Advances in Mathematics: Scientific Journal **9** (2020), no.8, 6229–6240
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.8.90 Special Issue on ICMA-2020

NEW RANKING OF GENERALIZED HEXAGONAL FUZZY NUMBER USING CENTROIDS OF CENTROIDED METHOD

A. THIRUPPATHI AND C. K. KIRUBHASHANKAR[1]

ABSTRACT. Ranking fuzzy numbers plays an important role in optimization techniques like assignment problem, transportation problem, project schedules, network flow analysis and uncertain environment in organizational economics, etc. In this paper, we introduce a new raking method based on the hexagonal fuzzy number using the centroid of the triangle and rectangle. Thus, the hexagonal fuzzy number is transformed into a crisp number using new ranking methods.

1. INTRODUCTION

The concept of fuzzy sets, fuzzy numbers and arithmetic operations were introduced by Lotfi. A.Zadeh ranking fuzzy numbers has a wide variety of applications in the various sciences and engineering. Several methods for ranking fuzzy numbers have been proposed by various researchers presented till now. P. Rajarajeswari and A. Sahayasudha et al. in [18] introduced a new operation of hexagonal fuzzy numbers. H. Shirmardi [13] introduced ranking of hexagonal fuzzy numbers using average and standard deviation method and then applying linear programing problem. A. S. Sudha and K. R. Vijayalakshmi [3] introduced

[1]*Corresponding author*

2010 *Mathematics Subject Classification.* 90C05.
Key words and phrases. fuzzy number, hexagonal fuzzy number, ranking methods, centroid methods.

a new ranking of symmetric hexagonal fuzzy numbers. In this paper the hexagonal has been divided into two trapezoids and two triangles. Then the ranking function is taken for trapezoids and triangle and the average has been taken. A. S. Sudha and M. Revathy in [12] in this paper separated the hexagon into 4 triangles and one rectangle. By using the centroid formula of triangle and rectangle centroid has been calculated and then adding all the centroid points is called reference point. K. Dhurai, A. Karpagam in [15] in this paper presented a new ranking technique of hexagonal fuzzy numbers with membership function and then solving the transportation problem. S. M. Ingle and K. P. Ghadle in [**?**], introduced a new ranking of hexagonal fuzzy numbers by means of sum of all six numbers divided by three. C. Muralidaran and B. Venkateswarlu in [7] introduced left, right and equally spread ranking methods. A. Thamaraiselvi, R. Santhi brought ranking of a hexagonal fuzzy number. The hexagon is split into triangles and a hexagon. The centroid of the hexagonal fuzzy number is the center point of the hexagon. A new approach for ranking based on rank, mode, divergence and spread was introduced in this paper by A.Virgin Raj and V. Ezhilarasi in [4]. Rajarajeswari and Sahayasudha in [17] introduced, splitting the generalized hexagonal fuzzy numbers into three plane figures and then calculating the centroid of every point determine accompanied by using the in-center of the centroid and then using the Euclidean distance formula.

In this paper, first we have reviewed the basic preliminaries of Hexagonal fuzzy numbers and its operations. A brand new method is introduced that is based totally on centroids of centroid. In a hexagonal fuzzy range, the hexagonal is split into three plane figures where the first part being a two triangle and the second part being rectangle and then triangle formed using centroid point, again using the centroid formula centroid has been calculated and then applying Euclidean distance formula. Finally, the proposed approach is compared with different existing approaches.

2. Preliminaries

2.1. **Fuzzy number.** There are various methods to define fuzzy numbers. This paper sets up the concept of fuzzy numbers as follows [5–7, 18].

Definition 2.1. *Generalized regular real number with membership function* $\mu_{\breve{A}} : X \to [0,1]$ *is a fuzzy number* $\tilde{A}$ *and satisfying the following conditions:*

(i) *Fuzzy set* $\tilde{A}$ *is a normal and convex set.*
(ii) $\mu_{\breve{A}}(x) = 1$ *then there exist at least a* $x \in R$
(iii) $\mu_{\breve{A}}(x)$ *is a piecewise continuous in the range* $[0,1]$

3. Hexagonal fuzzy number

$\breve{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$ Where $\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6}$ real numbers and its membership function are $\mu_{\breve{\mathcal{A}}_{\mathcal{H}}}(x)$ is a Hexagonal fuzzy number with six points.

A hexagonal fuzzy number with six points $\tilde{\mathcal{A}}_{\mathcal{H}}$ is indicated as $\tilde{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$, where $\hbar_{a_1} \le \hbar_{a_2} \le \hbar_{a_3} \le \hbar_{a_4} \le \hbar_{a_5} \le \hbar_{a_6}$. Real numbers satisfy $\hbar_{a_2} - \hbar_{a_1} \le \hbar_{a_3} - \hbar_{a_2}$ and $\hbar_{a_6} - \hbar_{a_5} \le \hbar_{a_5} - \hbar_{a_4}$ if its membership function is $\mu_{\tilde{\mathcal{A}}_{\mathcal{H}}}(x)$, [3, 10–12, 15, 18].

$$\mu_{\tilde{\mathcal{A}}_{\mathcal{H}}}(x) = \begin{cases} 0 & for \quad x < \hbar_{a_1} \\ \frac{1}{2}\left(\frac{x - \hbar_{a_1}}{\hbar_{a_2} - \hbar_{a_1}}\right) & for \quad \hbar_{a_1} \le x \le \hbar_{a_2} \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - \hbar_{a_2}}{\hbar_{a_3} - \hbar_{a_2}}\right) & for \quad \hbar_{a_2} \le x \le \hbar_{a_3} \\ 1 & for \quad \hbar_{a_3} \le x \le \hbar_{a_4} \\ \frac{1}{2} - \frac{1}{2}\left(\frac{x - \hbar_{a_4}}{\hbar_{a_5} - \hbar_{a_4}}\right) & for \quad \hbar_{a_4} \le x \le \hbar_{a_5} \\ \frac{1}{2}\left(\frac{\hbar_{a_6} - x}{\hbar_{a_6} - \hbar_{a_5}}\right) & for \quad \hbar_{a_5} \le x \le \hbar_{a_6} \\ 0 & for \quad x > \hbar_{a_6} \end{cases}$$

3.1. **Generalized Hexagonal fuzzy number,** [1–4, 8, 12, 13, 16, 17] A generalized hexagonal fuzzy number is defined as $\tilde{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$, where $\hbar_{a_1} \le \hbar_{a_2} \le \hbar_{a_3} \le \hbar_{a_4} \le \hbar_{a_5} \le \hbar_{a_6}$ are real and ω is its maximum degree of membership function. The membership function is given below:

$$\mu_{\tilde{\mathcal{A}}_{\mathcal{H}}}(x)=\begin{cases} 0 & for \quad x<\hbar_{a_1} \\ \frac{\omega}{2}\left(\frac{x-\hbar_{a_1}}{\hbar_{a_2}-\hbar_{a_1}}\right) & for \quad \hbar_{a_1}\leq x\leq \hbar_{a_2} \\ \frac{\omega}{2}+\frac{\omega}{2}\left(\frac{x-\hbar_{a_2}}{\hbar_{a_3}-\hbar_{a_2}}\right) & for \quad \hbar_{a_2}\leq x\leq \hbar_{a_3} \\ 1 & for \quad \hbar_{a_3}\leq x\leq \hbar_{a_4} \\ \frac{\omega}{2}-\frac{1}{2}\left(\frac{x-\hbar_{a_4}}{\hbar_{a_5}-\hbar_{a_4}}\right) & for \quad \hbar_{a_4}\leq x\leq \hbar_{a_5} \\ \frac{\omega}{2}\left(\frac{\hbar_{a_6}-x}{\hbar_{a_6}-\hbar_{a_5}}\right) & for \quad \hbar_{a_5}\leq x\leq \hbar_{a_6} \\ 0 & for \quad x>\hbar_{a_6} \end{cases}$$

Definition 3.1. A fuzzy set $\tilde{\mathcal{A}}_{\mathcal{H}}$ is described at the regular set of real numbers as a generalized Hexagonal fuzzy number with its membership function that has the following properties: [5, 7, 16, 18].

1. Left continuous non- decreasing function is denoted as $L_L(u)$ within the range [0,0.5];
2. Left continuous non- decreasing function is denoted as $L_L(v)$ within the range $[0.5,\omega]$;
3. Right continuous non- increasing function is denoted as $U_R(v)$ within the range $[\omega,0.5]$;
4. Right continuous non- increasing function is denoted as $U_R(u)$ within the range [0.5,0].

Remark 3.2. If $\omega=1$, then it is said to be a normal hexagonal fuzzy number.

Remark 3.3. Membership functions $\mu_{\tilde{\mathcal{A}}_{\mathcal{H}}}$ are continuous functions.

Remark 3.4. Hexagonal fuzzy number $\tilde{\mathcal{A}}_{\mathcal{H}}$ is the ordered quadruple $L_L(u)$, $U_L(v)$, $U_R(v)$, $L_R(u)$ length of $u\in[0,0.5]$ and length of $v\in[0.5,1]$ where

$$L_L(u)=\frac{1}{2}\left(\frac{u-\hbar_{a_1}}{\hbar_{a_2}-\hbar_{a_1}}\right), U_L(v)=\frac{1}{2}+\frac{1}{2}\left(\frac{v-\hbar_{a_2}}{\hbar_{a_3}-\hbar_{a_2}}\right)$$

$$U_R(v)=1-\frac{1}{2}\left(\frac{v-\hbar_{a_4}}{\hbar_{a_5}-\hbar_{a_4}}\right), L_R(u)=\frac{1}{2}\left(\frac{\hbar_{a_6}-u}{\hbar_{a_6}-\hbar_{a_5}}\right)$$

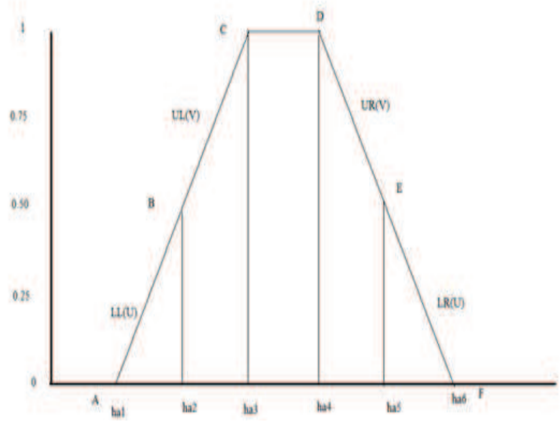

FIGURE 1. Diagram for Normal Hexagonal fuzzy numbers

Definition 3.5. A fuzzy number $\tilde{\mathcal{A}}_{\mathcal{H}}$ is said to be Positive hexagonal Fuzzy number and it's denoted by $\tilde{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$ where all $\hbar_{a_i} > 0$ for all $i = 1, 2, 3, 4, 5, 6$.

3.2. **Alpha cut of hexagonal fuzzy number: [14, 16].** The alpha cut of a Hexagonal fuzzy set $\tilde{\mathcal{A}}_{\alpha}$ is a crisp set and it is defined by
$\tilde{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$

$$\mathcal{A}_{\alpha} = \{x \in X / \mu_{\tilde{\mathcal{A}}_{\mathcal{H}}}(x) \geq \alpha\} = \begin{cases} L_L(\alpha), L_R(\alpha) & for \quad \alpha \in [0, 0.5) \\ U_L(\alpha), U_R(\alpha) & for \quad \alpha \in [0.5, 1] \end{cases} .$$

4. ORDERING OF HEXAGONAL FUZZY NUMBER

Let $\check{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6})$ and $\check{\mathcal{B}}_{\mathcal{H}} = (\hbar_{b_1}, \hbar_{b_2}, \hbar_{b_3}, \hbar_{b_4}, \hbar_{b_5}, \hbar_{b_6})$ be in $F(\rho)$ be the set of all real hexagonal fuzzy numbers, [6, 13, 16, 18]:

i. $\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$ so that $\hbar_{a_i} = \hbar_{a_i}$, $i = 1, 2, 3, 4, 5, 6$;
ii. $\check{\mathcal{A}}_{\mathcal{H}} \leq \check{\mathcal{B}}_{\mathcal{H}}$ so that $\hbar_{a_i} \leq \hbar_{a_i}$, $i = 1, 2, 3, 4, 5, 6$;
iii. $\check{\mathcal{A}}_{\mathcal{H}} \geq \check{\mathcal{B}}_{\mathcal{H}}$ so that $\hbar_{a_i} \geq \hbar_{a_i}$, $i = 1, 2, 3, 4, 5, 6$.

5. PROPOSED RANKING OF HEXAGONAL FUZZY NUMBERS

Let's consider a generalized Hexagonal fuzzy number $\overline{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6}; \omega)$. The centroid of hexagonal fuzzy number

is viewed to be the adjusting point. The Hexagonal fuzzy number is divided vertically into three plane figures and horizontally into two equal parts. The plane figures are two triangles Triangle ABC, Triangle DEF and one Rectangle GHIJ respectively. The in-centers of the centroids of triangle are $\mathbb{G}_1, \mathbb{G}_2$ and the center of rectangle $\mathbb{G}_3$ is taken as the reference point of Hexagonal fuzzy number. Let's consider a generalized Hexagonal fuzzy number
$\overline{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6}; \omega)$
The centroid of triangle ABC is $\mathbb{G}_1 = \left(\dfrac{\hbar_{a_1} + 2\hbar_{a_2}}{3}, \dfrac{w}{6}\right)$;
The centroid of triangle DEF is $\mathbb{G}_2 = \left(\dfrac{\hbar_{a_6} + 2\hbar_{a_5}}{3}, \dfrac{w}{6}\right)$;
The centroid of rectangle $GHIJ$ is $\mathbb{G}_3 = \left(\dfrac{\hbar_{a_4} + 2\hbar_{a_3}}{2}, \dfrac{3w}{4}\right)$;
The centroid points of triangle ABC and DEF of $\mathbb{G}_1$ and $\mathbb{G}_2$ is passing through the line $y = \dfrac{\omega}{6}$ and $\mathbb{G}_3$ is passing $y = \dfrac{3\omega}{4}$. So $\mathbb{G}_3$ do not lie on the line $\overline{\mathbb{G}_1\mathbb{G}_2}$. Hence $\mathbb{G}_1, \mathbb{G}_2$ and $\mathbb{G}_3$ forms a triangle. Now, the center of the triangle with vertices of the triangle $\mathbb{G}_1, \mathbb{G}_2$and $\mathbb{G}_3$ of the generalized hexagonal fuzzy number is $(\overline{x}_{\overline{\mathcal{A}}_{\mathcal{H}}}, \overline{y}_{\overline{\mathcal{A}}_{\mathcal{H}}}) = \left(\dfrac{2\hbar_{a_1} + 4\hbar_{a_2} + 3\hbar_{a_3} + 3\hbar_{a_4} + 4\hbar_{a_5} + 2\hbar_{a_6}}{18}, \dfrac{13\omega}{36}\right)$.
The generalized hexagonal fuzzy number ranking function,
$\overline{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6}; \omega)$ maps the set of all fuzzy number to a set of real number as $\rho(\overline{\mathcal{A}}_{\mathcal{H}}) = \sqrt{x^2_{\overline{\mathcal{A}}_{\mathcal{H}}} + y^2_{\overline{\mathcal{A}}_{\mathcal{H}}}}$.
This is the Euclidean distance from the in-center of the centroids points.

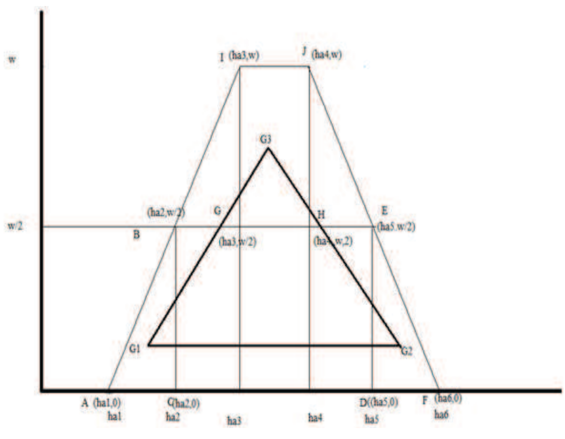

FIGURE 2. Diagram for Generalized Hexagonal fuzzy numbers

5.1. **The ranking function of the generalized hexagonal fuzzy numbers.** Let $\overline{\mathcal{A}}_{\mathcal{H}} = (\hbar_{a_1}, \hbar_{a_2}, \hbar_{a_3}, \hbar_{a_4}, \hbar_{a_5}, \hbar_{a_6}; \omega_1)$ and $\overline{\mathcal{B}}_{\mathcal{H}} = (\hbar_{b_1}, \hbar_{b_2}, \hbar_{b_3}, \hbar_{b_4}, \hbar_{b_5}, \hbar_{b_6}; \omega_2)$ be two generalized hexagonal fuzzy number numbers then $\rho(\overline{\mathcal{A}}_{\mathcal{H}}) = \sqrt{x_{\overline{\mathcal{A}}}^2 + y_{\overline{\mathcal{A}}}^2}$,
where $\overline{\mathcal{A}} = \dfrac{2\hbar_{a_1} + 4\hbar_{a_2} + 3\hbar_{a_3} + 3\hbar_{a_4} + 4\hbar_{a_5} + 2\hbar_{a_6}}{18}$ and $\overline{\mathcal{A}} = \dfrac{13\omega_1}{36}$ and
$\rho(\overline{\mathcal{B}}_{\mathcal{H}}) = \sqrt{x_{\overline{\mathcal{B}}}^2 + y_{\overline{\mathcal{B}}}^2}$,
where $\overline{\mathcal{B}} = \dfrac{2\hbar_{b_1} + 4\hbar_{b_2} + 3\hbar_{b_3} + 3\hbar_{b_4} + 4\hbar_{b_5} + 2\hbar_{b_6}}{18}$ and $\overline{\mathcal{A}} = \dfrac{13\omega_2}{36}$.
Find $\rho(\check{\mathcal{A}}_{\mathcal{H}})$ and $\rho(\check{\mathcal{B}}_{\mathcal{H}})$ and then using the following for the ranking of fuzzy numbers:

(1) If $\rho(\check{\mathcal{A}}_{\mathcal{H}}) \approx \rho(\check{\mathcal{B}}_{\mathcal{H}})$ then generalizes hexagonal fuzzy number is $\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$;
(2) If $\rho(\check{\mathcal{A}}_{\mathcal{H}}) \leq \rho(\check{\mathcal{B}}_{\mathcal{H}})$ then generalizes hexagonal fuzzy number is $\check{\mathcal{A}}_{\mathcal{H}} \leq \check{\mathcal{B}}_{\mathcal{H}}$;
(3) If $\rho(\check{\mathcal{A}}_{\mathcal{H}}) \geq \rho(\check{\mathcal{B}}_{\mathcal{H}})$ then generalizes hexagonal fuzzy number is $\check{\mathcal{A}}_{\mathcal{H}} \geq \check{\mathcal{B}}_{\mathcal{H}}$.

6. Numerical example

Example 1. *Let* $\check{\mathcal{A}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.6, 0.7, 0.9; 1)$ *and* $\check{\mathcal{B}}_{\mathcal{H}} = (0.2, 0.4, 0.6, 0.7, 0.8, 0.9; 1)$ *be two generalized hexagonal fuzzy numbers, then using Rajarajeswari and Sahayasudha, [17]*

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = 0.48, \rho(\check{\mathcal{B}}_{\mathcal{H}}) = 0.59$$

$$\textit{So} \quad \rho(\check{\mathcal{A}}_{\mathcal{H}}) < \rho(\check{\mathcal{A}}_{\mathcal{H}}) \quad \textit{then}, \quad \check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}.$$

Using T. C. Chu and C. T. Tsao approach, [9]

$$\overline{x}(\mathcal{A}_{\mathcal{H}}) = \frac{\int\limits_{0.1}^{0.2}(5x^2 - 0.5x)dx + \int\limits_{0.2}^{0.4} 2.5x^2 dx + \int\limits_{0.4}^{0.6} x dx + \int\limits_{0.6}^{0.7} -5x^2 + 4x dx + \int\limits_{0.7}^{0.8} -2.5x^2 + 2.25x dx}{\int\limits_{0.1}^{0.2}(5x - 0.5)dx + \int\limits_{0.2}^{0.4} 2.5x dx + \int\limits_{0.4}^{0.6} dx + \int\limits_{0.6}^{0.7} -5x + 4 dx + \int\limits_{0.7}^{0.8} -2.5x + 2.25 dx} = 0.41$$

$$\overline{y}(\mathcal{A}_{\mathcal{H}}) = \frac{1/5 \int\limits_{0}^{0.5} y^2 + 05y dy + 1/2.5 \int\limits_{0.5}^{1} y^2 dy + 1/5 \int\limits_{0.5}^{1} 4y - y^2 dy + 1/2.5 \int\limits_{0}^{0.5} 2.25y - y^2 dy}{1/5 \int\limits_{0}^{0.5} y + 05 dy + 1/2.5 \int\limits_{0.5}^{1} y dy + 1/5 \int\limits_{0.5}^{1} 4 - y dy + 1/2.5 \int\limits_{0}^{0.5} 2.25 - y dy}$$

$= 0.46$

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = \overline{x}(\mathcal{A}_{\mathcal{H}}) \times \overline{y}(\mathcal{A}_{\mathcal{H}}) = 0.19$$

Similarly $\overline{x}(\mathcal{B}_{\mathcal{H}}) =$

$$\frac{\int_{0.2}^{0.4}(2.5x^2-0.5x)dx + \int_{0.4}^{0.6}(2.5x^2-0.5x)dx + \int_{0.6}^{0.7} xdx + \int_{0.7}^{0.8} -5x^2+4.5xdx + \int_{0.8}^{0.9} -5x^2+4.5xdx}{\int_{0.2}^{0.4}(2.5x-0.5)dx + \int_{0.4}^{0.6}(2.5x-0.5)dx + \int_{0.6}^{0.7} dx + \int_{0.7}^{0.8} -5x+4.5dx + \int_{0.8}^{0.9} -5x+4.5dx}$$

$= 0.58$

$\overline{y}(\mathcal{B}_{\mathcal{H}})$

$$= \frac{1/5\int_{0}^{1/2}(2y^2+y)dy + 1/5\int_{1/2}^{1}(2y^2+y)dy + \int_{0}^{0.5}(0.9y-0.2y^2)dy + \int_{0.5}^{1}(0.9y-0.2y^2)dy}{1/5\int_{0}^{1/2}(2y+1)dy + 1/5\int_{1/2}^{1}(2y+1)dy + \int_{0}^{0.5}(0.9-0.2y)dy + \int_{0.5}^{1}(0.9-0.2y)dy}$$

$= 0.55$

$$\rho(\check{\mathcal{B}}_{\mathcal{H}}) = \overline{x}(\mathcal{B}_{\mathcal{H}}) \times \overline{y}(\mathcal{B}_{\mathcal{H}}) = 0.32$$

So $\rho(\check{\mathcal{A}}_{\mathcal{H}}) < \rho(\check{\mathcal{B}}_{\mathcal{H}})$ *then* $\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$.

Our new method

$\check{\mathcal{A}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.6, 0.7, 0.9; 1)$

$x_0 = \dfrac{2\hbar_{a_1} + 4\hbar_{a_2} + 3\hbar_{a_3} + 3\hbar_{a_4} + 4\hbar_{a_5} + 2\hbar_{a_6}}{18}$ *and* $y_0 = \dfrac{13\omega_1}{36}$

$x_0 = \dfrac{2(0.1) + 4(0.2) + 3(0.4) + 3(0.6) + 4(0.7) + 2(0.9)}{18} = 0.48,$

$y_0 = \dfrac{13(1)}{36} = 0.36$

$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = \sqrt{(0.48)^2 + (0.36)^2} = 0.60$

$\check{\mathcal{B}}_{\mathcal{H}} = (0.2, 0.4, 0.6, 0.7, 0.8, 0.9; 1)$

$x_0 = \dfrac{2\hbar_{b_1} + 4\hbar_{b_2} + 3\hbar_{b_3} + 3\hbar_{b_4} + 4\hbar_{b_5} + 2\hbar_{b_6}}{18}$ *and* $y_0 = \dfrac{13\omega_2}{36}$

$x_0 = \dfrac{2(0.2) + 4(0.4) + 3(0.6) + 3(0.7) + 4(0.8) + 2(0.9)}{18} = 0.61,$

$y_0 = \dfrac{13(1)}{36} = 0.36$

$\rho(\check{\mathcal{B}}_{\mathcal{H}}) = \sqrt{(0.61)^2 + (0.36)^2} = 0.71$

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = 0.60, \rho(\check{\mathcal{B}}_{\mathcal{H}}) = 0.71$$

$$So \quad \rho(\check{\mathcal{A}}_{\mathcal{H}}) < \rho(\check{\mathcal{B}}_{\mathcal{H}}), \check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$$

Example 2. *Let* $\check{\mathcal{A}}_{\mathcal{H}} = (0.2, 0.3, 0.5, 0.6, 0.7, 0.9; 0.35)$ *and* $\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.5, 0.6, 0.9; 0.7)$ *be two generalized hexagonal fuzzy numbers, then using Rajarajeswari and Sahayasudha, [17]*

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = 0.44, \rho(\check{\mathcal{B}}_{\mathcal{H}}) = 0.42$$

$$\textit{So} \quad \rho(\check{\mathcal{A}}_{\mathcal{H}}) > \rho(\check{\mathcal{A}}_{\mathcal{H}}) \quad \textit{then}, \ \check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}.$$

Using T. C. Chu and C. T. Tsao approach, [9]
$\overline{x}(\mathcal{A}_{\mathcal{H}}) = 0.5$, $\overline{y}(\mathcal{A}_{\mathcal{H}}) = 0.49$

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = \overline{x}(\mathcal{A}_{\mathcal{H}}) \times \overline{y}(\mathcal{A}_{\mathcal{H}}) = 0.24$$

Similarly $\overline{x}(\mathcal{B}_{\mathcal{H}}) = 0.57$, $\overline{y}(\mathcal{B}_{\mathcal{H}}) = 0.58$

$$\rho(\check{\mathcal{B}}_{\mathcal{H}}) = \overline{x}(\mathcal{B}_{\mathcal{H}}) \times \overline{y}(\mathcal{B}_{\mathcal{H}}) = 0.18$$

So $\rho(\check{\mathcal{A}}_{\mathcal{H}}) > \rho(\check{\mathcal{B}}_{\mathcal{H}})$ *then* $\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$.

Our new method

$\check{\mathcal{A}}_{\mathcal{H}} = (0.2, 0.3, 0.5, 0.6, 0.7, 0.9; 0.35)$

$x_0 = \dfrac{2\hbar_{a_1} + 4\hbar_{a_2} + 3\hbar_{a_3} + 3\hbar_{a_4} + 4\hbar_{a_5} + 2\hbar_{a_6}}{18}$ *and* $y_0 = \dfrac{13\omega_1}{36}$

$x_0 = \dfrac{2(0.2) + 4(0.3) + 3(0.5) + 3(0.6) + 4(0.7) + 2(0.9)}{18} = 0.53,$

$y_0 = \dfrac{13(0.35)}{36} = 0.13$

$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = \sqrt{(0.53)^2 + (0.13)^2} = 0.54$

$\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.5, 0.6, 0.9; 0.7)$

$x_0 = \dfrac{2\hbar_{b_1} + 4\hbar_{b_2} + 3\hbar_{b_3} + 3\hbar_{b_4} + 4\hbar_{b_5} + 2\hbar_{b_6}}{18}$ *and* $y_0 = \dfrac{13\omega_2}{36}$

$x_0 = \dfrac{2(0.1) + 4(0.2) + 3(0.4) + 3(0.5) + 4(0.6) + 2(0.9)}{18} = 0.44,$

$y_0 = \dfrac{13(0.35)}{36} = 0.25$

$\rho(\check{\mathcal{B}}_{\mathcal{H}}) = \sqrt{(0.306)^2 + (0.253)^2} = 0.51$

$$\rho(\check{\mathcal{A}}_{\mathcal{H}}) = 0.54, \rho(\check{\mathcal{B}}_{\mathcal{H}}) = 0.51$$

$$So \quad \rho(\check{\mathcal{A}}_{\mathcal{H}}) > \rho(\check{\mathcal{B}}_{\mathcal{H}}), \ \check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$$

Approaches	Set-1	Set 2	Set 3	Set 4	Set 5
Cheng	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	-	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
Chu and Tsao	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	-	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
Chen and Chen	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
Abbasbandy and Hajjari	-	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
Kumar et.al	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
A.Virgin Raj V.Ezhilarasi	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
A.SahayaSudha M.Revathy	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$
Our methods	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} \approx \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} < \check{\mathcal{B}}_{\mathcal{H}}$	$\check{\mathcal{A}}_{\mathcal{H}} > \check{\mathcal{B}}_{\mathcal{H}}$

6.1. **Results and Discussions.** In this section seven sets of fuzzy number are compared using the our new proposed method and existing approaches, and result is given in table -1, [4].
Set-1 Let $\check{\mathcal{A}}_{\mathcal{H}} = (0.2, 0.4, 0.6, 0.8, 0.9, 0.9; 0.35)$,$\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6; 0.7)$
Set-2 Let $\check{\mathcal{A}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.5, 0.6, 0.7; 1)$, $\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.1, 0.5, 0.5, 0.6, 0.7; 1)$
Set-3 Let $\check{\mathcal{A}}_{\mathcal{H}} = (0.1, 0.2, 0.4, 0.5, 0.6, 0.7; 1)$, $\check{\mathcal{B}}_{\mathcal{H}} = (1, 1, 1, 1, 1, 1; 1)$
Set-4 Let $\check{\mathcal{A}}_{\mathcal{H}} = (0.1, 0.3, 0.3, 0.4, 0.5, 0.6; 1)$, $\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.3, 0.3, 0.6, 0.7, 0.8; 1)$
Set-5 Let $\check{\mathcal{A}}_{\mathcal{H}} = (0.3, 0.5, 0.5, 0.6, 0.6, 0.7; 1)$, $\check{\mathcal{B}}_{\mathcal{H}} = (0.1, 0.2, 0.2, 0.3, 0.4, 0.5; 1)$

The advantage of our method is providing the correct ordering of generalized hexagonal fuzzy numbers. Also very easy to apply in any problem.

7. Conclusion

This method provides the correct ordering of generalized hexagonal fuzzy numbers also which is simple in calculation and gives satisfactory results. In a hexagonal fuzzy number, the hexagonal is split into three plane figures where the first part being a two triangles and the second part being rectangle and then triangle formed using centroid point. Again using the centroid formula centroid has been calculated and then applying Euclidean distance formula. Finally the proposed approach is compared with different existing approaches.

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