshipment problem is equal to 6 (i.e 4+2=6). We also have*

$$\sum_{i=1}^{4} \widetilde{a_i} = (95, 100, 105) \oplus (198, 200, 202) \oplus (145, 150, 155)$$
$$\oplus (349, 350, 351) = (787, 800, 813)$$
$$\sum_{j=1}^{2} \widetilde{b_j} = (320, 350, 380) \oplus (430, 450, 470) = (750, 800, 850)$$
$$\sum_{i=1}^{4} R(\widetilde{a_i}) = 800, \sum_{j=1}^{2} R(\widetilde{b_j}) = 800 \quad \textit{[by definition 2.3]}$$
$$\widetilde{T} = \sum_{i=1}^{4} \widetilde{a_i} \approx \sum_{j=1}^{2} \widetilde{b_j}, R(\widetilde{T}) = \sum_{i=1}^{4} R(\widetilde{a_i}) = \sum_{j=1}^{2} R(\widetilde{b_j}).$$

TABLE 2. Converted fuzzy transportation problem

Sources	*Destinations*						
	S_1	S_2	S_3	S_4	D_1	D_2	*Fuzzy Supply (FS)*
S_1	*(1,1,1)*	*(2,4,6)*	*(15,20,25)*	*(4,5,6)*	*(20,25,30)*	*(10,12,14)*	*(95,100,105)* $\oplus$*(750,800,850)* =*(845,900,955)*
S_2	*(5,10,15)*	*(1,1,1)*	*(3,6,9)*	*(5,10,15)*	*(4,5,6)*	*(15,20,25)*	*(198,200,202)* $\oplus$*(750,800,850)* =*(948,1000,1052)*
S_3	*(10,15,20)*	*(15,20,25)*	*(1,1,1)*	*(6,8,10)*	*(40,45,50)*	*(6,7,8)*	*(145,150,155)* $\oplus$*(750,800,850)* =*(895,950,1005)*
S_4	*(15,20,25)*	*(20,25,30)*	*(5,10,15)*	*(1,1,1)*	*(25,30,35)*	*(3,6,8)*	*(349,350,351)* $\oplus$*(750,800,850)* =*(1099,1150,1201)*
D_1	*(15,20,25)*	*(16,18,20)*	*(55,60,65)*	*(10,15,20)*	*(1,1,1)*	*(5,10,15)*	*(750,800,850)*
D_2	*(5,10,15)*	*(20,25,30)*	*(25,30,35)*	*(22,23,24)*	*(2,4,6)*	*(1,1,1)*	*(750,800,850)*
Fuzzy Demand(FD)	*(750,800,850)*	*(750,800,850)*	*(750,800,850)*	*(750,800,850)*	*(320,350,380)* $\oplus$*(750,800,850)* =*(1070,1150,1230)*	*(430,450,470)* $\oplus$ *(750,800,850)* =*(1180,1250,1320)*	

Here i= 1 to 6 and j=1 to 6. By applying the procedure for the 1st stage we obtain the optimal table as follows.

TABLE 3. The final optimal table for the 1^{st} stage

Sources	*Destinations*							
	S_1	S_2	S_3	S_4	D_1	D_2	*Dummy Column*	*Supply*
S_1	*750* 1	*95* 2						*845*
S_2		*655* 1			*293* 4			*948*
S_3			*750* 1			*108* 6	*37* 1	*895*
S_4				*750* 1		*349* 3		*1099*
D_1					*750* 1			*750*
D_2					*27* 2	*723* 1		*750*
Demand	*750*	*750*	*750*	*750*	*1070*	*1180*	*37*	*5287*

The optimal objective value in the 1^{st}*stage is Minimize* $Z_1 = 7,526 - 4415 = 3,111.$

Repeat the procedure for k=2,3.

The optimal fuzzy objective value is $\widetilde{Z}$ *=(3111,5250,9382).*

FIGURE 1. Simulation result using TORA software for the 1^{st} stage

Example 2. Numerical Example (Intermediate nodes acting as transient nodes)

A multi-plant organization has three plants (P_1, P_2, P_3) *and three market places* (M_1, M_2, M_3)*. The items from the plants are transported to the market places through two intermediate finished goods warehouses. The details on cost of transportation per unit for different combinations between the plants and warehouses, between warehouses and markets, between warehouses, supply values of the plants and demand values of the markets are summarized in table 4.*

TABLE 4. Fuzzy Transshipment Problem

	M_1	M_2	M_3	W_1	W_2	*Fuzzy Supply*
P_1	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	*(10,15,20)*	*(15,30,45)*	*(50,200,350)*
P_2	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	*(25,28,31)*	*(5,10,15)*	*(200,300,400)*
P_3	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	*(15,30,45)*	*(10,15,20)*	*(200,400,600)*
W_1	*(5,10,15)*	*(20,40,60)*	*(15,30,45)*	*(1,1,1)*	*(10,20,30)*	-
W_2	*(20,25,30)*	*(10,15,20)*	*(30,35,40)*	*(20,25,30)*	*(1,1,1)*	-
Fuzzy Demand	*(50,100,150)*	*(200,400,600)*	*(200,400,600)*	-	-	

Solution. *Here the number of sources is 3 and the number of destinations is 3.*
$\sum_{i=1}^{3} \widetilde{a}_i = (50, 200, 350) \oplus (200, 300, 400) \oplus (200, 400, 600) = (450, 900, 1350)$
$\sum_{j=1}^{3} \widetilde{b}_j = (50, 100, 150) \oplus (200, 400, 600) \oplus (200, 400, 600) = (450, 900, 1350)$
$\sum_{i=1}^{3} R(\widetilde{a}_i) = 900, \sum_{j=1}^{3} R(\widetilde{b}_j) = 900$ *[by definition 2.3]*
$\widetilde{T} = \sum_{i=1}^{3} \widetilde{a}_i \approx \sum_{j=1}^{3} \widetilde{b}_j, R(\widetilde{T}) = \sum_{i=1}^{3} R(\widetilde{a}_i) = \sum_{j=1}^{3} R(\widetilde{b}_j)$.

TABLE 5. Converted fuzzy transportation problem

	M_1	M_2	M_3	W_1	W_2	*Fuzzy Supply*
P_1	(∞, ∞, ∞)	(∞, ∞, ∞)	(∞, ∞, ∞)	*(10,15,20)*	*(15,30,45)*	*(50,200,350)*
P_2	(∞, ∞, ∞)	(∞, ∞, ∞)	(∞, ∞, ∞)	*(25,28,31)*	*(5,10,15)*	*(200,300,400)*
P_3	(∞, ∞, ∞)	(∞, ∞, ∞)	(∞, ∞, ∞)	*(15,30,45)*	*(10,15,20)*	*(200,400,600)*
W_1	*(5,10,15)*	*(20,40,60)*	*(15,30,45)*	*(1,1,1)*	*(10,20,30)*	*(450,900,1350)*
W_2	*(20,25,30)*	*(10,15,20)*	*(30,35,40)*	*(20,25,30)*	*(1,1,1)*	*(450,900,1350)*
Fuzzy Demand	*(50,100,150)*	*(200,400,600)*	*(200,400,600)*	*(450,900,1350)*	*(450,900,1350)*	

Here i= 1 to 5 and j=1 to 5.
By applying the procedure for the 1st stage we obtain the optimal table as follows.

TABLE 6. The final optimal table for the 1^{st} stage

	M_1	M_2	M_3	W_1	W_2	*Supply*
P_1				*50* 10		*50*
P_2					*200* 5	*200*
P_3				*200* 15		*200*
W_1	*50* 5		*200* 15	*200* 1		*450*
W_2		*200* 10			*250* 1	*450*
Demand	*50*	*200*	*200*	*450*	*450*	*1350*

The optimal objective value in the 1^{st}*stage is Minimize* $Z_1 = 10200 - 450 = 9750$.
Repeat the procedure for k=2,3.
The optimal fuzzy objective value is $\widetilde{Z} = (9750, 32500, 64250)$.

Example 3. ***Numerical Example (Intermediate and Destination nodes acting as transient nodes)***
Two automobile plants P_1 *and* P_2 *are linked to three dealers* D_1,D_2 *and* D_3 *by way of two transit centers,* T_1 *and* T_2 *according to the network. The supply amounts*

FIGURE 2. Simulation result using TORA software for the 1^{st} stage

at plants P_1 and P_2 are (800,1000,1200) and (1000,1200,1400) cars, and the demand amounts at dealers D_1, D_2 and D_3 are (600,800,1000),(800,900,1000) and (400,500,600) cars. The shipping costs per car (in hundreds of dollars) between pairs of nodes are shown in the table 7.

TABLE 7. Fuzzy Transshipment Problem

	T_1	T_2	D_1	D_2	D_3
P_1	$(1,3,6)$	$(2,4,6)$	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)
P_2	$(1,2,3)$	$(3,5,7)$	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)
T_1	$(1,1,1,)$	$(5,7,9)$	$(6,8,10)$	$(5,6,7)$	(∞,∞,∞)
T_2	(∞,∞,∞)	$(1,1,1)$	(∞,∞,∞)	$(2,4,6)$	$(8,9,10)$
D_1	(∞,∞,∞)	(∞,∞,∞)	$(1,1,1)$	$(3,5,7)$	(∞,∞,∞)
D_2	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	$(1,1,1)$	$(1,3,6)$

Solution. *Here the number of sources is 2 and the number of destinations is 3.*
$\sum_{i=1}^{2} \widetilde{a}_i = (800, 1000, 1200) \oplus (1000, 1200, 1400) = (1800, 2200, 2600)$
$\sum_{j=1}^{3} \widetilde{b}_j = (600, 800, 1000) \oplus (800, 900, 1000) \oplus (400, 500, 600) = (1800, 2200, 2600)$
$\sum_{i=1}^{2} R(\widetilde{a}_i) = 2200, \sum_{j=1}^{3} R(\widetilde{b}_j) = 2200$ *[by definition 2.3]*
$\widetilde{T} = \sum_{i=1}^{2} \widetilde{a}_i \approx \sum_{j=1}^{3} \widetilde{b}_j, R(\widetilde{T}) = \sum_{i=1}^{2} R(\widetilde{a}_i) = \sum_{j=1}^{3} R(\widetilde{b}_j).$

TABLE 8. Converted fuzzy transportation problem

	T_1	T_2	D_1	D_2	D_3	FS
P_1	(1,3,6)	(2,4,6)	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	(800,1000,1200)
P_2	(1,2,3)	(3,5,7)	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	(1000,1200,1400)
T_1	(1,1,1,)	(5,7,9)	(6,8,10)	(5,6,7)	(∞,∞,∞)	(1800,2200,2600)
T_2	(∞,∞,∞)	(1,1,1)	(∞,∞,∞)	(2,4,6)	(8,9,10)	(1800,2200,2600)
D_1	(∞,∞,∞)	(∞,∞,∞)	(1,1,1)	(3,5,7)	(∞,∞,∞)	(1800,2200,2600)
D_2	(∞,∞,∞)	(∞,∞,∞)	(∞,∞,∞)	(1,1,1)	(1,3,6)	(1800,2200,2600)
FD	(1800,2200, 2600)	(1800,2200, 2600)	(600,800,1000)$\oplus$ (1800,2200,2600)= (2400,3000,3600)	(800,900,1000)$\oplus$ (1800,2200,2600) = (2600,3100,3600)	(400,500,600)	

Here i= 6 and j=5.

By applying the procedure for the 1^{st} stage we obtain the optimal table as follows.

TABLE 9. The final optimal table for the 1^{st} stage

	T_1	T_2	D_1	D_2	D_3	Supply
P_1		800 ^{2}				800
P_2	1000 ^{1}					1000
T_1	800 ^{1}		600 ^{6}	400 ^{5}		1800
T_2		1000 ^{1}		800 ^{2}		1800
D_1			1800 ^{1}			1800
D_2				1400 ^{1}	400 ^{1}	1800
Demand	1800	1800	2400	2600	400	9000

The optimal objective value in the 1^{st}stage is Minimize $Z_1 = 15200 - 5000 = 10200$.

Repeat the procedure for k=2,3.

The optimal fuzzy objective value is $\widetilde{Z} = (10200, 21700, 33800)$.

5. CONCLUSION

Fuzzy transportation and fuzzy transshipment problems are treated as a special case of the problem of the cheapest flow in a network. Since the economic growth of a country depends on the increases of the capacity of transport, the study of both the problems are essential. In the literature, the fuzzy

FIGURE 3. Simulation result using TORA software for the 3^{rd} stage

transshipment problems are solved by converting fuzzy transshipment to fuzzy transportation where conventional procedure like tabular methods such as Northwest corner rule / Least cost method / Vogel's approximation method are used to obtain the Initial basic feasible solution and Modified distribution (MODI) method to obtain the optimal solution. In this paper we have used fuzzy one point method to obtain the optimal solution that minimize the total fuzzy transshipment cost, which are independent of the conventional procedure. The procedure proposed in this paper can be treated as an alternative method to obtain the optimal solution for fuzzy transshipment problem.

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