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CODING MESSAGES THROUGH PRIME CORDIAL LABELING ON MCGEE GRAPH

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ABSTRACT. In this paper, the McGee graph is proved to be a Prime Cordial Graph. A coding technique is developed using McGee graph, GMJ code and Prime Cordial Labeling.

1. INTRODUCTION

Inspired by the variety of papers on graph labelings given in the Dynamic Survey of Graph Labelings by J. A. Gallian [1], the authors have planned this paper on coding with a selection of a suitable graph labeling and a graph and the outcome is presented here. Much work is done by S. K. Vaidya and many authors on Prime Cordial Labeling, [2, 3]. Uma Maheswari et al. have introduced GMJ coding and used it on different graphs with different labelings such as Super Mean Labeling, Vertex Product, Edge Product and Difference Cordial Labelings [4, 5]. Motivated by these, the authors decided to work on McGee Graph assigning the Prime Cordial Labeling for coding messages and hence this paper.

2. PREREQUISITES

Definition 2.1 (McGee Graph)**.** *The unique* $(3, 7)$ *cage, the smallest cubic graph of girth* 7 *is the McGee graph with* 24 *vertices and* 36 *edges, [6].*

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Definition 2.2 (Prime Cordial Labeling)**.** *Prime Cordial Labeling on a graph G with the vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ and f^*, the induced function is such that $f^* : E(G) \to \{0, 1\}$ where*

$$f^*(uv) = \begin{cases} 1 & \text{if } gcd(f(u), f(v)) = 1 \\ 0 & \text{if } gcd(f(u), f(v)) > 1 \end{cases},$$

with the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1, where $e_f(0)$ denotes the number of edges with value 0 and $e_f(1)$ denotes the number of edges with value 1 such that $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a Prime Cordial Labeling is called Prime Cordial graph.

3. Main Results

Notations

The configuration chosen for the McGee graph consists of 24 vertices on the circumference and 36 edges. The 24 vertices are numbered starting from the topmost (an extremity of the vertical diameter) as v_1, proceeding in the clockwise direction. There are 24 edges on the circumference, 4 edges along the diameters and 8 edges along the chords. They are called circular edges, diametric edges and chordal edges. They are denoted by CIRE, DIAE and CHDE. The vertices can be called as circular vertices denoted by CIRV or HDIAV (Head of the diameter), TDIAV (Tail of the diameter), HCHDV or TCHDV. These abbreviations are used in the coding of letters.

An illustration is given by using McGee graph and the Prime Cordial Labeling. Here it is proved that McGee Graph is a Prime Cordial Graph. The vertex and the edge labelings are provided below and so, numbers can be assigned on the McGee Graph using Prime Cordial Labeling and a new coding technique can be developed using them.

Theorem 3.1. *The McGee graph admits the Prime Cordial Labeling.*

Proof. There are 24 vertices $v_1, v_2, \ldots, v_{24}$ taken in the clockwise direction for the McGee Graph G.
Define the labelling for the vertices $f : V(G) \to \{1, 2, \ldots, 24\}$ as follows:

$$f(v_i) = 2i \text{ for } i = 1 \text{ to } 12,$$
$$f(v_{24-j}) = 23 - 2j \text{ for } j = 0, 1, 2, 3, 4, 6, 7, 9.$$
$$\text{and } f(v_{19}) = 7 \text{ for } j = 5,$$
$$f(v_{16}) = 13, \text{ for } j = 8,$$
$$f(v_{14}) = 1, \text{ for } j = 10,$$
$$f(v_{13}) = 3, \text{ for } j = 11.$$

Define the labeling for the edges $f^* : E(G) \to \{0, 1\}$ as follows.

Here the edges are classified as circular edges, diametric edges and chordal edges respectively.

(i) **Labeling for Circular Edges**

$$f^*(v_i v_{i+1}) = gcd(2i, 2(i+1)) \text{ for } i = 1 \text{ to } 11$$
$$f^*(v_{24-i} v_{24-(i+1)}) = gcd\,(23 - 2i, 23 - 2(i-1)) \text{ for } i \neq 5, 8, 10, 11$$
$$\text{and } f^*(v_{19} v_{18}) = gcd(7, 11) \text{ for } i = 5$$
$$f^*(v_{16} v_{15}) = gcd(13, 5) \text{ for } i = 8$$
$$f^*(v_{14} v_{13}) = gcd(1, 3) \text{ for } i = 10$$
$$f^*(v_{13} v_{12}) = gcd(3, 24) \text{ for } i = 11$$

(ii) **Labeling for Diametric Edges**

$$f^*(v_1 v_{13}) = gcd(2i, 3) \text{ for } i = 1$$
$$f^*(v_4 v_{15}) = gcd\,(2i, 23 - 2j) \text{for } i = 4, j = 9$$
$$f^*(v_7 v_{19}) = gcd(14, 7)$$
$$f^*(v_{10} v_{22}) = gcd\,(2i, 23 - 2j) \text{ for } i = 0, j = 2$$

(iii) **Labeling for Chordal Edges**

$$f^*(v_2 v_9) = gcd\,\{2i, 2(j+1)\} \text{ for } i = 1, j = 8$$
$$f^*(v_3 v_{20}) = gcd(2i, 23 - 2j) \text{ for } i = 3, j = 4$$
$$f^*(v_5 v_{12}) = gcd\,\{2i, 2(j+1)\} \text{ for } i = 5, j = 11$$
$$f^*(v_6 v_{23}) = gcd(2i, 23 - 2j) \text{ for } i = 6, j = 1$$
$$f^*(v_8 v_{15}) = gcd(2i, 23 - 2j) \text{ for } i = 8, j = 9$$
$$f^*(v_{11} v_{18}) = gcd(2i, 23 - 2j) \text{ for } i = 11, j = 6$$
$$f^*(v_{14} v_{21}) = gcd(1, 23 - 2j) \text{ for } j = 3$$
$$f^*(v_{17} v_{24}) = gcd(23 - 2i, 23 - 2j) \text{ for } i = 7, j = 0$$

Counting $e_f(0)$ and $e_f(1)$:

(i) **Along the circular edges**

$$f^*(v_i v_{i+1}) = 0 \text{ as } gcd(2i, 2(i+1)) \neq 1 \text{ for } i = 1to11$$

$$f^*(v_{24-i}v_{24-(i+1)}) = 1 \text{ as } gcd(23-2i, 23-2(i-1)) = 1 \text{for } i \neq 5, 8, 10, 11$$
$$\text{and} f^*(v_{19}v_{18}) = 1 \text{ as } gcd(7, 11) = 1 \text{ for } i = 5$$
$$f^*(v_{16}v_{15}) = 1 \text{ as } gcd(13, 5) = 1 \text{ for} i = 8$$
$$f^*(v_{14}v_{13}) = 1 \text{ as } gcd(1, 3) = 1 \text{ for } i = 10$$
$$f^*(v_{13}v_{12}) = 0 \text{ as } gcd(3, 24) \neq 1 \text{ for } i = 11$$

Here

$$e_f(0) = 12 \quad \text{and} \quad e_f(1) = 12 \tag{3.1}$$

(ii) **Along the diametric edges**

$$f^*(v_1v_{13}) = 1 \text{ as } gcd(2i, 3) = 1 \text{ for } i = 1$$
$$f^*(v_4v_{15}) = 1 \text{ as } gcd(2i, 23-2j) = 1 \text{ for } i = 4, j = 9$$
$$f^*(v_7v_{19}) = 0 \text{ as } gcd(14, 7) \neq 1$$
$$f^*(v_{10}v_{22}) = 1 \text{ as } gcd(2i, 23-2j) = 1 \text{ for } i = 0, j = 2$$

Here

$$e_f(0) = 1 \quad \text{and} \quad e_f(1) = 3 \tag{3.2}$$

(iii) **Along the chordal edges**

$$f^*(v_2v_9) = 0 \text{ as } gcd\{2i, 2(j+1)\} \neq 1 \text{ for } i = 1, j = 8$$
$$f^*(v_3v_{20}) = 0 \text{as } gcd(2i, 23-2j) \neq 1 \text{ for } i = 3, j = 4$$
$$f^*(v_5v_{12}) = 0 \text{ as } gcd\{2i, 2(j+1)\} \neq 1 \text{for } i = 5, j = 11$$
$$f^*(v_6v_{23}) = 0 \text{ as } gcd(2i, 23-2j) \neq 1 \text{ for } i = 6, j = 1$$
$$f^*(v_8v_{15}) = 1 \text{ as } gcd(2i, 23-2j) = 1 \text{ for } i = 8, j = 9$$
$$f^*(v_{11}v_{18}) = 0 \text{ as } gcd(2i, 23-2j) \neq 1 \text{ for } i = 11, j = 6$$
$$f^*(v_{14}v_{21}) = 1 \text{ as } gcd(1, 23-2j) = 1 \text{for } j = 3$$
$$f^*(v_{17}v_{24}) = 1 \text{ as } gcd(23-2i, 23-2j) = 1 \text{ for } i = 7, j = 0$$

Here

$$e_f(0) = 5 \quad \text{and} \quad e_f(1) = 3 \tag{3.3}$$

From (3.1), (3.2) and (3.3),
we get $e_f(0) = 18, e_f(1) = 18$ and hence
$|e_f(0) - e_f(1)| \leq 1$ is verified.
Hence the McGee graph G is a Prime Cordial graph. □

Steps for Coding a message

(1) A clue, one each, mathematical or non-mathematical is given to recognize the labeling and the graph used for coding.
(2) The numbering of alphabets is stated using abbreviation (such as PSNF), [7].
(3) The graph is drawn and numbers are assigned according to the labeling chosen.
(4) The message to be coded is written. For each letter in the message, the number assigned from numbering of alphabets is noted. This number is located in the graph which is labeled. Using certain notation with respect to the graph, the number is expressed. The letter now is coded.
(5) The coded form can be presented in a particular manner.
(6) The coding can also be presented in the picture form.

Illustration (Through McGee graph and Prime Cordial labeling)
Message: NUMBERS RULE THE UNIVERSE
Clue for the graph: A named graph unique with respect to regularity and girth.
Clue for the labeling: The foremost characteristic in a human relationship.

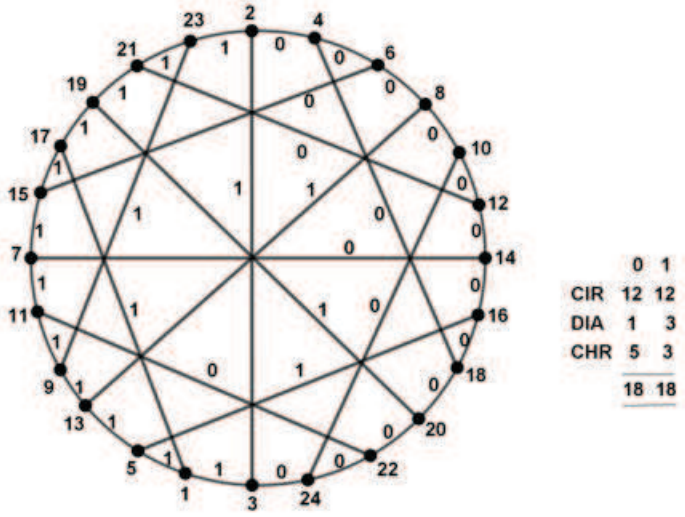

FIGURE 1. McGee graph with prime Cordial Labeling.

1	6	7	2	8	9	10	11	3	12	13	14	15
A	B	C	D	E	F	G	H	I	J	K	L	M
16	17	4	18	19	20	21	22	23	24	25	5	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

TABLE 1. Numbering of alphabets : PSNF (Perfect Square Numbered First)

The alphabets in the position$1^2, 2^2, 3^2, 4^2$,and 5^2 are given the numbers$1, 2, 3, 4, 5$. Then the letter B gets the number6. C gets the number 7 and E gets the number 8 and so on. This method is named as PSNF.

Method of coding:

The letter N from numbering of alphabets PSNF takes the value 16. In the labelled graph, 16 occurs as the 8th vertex, it is the head vertex of fifth chordal edge. So it is written as HCHD5. In the same way, the coding for letters U, M, B, E, R, S are done and listed below.

U - HCHD6 M - TCHD2 B - HCHD2

E - HDIA2 R - TDIA3 S - HDIA3

The word 'NUMBERS' in the coded form is as follows

HCHD5 HCHD6 TCHD2 HCHD2 HDIA2 TDIA3 HDIA3

Similarly all the other letters are coded.

It should be noted that as there are only 24 vertices on the McGee graph, the letters corresponding to numbers 25 and 26 can be coded in the following way. Express 25 as the sum of 2 integers, locate them in the labelled graph and write it as a sum.

$25 = 23 + 2, 23 - TCHD3, 2 - TDIA1$

The letter X corresponds to the number 25 is denoted by TCHD3 + TDIA1.

Coding wordwise:

NUMBERS - HCHD5 HCHD6 TCHD2 HCHD2 HDIA2 TDIA3 HDIA3

RULE - TDIA3 HCHD6 HDIA3 HDIA2

THE - TCHD4 TCHD6 HDIA2

UNIVERSE - HCHD6 HCHD5 TDIA1 TCHD8 HDIA2 TDIA3 HDIA3 HDIA2

Horizontal string:

HCHD5 TDIA3 TCHD4 HCHD6 HCHD6 HCHD6 TCHD6 HCHD5 TCHD2

HDIA3 HDIA2 TDIA1 HCHD2 HDIA2 TCHD8 HDIA2 HDIA2 TDIA3
HDIA3 TDIA3 HDIA3 HDIA2

The sender and the receiver must be familiar with graph labelings to a certain extent. The decoding is done by receiving the process.

4. Conclusion

The coding technique discussed in this paper is expected to be useful in communicating a message in any level, personal or official as it maintains a very high degree of secrecy and intricacy.

The authors have planned to work on a variety of labelings and produce more techniques on coding.

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