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IMPACT AND STEADY STATE ANALYSIS $M/G/1$ QUEUING SYSTEM WITH RANDOM REVOLUTION SUBJECT TO CATASTROPHICS EVENT

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ABSTRACT. This paper manages the consistent state conduct of single stage $M/G/1$ line(queue) in random revolution subject to Catastrophics event. This model set non-markovian queuing model. The appearance happen in a poisson procedure with parameter and the administration times are accepted to have an overall help time dissemination. This model contain a Random revolution when the Catastrophics event happens the framework. This model centers the consistent state arrangement of queuing model, we infer the probability generating capacity of Line size appropriation of Random age and we determine the Laplace Stieltjes transform of Random revolution.

1. INTRODUCTION

A non-markovian queuing models have a single stage appearance line with breakdown and Random revolution. In this $M/G/1$ queuing model once the client enter they will be given the administration in the phases of administration office individually in progression. When the administration gets finished, clients withdraws from the framework. One the administration is abrupt breakdown, there will be an inaccessibility of administration, all things considered Catastrophics event happens. Catastrophics is expected to follows an exponential distribution. Appearance follows a poisson procedure and administration

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time follows general distribution. Utilizing supplementary variable procedure we determine the steady state probabilities and execution measures. At that point the numerical portrayal of the model is given, the documentations are depicted. The conditions administering the framework are given detail utilizing birth and passing procedure, the line size circulation of Random age is featured. A Complete thought regarding the model wherein bit by bit technique is followed for better understanding which helps in any expansion of this model.

2. Mathematical Analysis of the System

Accept that the client show up at the framework as per a Poisson procedure with a pace of appearance λ . Let $\lambda d_j dt (j = 1, 2)$ be the probability that jth client shows up at the framework during short time frame $(y + dy)$, where $0 \leq d_j \leq 1$ and $\sum_{j=1}^{\infty} d_j$ and $\lambda > 0$ is the mean appearance rate. We accept that the administration time dispersion with administration rate $\eta > 0$ and mean help time $1/\eta$. There is a solitary server and the administration time follows general distribution with Probability dissemination work $B_j(y)$ and probability density work $b_j(y), j = 1, 2,$

(2.1)

$$\text{Let } \alpha_j(y) = \frac{b_j(y)}{1 - B_j(y)}, j = 1, 2, \quad b_j(s) = \alpha_j exp\left[-\int_0^v \alpha_j(y)dy\right], j = 1, 2.$$

Consider a solitary server queueing framework where the server is breakdown, on account of Catastrophics event happens then the Random revolution shows up. Allow us to anticipate the Catastrophics events, after the overall restrictive probability law with distribution work $W(y)$ and thickness work w(y). Here, let us think about that (y) is the restrictive probability of a period in the midst of the stretch $(y, y + dy)$, given that the sneaked past time is y, which can given as

$$r(y) = \frac{w(y)}{1 - W(y)}, \qquad w(t) = r(v)e\left[-\int_0^v r(y)dy\right]. \tag{2.2}$$

3. NOTATIONS

$\Theta_n(y,t)$: Probability that at time t , the server is dynamic offering support and there are $n(n \geq 0)$ clients in the line barring the one being served in the j^{th} phase of administration and the passed administration time for this client is y. Thus $\Theta_n(t) = \int\limits_0^\infty \Theta_n(y,t)dy$ means probability that at time there are n clients in the line barring the one client in the single stage regardless of the estimation of y. $K(y,t)$; Probability that at time t, the administration is on breakdown with Catastrophics event time y and there are $n(n > 0)$customers sitting tight in the line for administration. Thus , $K_n(t) = \int\limits_0^\infty K_n(y,t)dy$ indicates the probaiblity that at time t there are n clients in the line and the administration is on breakdown with Disastrous events regardless of the estimation of y. $I(t)$: indicate the probability that there are no clients in the framework and the server is out of gear period, administration period and breakdown with Catastrophics.

4. STEADY STATE CONDITIONS ADMINISTERING THE SYSTEM

In this section, we initially build up the steady state differential-difference conditions for the model under contemplations are,

$$\frac{d}{dy}\Theta_n(y,t) + (\lambda + \alpha(y))\Theta_n(y) = \lambda \sum_{j=1}^{n-1} d_j \Theta_{n-j}(y), \quad n \geq 1 \tag{4.1}$$

$$\frac{d}{dy}\Theta_n(y,t) + (\lambda + \alpha(y))\Theta_n(y) = 0 \tag{4.2}$$

$$\frac{d}{dy}K_n(y) + (\lambda + r(y) + \eta)K_n(y) = \lambda \sum_{j=1}^{n-1} d_j K_{n-j}(y) + \eta K_{n+1}(y) \tag{4.3}$$

$$\frac{d}{dy}K_0(y) + (\lambda + r(y) + \eta)K_0(y) = \eta K_1(y) \tag{4.4}$$

$$\lambda I = \int\limits_0^\infty K_0(y) r(y) dy \tag{4.5}$$

The above conditions are to be explained to the limit condition given at $y = 0$.

$$\Theta_n(0) = \int_0^\infty K_{n+1}(y)r(y)dy + \lambda d_{n+1}I, \quad n \geq 0, \tag{4.6}$$

$$K_n(0) = \int_0^\infty \Theta_n(y)\alpha(y)dy. \tag{4.7}$$

5. Steady State Solution

We characterize the probability generation function as follows:

$$\Theta_q^{(j)}(y,z) = \sum_{n=0}^\infty z^n \Theta_q^{(j)}(y), \quad \Theta_q^{(j)}(z,t) = \sum_{n=0}^\infty z^n \Theta_q^{(j)}(y) \qquad (j = 1,2)$$

$$K_q(y,z) = \sum_{n=0}^\infty z^n K_n(y,t) \Rightarrow K_q(z,t) = \sum_{n=0}^\infty z^n K_n,\ d(z) = \sum_{n=0}^\infty d_j z^j \quad (|z| \leq 1).$$

6. The Steady state queue size distribution

Cox [3](1955) has investigated non-Markovian models by changing them into Markovian ones, through the presentation of at least one supplementary factors. A stable recursive plan for the estimation of the restricting probabilities can be created, in view of an adaptable regenerative approach, Multiplying (4.1) by z^n, summing over n and adding the result to eq (4.3) and again using probability generating function, we get

$$\sum_{n=0}^\infty z^n \frac{d}{dy}\Theta_n(y,t) + \sum_{n=0}^\infty z^n(\lambda + \alpha(y))\Theta_n(y) = \sum_{n=0}^\infty z^n \left(\lambda \sum_{j=1}^{n-1} d_j \Theta_{n-j}(y)\right)$$

$$\frac{d}{dy}\Theta_q(y,z) + (\lambda + \alpha(y) - \lambda d(z))\Theta_q(y,z) = 0 \tag{6.1}$$

$$\begin{aligned}\sum_{n=0}^\infty z^n \frac{d}{dy}K_n(y) + \sum_{n=0}^\infty z^n(\lambda + r(y) + \eta)K_n(y) \\ = \sum_{n=0}^\infty z^n \left(\lambda \sum_{i=1}^{n-1} d_i K_{n-i}(y) + \eta \sum_{n=0}^\infty z^n K_{n+1}(y)\right)\end{aligned}$$

$$\frac{d}{dy}K_q(y,z) + (\lambda + r(y) + \eta - \frac{\eta}{z} - \lambda d(z))K_q(y,z) = 0. \tag{6.2}$$

Next, Similar operation are carried out on the boundary condition(4.6), we get type equation here:

$$\sum_{n=0}^{\infty} z^n \Theta_n(0) = \int_0^{\infty} \sum_{n=0}^{\infty} z^n K_{n+1}(y) r(y) dy + \sum_{n=0}^{\infty} z^n \lambda d_{n+1} I$$

(6.3)
$$z\Theta_q(0,z) = \int_0^{\infty} K_q(y,z) r(z) dy + \lambda(d(z)-1)I$$

$$\sum_{n=0}^{\infty} z^n K_n(0) = \int_0^{\infty} \sum_{n=0}^{\infty} z^n \Theta_n(y) \alpha(y) dy$$

(6.4)
$$K_q(0,z) = \int_0^{\infty} \Theta_q(y,z) \alpha(y) dy$$

Now we integrate equations (6.1) - (6.2) between the limits 0 to y

(6.5)
$$Ye^{\int P dy} = \int \Theta e^{\int P dy} dy + C$$

(6.6)
$$\Theta_q(y,z) = \Theta_q(0,z) e^{-(\lambda - \lambda d(z))y - \int_0^y \alpha(t) dt}$$

(6.7)
$$K_q(y,z) = K_q(0,z) e^{-\psi y - \int_0^y r(t) dt}$$

Here, $\psi = (\lambda + \eta - \frac{\eta}{2} - \lambda d(z))$, $\Theta_q(0,z), K_q(0,z)$ are given by equations (6.3) and (6.4).

Next we integrate (6.6) and (6.7) with respect to y, by parts we get:

$$\Theta_q(z) = \int_0^{\infty} \Theta_q(y,z) dy$$

(6.8)
$$\Theta_q(z) = \Theta_q(0,z) \frac{1 - \overline{B_1}(\lambda - \lambda d(z))}{\lambda - \lambda d(z)},$$

where $\int\limits_0^\infty e^{-Ky}(1-B(y))dy = \frac{1-\overline{B_1}(K)}{K}$,

$$K_q(z) = \int\limits_0^\infty K_q(y,z)dy$$

$$K_q(z) = K_q(0,z)\frac{1-\overline{W}(\psi)}{\psi}, \tag{6.9}$$

where $\overline{B_j}(\lambda - \lambda d(z)) = \int\limits_0^\infty e^{-(\lambda-\lambda d(z))y}dB_j(y), j = 1,2$ is the Laplace stieltjes change of the j^{th} administration time and $\overline{W}(\psi) = \int\limits_0^\infty e^{-(\psi)x}dW(y)$ is the Laplace stieltjes change at the random revolution.

Let find $\int\limits_0^\infty \Theta_q(y,z)\alpha(y)dy$, and $\int\limits_0^\infty K_q(y,z)r(y)dy$. For this purpose, we multiply the equation (6.6) (6.7)by $\alpha(y)$ and $r(y)$ respectively and integrate each with respect to y.

$$\int\limits_0^\infty \Theta_q(y,z)\alpha(y)dy = \int\limits_0^\infty \Theta_q(0,z)e^{-(\lambda-\lambda d(z))y - \int\limits_0^y \alpha(t)dt}\alpha(y)dy$$
$$= \Theta_q(0,z)\int\limits_0^\infty e^{-(\lambda-\lambda d(z))y}dB_j(y).$$

Hence, we get

$$\int\limits_0^\infty \Theta_q(y,z)\alpha(y)dy = \Theta_q(0,z)\overline{B_1}(\lambda - \lambda d(z)), \tag{6.10}$$

$$\int\limits_0^\infty K_q(y,z)r(y)dy = \int\limits_0^\infty K_q(0,z)e^{-\psi y - \int\limits_0^y r(t)dt}r(y)dy,$$

and further,

$$\int\limits_0^\infty K_q(y,z)r(y)dy = K_q(0,z)\overline{W}(\psi). \tag{6.11}$$

Using the equations (6.10) - (6.11) into (6.3) - (6.4), we get

$$\Theta_q(0,z) = \frac{(\lambda d(z) - \lambda)I}{\left(z - \overline{B_1}(\lambda - \lambda d(z))\overline{W}(\psi)\right)}.$$

From equation (6.8), we obtain

$$\Theta_q(z) = \frac{-I(1-\overline{B_1}(\lambda-\lambda d(z))}{z-\overline{B_1}(\lambda-\lambda d(z))\overline{W}(\psi)}. \tag{6.12}$$

We perform the similar operations on (6.8) and (6.9), we obtain

$$K_q(z) = \frac{-I(1-\overline{W}(\psi))(\lambda-\lambda d(z))(\overline{B_1}(\lambda-\lambda d(z)))}{(\psi)(z-\beta\overline{B_1}(\lambda-\lambda d(z))\overline{W}(\psi))}. \tag{6.13}$$

Let us now we characterize H_q as the generating function the line size

$$H_q(z) = \Theta_q(z) + K_q(z).$$

To decide the inactive (idle) time I.
The normalizing condition is given by the condition $I + H_q(1) = 1$.
Furthermore, we acquire usage certainty of $\rho = 1 - I$.
The normal number of client in the framework $L = L_q + \rho$.
The normal holding up time in the line $W_q = L_q/\lambda$.
The holding up time in the framework $W_s = L/\lambda$.

7. Conclusion

In this paper, the generating function procedure is utilized to determine another exquisite explict answer for a $M/G/1$ queuing framework with random revolution subject to Catastrohes events. The Steady State arrangements are gotten by utilizing supplementary variable procedure and the fixed line size distribution at a Random epoch has been determined.

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