

SOME PROPERTIES OF INTUITIONISTIC DODECAGONAL FUZZY NUMBER AND ITS APPLICATION

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ABSTRACT. In this paper, we have introduced the basic arithmetic operations of Intuitionistic Dodecagonal Fuzzy Number (IDFN). Various properties of the Intuitionistic Dodecagonal Fuzzy Number are also introduced. It has been proved with numerical examples which help us to understand and deal the concept to gain better results. This paper entirely brings out the characteristics of an Intuitionistic Dodecagonal Fuzzy Number which plays a vital role in dealing uncertainty. We have found the ranking for cricket batsmen with the help of our newly defined IDFN using the new distance measure which is already an existing method.

1. INTRODUCTION

Zadeh invented fuzzy sets. Fuzzy set helps us to deal with information which is ambiguous in nature. It operates on data which are not clear and uncertain. Certain problems which were found fuzzy were dealt with the concepts of fuzzy set. Later, to betterment the results of such ambiguous problems, Atanassov [1] came out with the concept of intuitionistic fuzzy set which is a generalization of a FS. Atanassov. K and G. Gargov [2] dealt with the concept "Interval-Valued intuitionistic fuzzy sets". It was a pathway to many problems which ended up due to uncertainty. Burillo et al [3] conceptualized with the definition of an intuitionistic fuzzy number (IFN). J. Q. Wang, Z. Zhang [4, 5] used the concept of intuitionistic fuzzy number to multi criteria problems with aggregation operators as tools. Xia, M. M. Xu et al. [6] discussed on some new similarity measures for intuitionistic fuzzy values and their application in group decision making. The boosting factor

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in bringing up this paper lies on New Distance Measure for Atanassov's intuitionistic fuzzy sets and its application in decision making by Di Ke, Yafei Song and Wen Quan [7]. The main aim of the paper is to reduce the amount of vagueness by introducing Intuitionistic Dodecagonal Fuzzy Number (DIFN). We have used the distance measure formula for intuitionistic fuzzy sets using the newly defined intuitionistic dodecagonal fuzzy number and have ranked the players for choosing them in an IPL team. Also we have discussed the characteristics and properties of the IDFN with some numerical examples.

2. ARITHMETIC OPERATIONS OF INTUITIONISTIC DODECAGONAL FUZZY NUMBER

Consider two IDFN, $\tilde{A}_{DD}I$ and $\tilde{B}_{DD}$ denoted by:

$$\begin{aligned}\tilde{A}_{DD}I = (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \theta_5, \vartheta_6, \vartheta_7, \vartheta_8, \vartheta_9, \vartheta_{10}, \vartheta_{11}, \vartheta_{12}; \\ \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8, \tau_9, \tau_{10}, \tau_{11}, \tau_{12})\end{aligned}$$

and

$$\begin{aligned}\tilde{B}_{DD}I = (\varphi_1, \varphi_2, \varphi_2, \varphi_4, \varphi_5, \varphi_6, \varphi_7, \varphi_9, \varphi_9, \varphi_{10}, \varphi_{11}, \varphi_{12}; \\ \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}, \omega_{12}, \omega_{12})\end{aligned}$$

Then with their addition we have:

$$\begin{aligned}\tilde{A}_{DD}I + \tilde{B}_{DD}I = (\vartheta_1 + \varphi_1, \vartheta_2 + \varphi_2, \vartheta_2 + \varphi_3, \vartheta_4 + \varphi_4, \vartheta_5 + \varphi_5, \vartheta_6 + \varphi_6, \vartheta_7 \\ + \varphi_7, \vartheta_8 + \varphi_8, \vartheta_9 + \varphi_9, \vartheta_{10} + \varphi_{10}, \vartheta_{11} + \varphi_{11}, \vartheta_{12} + \varphi_{12}; \tau_1 \\ + \omega_1, \tau_2 + \omega_2, \tau_3 + \omega_3, \tau_4 + \omega_4, \tau_5 + \omega_5, \tau_6 + \omega_6, \tau_7 + \omega_7, \tau_8 \\ + \omega_8, \tau_9 + \omega_9, \tau_{10} + \omega_{10}, \tau_{11} + \omega_{11}, \tau_{12} + \omega_{12})\end{aligned}$$

With their subtraction we have:

$$\begin{aligned}\tilde{A}_{DD}I - \tilde{B}_{DD}I = (\vartheta_1 - \varphi_1, \vartheta_2 - \varphi_2, \vartheta_3 - \varphi_3, \vartheta_4 - \varphi_4, \vartheta_5 - \varphi_5, \vartheta_6 - \varphi_6, \vartheta_7 \\ - \varphi_7, \vartheta_8 - \varphi_8, \vartheta_9 - \varphi_9, \vartheta_{10} - \varphi_{10}, \vartheta_{11} - \varphi_{11}, \vartheta_{12} - \varphi_{12}; \tau_1 \\ - \omega_1, \tau_2 - \omega_2, \tau_3 - \omega_3, \tau_4 - \omega_4, \tau_5 - \omega_5, \tau_6 - \omega_6, \tau_7 - \omega_7, \tau_8 \\ - \omega_8, \tau_9 - \omega_9, \tau_{10} - \omega_{10}, \tau_{11} - \omega_{11}, \tau_{12} - \omega_{12})\end{aligned}$$

With multiplication we have:

$$\begin{aligned}\tilde{A}_{DD}I * \tilde{B}_{DD}I = (\vartheta_1 * \varphi_1, \vartheta_2 * \varphi_2, \vartheta_3 * \varphi_3, \vartheta_4 * \varphi_4, \vartheta_5 * \varphi_5, \vartheta_6 * \varphi_6, \vartheta_7 * \varphi_7, \vartheta_8 \\ * \varphi_8, \vartheta_9 * \varphi_9, \vartheta_{10} * \varphi_{10}, \vartheta_{11} * \varphi_{11}, \vartheta_{12} * \varphi_{12}; \tau_1 * \omega_1, \tau_2 * \omega_2, \tau_3 \\ * \omega_3, \tau_4 * \omega_4, \tau_5 * \omega_5, \tau_6 * \omega_6, \tau_7 * \omega_7, \tau_8 \\ * \omega_8, \tau_9 * \omega_9, \tau_{10} * \omega_{10}, \tau_{11} * \omega_{11}, \tau_{12} * \omega_{12})\end{aligned}$$

And with division we have:

$$\tilde{A}_{DD}I/\tilde{B}_{DD}I = \left(\frac{\vartheta_1}{\varphi_1}, \frac{\vartheta_2}{\varphi_2}, \frac{\vartheta_3}{\varphi_3}, \frac{\vartheta_4}{\varphi_4}, \frac{\vartheta_5}{\varphi_5}, \frac{\vartheta_6}{\varphi_6}, \frac{\vartheta_7}{\varphi_7}, \frac{\vartheta_8}{\varphi_8}, \frac{\vartheta_9}{\varphi_9}, \frac{\vartheta_{10}}{\varphi_{10}}, \frac{\vartheta_{11}}{\varphi_{11}}, \frac{\vartheta_{12}}{\varphi_{12}}, \right. \\ \left. \frac{\tau_1}{\omega_1}, \frac{\tau_2}{\omega_2}, \frac{\tau_3}{\omega_3}, \frac{\tau_4}{\omega_4}, \frac{\tau_5}{\omega_5}, \frac{\tau_6}{\omega_6}, \frac{\tau_7}{\omega_7}, \frac{\tau_8}{\omega_8}, \frac{\tau_9}{\omega_9}, \frac{\tau_{10}}{\omega_{10}}, \frac{\tau_{11}}{\omega_{11}}, \frac{\tau_{12}}{\omega_{12}} \right)$$

3. BASIC OPERATIONS AND RELATIONS OF INTUITIONISTIC DODECAGONAL FUZZY NUMBER

The following operations are defined:

Union:

$$\tilde{A}_{DD} \cup \tilde{B}_{DD} = \{x, \max(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \min(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x)) / x \in X\}$$

Intersection:

$$\tilde{A}_{DD} \cap \tilde{B}_{DD} = \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x)) / x \in X\}$$

Inclusion:

$$\tilde{A}_{DD} \subseteq \tilde{B}_{DD} \Leftrightarrow \vartheta_{\tilde{A}_{DD}}(x) \leq \varphi_{\tilde{B}_{DD}}(x), \varphi_{\tilde{A}_{DD}}(x) \geq \vartheta_{\tilde{B}_{DD}}(x), \forall x \in X$$

Equality:

$$\tilde{A}_{DD} = \tilde{B}_{DD} \Leftrightarrow \vartheta_{\tilde{A}_{DD}}(x) = \varphi_{\tilde{B}_{DD}}(x), \varphi_{\tilde{A}_{DD}}(x) = \vartheta_{\tilde{B}_{DD}}(x)$$

3.1. Properties of intuitionistic dodecagonal fuzzy number.

1. Commutative:

- (a) $\tilde{A}_{DD} \cup \tilde{B}_{DD} = \tilde{B}_{DD} \cup \tilde{A}_{DD}$
- (b) $\tilde{A}_{DD} \cap \tilde{B}_{DD} = \tilde{B}_{DD} \cap \tilde{A}_{DD}$

Proof. (a) Let $x \in X$

$$\vartheta_{\tilde{A}_{DD} \cup \tilde{B}_{DD}}(x) = \{x, \max(\vartheta_{\tilde{A}_{DD}I}(x), \vartheta_{\tilde{B}_{DD}I}(x)), \min(\varphi_{\tilde{A}_{DD}I}(x), \varphi_{\tilde{B}_{DD}I}(x)) / x \in X\} \\ = \{x, \max(\vartheta_{\tilde{B}_{DD}I}(x), \vartheta_{\tilde{A}_{DD}I}(x)), \min(\varphi_{\tilde{B}_{DD}I}(x), \varphi_{\tilde{A}_{DD}I}(x)) / x \in X\} \\ = \vartheta_{\tilde{B}_{DD} \cup \tilde{A}_{DD}}(x)$$

(b) Let $x \in X$

$$\vartheta_{\tilde{A}_{DD} \cap \tilde{B}_{DD}}(x) = \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x)) / x \in X\} \\ = \{x, \min(\vartheta_{\tilde{B}_{DD}}(x), \vartheta_{\tilde{A}_{DD}}(x)), \max(\varphi_{\tilde{B}_{DD}}(x), \varphi_{\tilde{A}_{DD}}(x)) / x \in X\} \\ = \vartheta_{\tilde{B}_{DD} \cap \tilde{A}_{DD}}(x)$$

Hence proved. □

2. Associative:

$$(a) \left(\tilde{A}_{DD} \cup \tilde{B}_{DD} \right) \cup \tilde{C}_{DD} = \tilde{A}_{DD} \cup \left(\tilde{B}_{DD} \cup \tilde{C}_{DD} \right)$$

$$[(b)] \left(\tilde{A}_{DD} \cap \tilde{B}_{DD} \right) \cap \tilde{C}_{DD} = \tilde{A}_{DD} \cap \left(\tilde{B}_{DD} \cap \tilde{C}_{DD} \right)$$

Proof. Let

$$\begin{aligned}\tilde{A}_{DD} &= \{(1, 2, 3); (0, 1, 2)\}, \\ \tilde{B}_{DD} &= \{(1, 3, 5); (1, 2, 3)\}, \\ \tilde{C}_{DD} &= \{(2, 3, 4); (0, 2, 5)\}.\end{aligned}$$

$$\begin{aligned}M_{\vartheta}(\tilde{A}_{DD}) &= \frac{1}{4}(1 + 4 + 3) = 2 & M_{\varphi}(\tilde{A}_{DD}) &= \frac{1}{4}(0 + 2 + 2) = 1 \\ M_{\vartheta}(\tilde{B}_{DD}) &= \frac{1}{4}(1 + 6 + 5) = 3 & M_{\varphi}(\tilde{B}_{DD}) &= \frac{1}{4}(1 + 4 + 3) = 2 \\ M_{\vartheta}(\tilde{C}_{DD}) &= \frac{1}{4}(2 + 6 + 4) = 3 & M_{\varphi}(\tilde{C}_{DD}) &= \frac{1}{4}(0 + 4 + 5) = 2.25\end{aligned}$$

$$\tilde{A}_{DD} \cap \tilde{B}_{DD} = \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x)) / x \in X\}$$

$$\tilde{A}_{DD} \cap \tilde{B}_{DD} = \{\min(2, 3), \max(1, 2)\} = (2, 2)$$

$$\left(\tilde{A}_{DD} \cap \tilde{B}_{DD} \right) \cap \tilde{C}_{DD} = (2, 2.25)$$

$$\tilde{B}_{DD} \cap \tilde{C}_{DD} = \{x, \min(\vartheta_{\tilde{B}_{DD}}(x), \vartheta_{\tilde{C}_{DD}}(x)), \max(\varphi_{\tilde{B}_{DD}}(x), \varphi_{\tilde{C}_{DD}}(x)) / x \in X\}$$

$$\tilde{B}_{DD} \cap \tilde{C}_{DD} = \{\min(3, 3), \max(2, 2.25)\} = (3, 2.25)$$

$$\begin{aligned}\tilde{A}_{DD} \cap \left(\tilde{B}_{DD} \cap \tilde{C}_{DD} \right) &= \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD} \cap \tilde{C}_{DD}}(x)), \\ &\quad \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD} \cap \tilde{C}_{DD}}(x))\} \\ &= \{\min(2, 3), \max(1, 2.25)\}\end{aligned}$$

$$\left(\tilde{A}_{DD} \cap \tilde{B}_{DD} \right) \cap \tilde{C}_{DD} = (2, 2.25)$$

□

3. Idempotence:

$$(a) \tilde{A}_{DD} \cup \tilde{A}_{DD} = \tilde{A}_{DD}$$

$$(b) \tilde{A}_{DD} \cap \tilde{A}_{DD} = \tilde{A}_{DD}$$

4. Distributive:

$$(a) \tilde{A}_{DD} \cap \left(\tilde{B}_{DD} \cup \tilde{C}_{DD} \right) = \left(\tilde{A}_{DD} \cap \tilde{B}_{DD} \right) \cup \left(\tilde{A}_{DD} \cap \tilde{C}_{DD} \right)$$

$$(b) \tilde{A}_{DD} \cup \left(\tilde{B}_{DD} \cap \tilde{C}_{DD} \right) = \left(\tilde{A}_{DD} \cup \tilde{B}_{DD} \right) \cap \left(\tilde{A}_{DD} \cup \tilde{C}_{DD} \right)$$

5. De Morgan's Property:

$$(a) \overline{\tilde{A}_{DD} \cap \tilde{B}_{DD}} = \overline{\tilde{A}_{DD}} \cup \overline{\tilde{B}_{DD}}$$

$$(b) \overline{\tilde{A}_{DD} \cup \tilde{B}_{DD}} = \overline{\tilde{A}_{DD}} \cap \overline{\tilde{B}_{DD}}$$

Proof.

$$\tilde{A}_{DD} = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13; 0, 1, 3, 5, 6, 7, 9, 11, 14, 15, 16, 17)$$

$$\tilde{B}_{DD} = (0, 3, 6, 9, 10, 12, 14, 16, 18, 20, 22, 24; -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8)$$

$$\tilde{C}_{DD} = (4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48; 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)$$

L.H.S:

$$\tilde{A}_{DD} \cap \tilde{B}_{DD} = \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x)) / x \in X\}$$

$$\begin{aligned} M_{\vartheta}(\tilde{A}_{DD}) &= \frac{1}{4}[(a_1 + a_2 + a_3 + a_{10} + a_{11} + a_{12})k \\ &\quad + (a_4 + a_5 + a_6 + a_7 + a_8 + a_9)(1 - k)] \\ &\Rightarrow \frac{1}{4}[(2 + 3 + 4 + 11 + 12 + 13) * (0.5) \\ &\quad + (5 + 6 + 7 + 8 + 9 + 10) * (0.5)] \\ &\Rightarrow \frac{1}{4}[(45 * 0.5) + (45 * 0.5)] \\ &\Rightarrow \frac{1}{4}(22.5 + 22.5) \Rightarrow \frac{45}{4} \Rightarrow 11.25 \end{aligned}$$

$$\begin{aligned} M_{\varphi}(\tilde{A}_{DD}) &= 13; & M_{\vartheta}(\tilde{B}_{DD}) &= 19.25; & M_{\varphi}(\tilde{B}_{DD}) &= 3.75; \\ M_{\vartheta}(\tilde{C}_{DD}) &= 19.25; & M_{\varphi}(\tilde{C}_{DD}) &= 9.75 \end{aligned}$$

$$\tilde{A}_{DD} = (11.25, 13); \quad \tilde{B}_{DD} = (19.25, 3.75); \quad \tilde{C}_{DD} = (19.5, 9.75)$$

$$(a) \quad \overline{\tilde{A}_{DD} \cap \tilde{B}_{DD}} = \overline{\tilde{A}_{DD} \cup \tilde{B}_{DD}}$$

L.H.S:

$$\begin{aligned} \tilde{A}_{DD} \cap \tilde{B}_{DD} &= \{x, \min(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \max(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x))\} \\ &\Rightarrow \{\min(11.25, 19.25), \max(13, 3.75)\} \\ &\Rightarrow (11.25, 13) \end{aligned}$$

$$\overline{\tilde{A}_{DD} \cap \tilde{B}_{DD}} = \{x, \vartheta_{\tilde{A}_{DD}}(x), \varphi_{\tilde{A}_{DD}}(x), x \in X\} \Rightarrow (13, 11.25)$$

$$\tilde{A}_{DD} = \{x, \vartheta_{\tilde{A}_{DD}}(x), \varphi_{\tilde{A}_{DD}}(x), x \in X\} \Rightarrow (3.75, 11.25)$$

$$\tilde{B}_{DD} = \{x, \vartheta_{\tilde{B}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x), x \in X\} \Rightarrow (13, 19.25)$$

$$\begin{aligned} \overline{\tilde{A}_{DD} \cup \tilde{B}_{DD}} &= \{x, \max(\vartheta_{\tilde{A}_{DD}}(x), \vartheta_{\tilde{B}_{DD}}(x)), \min(\varphi_{\tilde{A}_{DD}}(x), \varphi_{\tilde{B}_{DD}}(x))\} \\ &\Rightarrow \{\max(3.75, 13), \min(11.25, 19.25)\} \\ &\Rightarrow (13, 11.25) \end{aligned}$$

Hence proved. □

3.2. Some more properties of IDFN on bounded sum and bounded product.

1. Commutative Law:

$$(a) \tilde{A}_{DD} \otimes \tilde{B}_{DD} = \tilde{B}_{DD} \otimes \tilde{A}_{DD}$$

$$(b) \tilde{A}_{DD} \oplus \tilde{B}_{DD} = \tilde{B}_{DD} \oplus \tilde{A}_{DD}$$

2. Associative Law:

$$(a) (\tilde{A}_{DD} \otimes \tilde{B}_{DD}) \otimes \tilde{C}_{DD} = \tilde{A}_{DD} \otimes (\tilde{B}_{DD} \otimes \tilde{C}_{DD})$$

$$(b) (\tilde{A}_{DD} \oplus \tilde{B}_{DD}) \oplus \tilde{C}_{DD} = \tilde{A}_{DD} \oplus (\tilde{B}_{DD} \oplus \tilde{C}_{DD})$$

3. De Morgan's Law:

$$(a) \bar{\tilde{A}}_{DD} \otimes \tilde{B}_{DD} = \bar{\tilde{A}_{DD} \otimes \tilde{B}_{DD}}$$

$$(b) \tilde{A}_{DD} \oplus \tilde{B}_{DD} = \tilde{A}_{DD} \otimes \tilde{B}_{DD}$$

4. Hamming Distance:

$$\begin{aligned} D'(A, B) &= \sum_{d=1}^n |\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)| + |\varphi_{\tilde{A}_{DD}}(u_d) - \varphi_{\tilde{B}_{DD}}(u_d)| \\ &\Rightarrow \sum_{d=1}^n |\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)| + |1 - \vartheta_{\tilde{A}_{DD}}(u_d) - 1 + \vartheta_{\tilde{B}_{DD}}(u_d)| \\ &\Rightarrow 2D(A, B) \end{aligned}$$

i. e. it is twice as large as the Hamming distance of a fuzzy set.

5. Normalized Hamming Distance:

$$N'(A, B) = \frac{1}{k} D'(A, B) = \frac{2}{k} \sum_{d=1}^n |\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)|$$

6. Euclidean Distance:

$$\begin{aligned} \sigma'(A, B) &= \sqrt{\sum_{d=1}^n [\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)]^2 + [\varphi_{\tilde{A}_{DD}}(u_d) - \varphi_{\tilde{B}_{DD}}(u_d)]^2} \\ &\Rightarrow \sqrt{\sum_{d=1}^n [\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)]^2 + [1 - \vartheta_{\tilde{A}_{DD}}(u_d) - 1 + \vartheta_{\tilde{B}_{DD}}(u_d)]^2} \\ &\Rightarrow \sqrt{2 \sum_{d=1}^n [\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)]^2} \end{aligned}$$

7. Normalized Euclidean Distance:

$$Q'(A, B) = \sqrt{\frac{1}{k} \sigma'(A, B)} = \sqrt{\frac{2}{k} \sum_{d=1}^n [\vartheta_{\tilde{A}_{DD}}(u_d) - \vartheta_{\tilde{B}_{DD}}(u_d)]^2}$$

3.3. Application of the distance measure to multi criteria problem.

Algorithm:

Step 1: Obtain the relative positive and negative solution

$$\sigma^+ = (\sigma_1^+, \sigma_2^+, \dots, \sigma_n^+) \quad \text{and} \quad \sigma^- = (\sigma_1^-, \sigma_2^-, \dots, \sigma_n^-)$$

of the attributes, where each value σ_k^+ and σ_k^- is calculated as:

$$\begin{aligned} \sigma_k^+ &= \max_{m=1,2,3,\dots,l} \{g_{mn}\} = \left\langle \max_{m=1,2,3,\dots,l} \{\vartheta_{mn}\}, \min_{m=1,2,3,\dots,l} \{\varphi_{mn}\} \right\rangle \\ &= \langle \vartheta_k^+, \varphi_k^+ \rangle, \quad k = 1, 2, \dots, n. \end{aligned}$$

$$\begin{aligned} \sigma_k^- &= \min_{m=1,2,3,\dots,l} \{g_{mn}\} = \left\langle \min_{m=1,2,3,\dots,l} \{\vartheta_{mn}\}, \max_{m=1,2,3,\dots,l} \{\varphi_{mn}\} \right\rangle \\ &= \langle \vartheta_k^-, \varphi_k^- \rangle, \quad k = 1, 2, \dots, n. \end{aligned}$$

Step 2: With the distance measure, the similitude measure $g_{mn} = \langle \vartheta_{mn}, \varphi_{mn} \rangle$ is calculated. Construct the positive similitude measure matrix $A^+ = (a_{mn}^+)_{i \times j}$. The similitude measure is expressed as

$$\begin{aligned} a_{mn}^+ &= 1 - G(g_{mn}, \rho_k^+) \\ &= 1 - \sqrt{\left(\frac{\vartheta_{mn} - \varphi_{mn}}{2}\right)^2 + \frac{1}{3} \left(\frac{\vartheta_{mn} + \varphi_{mn}}{2} - \frac{\vartheta_k^+ + \varphi_k^+}{2}\right)^2} \end{aligned}$$

Step 3: With the distance measure, the similitude measure $g_{mn} = \langle \vartheta_{mn}, \varphi_{mn} \rangle$ is calculated. Construct the negative similitude measure matrix $A^- = (a_{mn}^-)_{i \times j}$. The similitude measure is expressed as

$$\begin{aligned} a_{mn}^- &= 1 - G(g_{mn}, \rho_k^-) \\ &= 1 - \sqrt{\left(\frac{\vartheta_{mn} - \varphi_{mn}}{2}\right)^2 + \frac{1}{3} \left(\frac{\vartheta_{mn} + \varphi_{mn}}{2} - \frac{\vartheta_k^- + \varphi_k^-}{2}\right)^2} \end{aligned}$$

Step 4: With the given weights w_i and the distance matrices A^+ and A^- , we have to calculate the weighted positive score $U^+(p_i)$ and weighted negative score $U^-(p_i)$. The weighted positive score is calculated by $U^+(p_i) = \sum_{i=1}^m w_i g_{mn}^+$ and the weighted negative score is calculated by $U^-(p_i) = \sum_{i=1}^m w_i g_{mn}^-$

Step 5: Obtain the correlative nearness degree $V(x_i) = \frac{U^+(x_i)}{U^+(x_i) + U^-(x_i)}$ for each alternative $p_i, i = 1, 2, \dots, m$.

Step 6: Get the likeness sequence of all the alternatives by comparing their correlative nearness degree. Larger nearness degree indicates better likeness sequence.

4. NUMERICAL EXAMPLE

Five cricket players have been chosen randomly. The best player is to be analyzed for a team. These five players are expressed as p_1, p_2, p_3, p_4, p_5 . They will be analyzed on four attributes namely, strike rate, average, not out and runs scored which are represented as e_1, e_2, e_3, e_4 respectively. The weights of the attributes are $w_1 = 0.1$; $w_2 = 0.2$; $w_3 = 0.3$ and $w_4 = 0.4$. The decision matrix in intuitionistic fuzzy number is shown below:

TABLE 1

Players		e_1	e_2	e_3	e_4
Virat Kohli	p_1	$\langle 5.575; 1.725 \rangle$	$\langle 5.325; 2.325 \rangle$	$\langle 3.825; 0.975 \rangle$	$\langle 12.825; 9.825 \rangle$
AB de Villiers	p_2	$\langle 8.557; 4.2 \rangle$	$\langle 6.925; 3.825 \rangle$	$\langle 5.325; 0.975 \rangle$	$\langle 11.325; 8.325 \rangle$
CH Gayle	p_3	$\langle 7.25; 3.825 \rangle$	$\langle 12.825; 8.325 \rangle$	$\langle 11.325; 8.325 \rangle$	$\langle 11.325; 8.325 \rangle$
KA Pollard	p_4	$\langle 8.55; 3.45 \rangle$	$\langle 5.025; 0.975 \rangle$	$\langle 5.325; 0.975 \rangle$	$\langle 8.55; 3.45 \rangle$
KL Rahul	p_5	$\langle 7.35; 2.85 \rangle$	$\langle 7.25; 0.975 \rangle$	$\langle 13.425; 8.925 \rangle$	$\langle 5.325; 0.975 \rangle$

Step 1: The relative positive and negative solutions for each attribute:

$$\begin{aligned}
 \rho_1^+ &= \langle 8.557; 1.725 \rangle & \rho_2^+ &= \langle 12.825; 0.975 \rangle \\
 \rho_3^+ &= \langle 11.325; 0.975 \rangle & \rho_4^+ &= \langle 12.825; 0.975 \rangle \\
 \rho_1^- &= \langle 5.575; 4.2 \rangle & \rho_2^- &= \langle 5.025; 8.325 \rangle \\
 \rho_3^- &= \langle 3.825; 8.925 \rangle & \rho_4^- &= \langle 5.325; 9.825 \rangle
 \end{aligned}$$

Step 2: The positive and negative similarity matrix is calculated with the distance measure formula and is given below:

Positive Similarity Matrix

0.7220	1.3241	1.5919	0.9300
1.2711	0.7826	0.7803	0.7958
0.7506	0.8641	1.5980	1.2580
1.3040	2.2080	1.7800	1.6020
1.2375	2.5260	2.6710	2.0680

Negative Similarity Matrix

1.0530	1.2286	1.7010	0.8700
1.2897	0.7220	1.8630	0.7320
0.7530	1.5220	0.8430	0.9843
1.6296	1.9320	1.8630	1.7072
1.2533	2.4600	3.2190	2.3280

Step 3: According to the attribute weights $w_1 = 0.1$; $w_2 = 0.2$; $w_3 = 0.3$ and $w_4 = 0.4$ we can get the weighted positive scores of all alternatives as:

$$\begin{aligned}
 S^+(x_1) &= 1.1865; & S^+(x_2) &= 1.1360; & S^+(x_3) &= 1.2304; \\
 S^+(x_4) &= 1.7468; & S^+(x_5) &= 2.4734
 \end{aligned}$$

The weighted negative scores of all alternatives is calculated and given below:

$$\begin{aligned}
 S^-(x_1) &= 1.20932; & S^-(x_2) &= 1.12507; & S^-(x_3) &= 1.02632; \\
 S^-(x_4) &= 1.79114; & S^-(x_5) &= 2.51423
 \end{aligned}$$

Step 4: The relative closeness degree of each alternative is calculated and given below:

$$\begin{aligned} T(x_1) &= 0.4952; & T(x_2) &= 0.5024; & T(x_3) &= 0.5452; \\ T(x_4) &= 0.4937; & T(x_5) &= 0.4959 \\ T(x_3) &> T(x_2) > T(x_5) > T(x_1) > T(x_4) \end{aligned}$$

5. CONCLUSION

According to the ordering given above, we have obtained the ranking of each player and it is given below:

Players	Rank
Chris Gayle	1
AB de Villiers	2
KL Rahul	3
Virat Kohli	4
KA Pollard	5

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