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A COMPARATIVE STUDY OF TRANSPORTATION PROBLEM BY GRAPH THEORETICAL ALGORITHMS

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ABSTRACT. Graphs are used as a tool for modeling and description of real life network systems which are, transport, electricity, internet, social network etc. The content of these areas may differ but they have common features in certain items between them. In this paper, we have compared the results of the different algorithms for the transportation problem which has considered as network model which is also solved using Path labeling algorithm, and it provides the shortest transportation cost and also the possible number of shortest paths from the initial vertices.

1. INTRODUCTION

Transportation problem plays a vital role in real world network optimization problems in operation research, in which the main thing is to minimize the total cost by finding the shortest path. In network optimisation, a huge number of shortest path algorithms [1, 5, 8, 10, 12, 15–17] has been solved more exactly than any other algorithm. Some of these are better than others, some are more comfortable for a particular system than others and some are only small differences of previous algorithms. Some algorithms like the Dijkstra algorithm [5] can find the shortest path of problems where there are no negative weights, etc. In network, negative weights make the problem difficult and negative cycles

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make the problem unmanage-able. The algorithms presented by Bellman [2], Dreyfus [7] and Dijkstra algorithms [19] are some standard shortest path algorithms [6, 13]. The existence of irregular and inaccuracy in the day to day life cannot be avoided owing to some unpredictable place of activity. There is a chance of unclear information because of the wild factors. Therefore, fuzziness can be brought into a system of connections beyond edge capacities, edge weights. The fuzzy shortest path network problem was examined in [1, 4, 9] and solved by Floyds algorithm. Klien [14] gave a dynamical programming recursion-based fuzzy shortest path algorithm. Chuang and Kung [3], Mahdavi et al [11] and Hernandes et al [10] have examined and determined a fuzzy shortest path for several problems in many fuzzy related areas like fuzzy linear programing, discrete fuzzy arc length, linear multiple objective programming approach, generic algorithm on networks fuzzy numbers and so on.

In this paper the transportation problem which has been modeled as a network is considered. It is solved already by using Dijkstras algorithm and Kruskals Algorithm.

Also, the same model has been taken and verified whether the same result arrived or not by using the another graph algorithm named as Path labeling algorithm [18]. The optimum path of a transportation problem has got and it has been taken as anadjacency matrix and has plotted a graph using MATLAB software.

Kruskal algorithm. Kruskal's algorithm is a minimum spanning tree algorithm which finds an edge of the least possible weight that connects any two trees in the forest.

Dijsktra algorithm. It is an algorithm for finding the shortest paths between vertices in a graph.

Transportation model. It is used to calculate the shortest path from initial vertex to each of other vertices in the network.

2. Path labeling algorithm

Step 1: Collect all the vertices that start from S_1 in the network which is lying near to n_0. If S_1 is empty, then go to Step 7. Or else proceed Step 2.

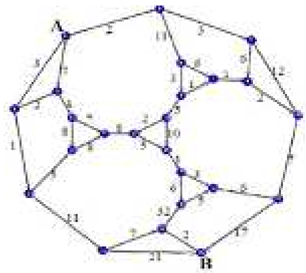

FIGURE 1. Network model (directed graph) of transportation problem

Step 2: Calculate the shortest path at each vertex of S_1 from n_0.
Step 3: Collect the vertices starts from the vertex S_2 in the network which is lying near S_1. If S_2 is empty, then go to Step 7. Or else proceed Step 4.
Step 4: Calculate the shortest path at each vertex of S_2 from n_0, using the Result 1. If the shortest path between n_0 and any one vertex of S_2 with negative weight is a cycle, go to the Step 7. Or else proceed Step 5.
Step 5: Repeat the Step 1 to the Step 4 until obtaining the set of collection of vertices in the network which are lying near to each of the shortest path vertices is empty.
Step 6: Calculate the shortest path from n_0 to each of vertices in the network in Step 5 and stop.
Step 7: There is no path from the vertex n_0 to the label vertex. Stop. When n_0 will be the initial vertex for the network model, the problem is to calculate the shortest path starts from the initial vertex to each and every vertex in the network.

Using the above mentioned algorithm, the transportation problem, Figure 1, have been solved below.

Now, $S_1 = \{2, 3, 4\}$ and using the Path labeling algorithm we have:

TABLE 1. Shortest path from the vertex 1 to S_1

Ending Vertex	Number of paths from the vertex 1	Shortest path
2	(2; 1-2)	1-2
3	(7; 1-3)	1-3
4	(3; 1-4)	1-4

Now $S_2 = \{5, 6, 11, 14\}$. By path labeling algorithm we have:

TABLE 2. Shortest path from the vertex 1 to S_2

Ending Vertex	Number of paths from the vertex 1	Shortest path
5	(5; 1-2-5)	1-2-5
6	(13; 1-2-6)	1-2-6
11	(8; 1-3-11)	1-3-11
14	(4; 1-4-14)	1-4-14

Now $S_3 = \{10, 9, 7, 8, 12, 13, 15\}$.

TABLE 3. Shortest path from the vertex 1 to S_3

Ending Vertex	Number of paths from the vertex 1	Shortest path
10	(17; 1-2-5-10)	1-2-5-10
9	(11; 1-2-5-9)	1-2-5-9
7	(19; 1-2-6-7)	1-2-6-7
8	(16; 1-2-6-8)	1-2-6-8
12	(15; 1-3-11-12)	1-3-11-12
13	(16; 1-3-11-13)	1-3-11-13
15	(15; 1-4-14-15)	1-4-14-15

Now $S_4 = \{23, 18, 17, 16, 24\}$

TABLE 4. Shortest path from the vertex 1 to S_4

Ending Vertex	Number of paths from the vertex 1	Shortest path
23	(20;1-2-5-9-10-23) (29;1-2-6-7-9-10-23)	1-2-5-10-23
18	(21;1-2-6-8-18)(25;1-2-6-7-8-18)	1-2-6-8-18
17	(23;1-3-11-12-17)	1-3-11-12-17
16	(22;1-4-14-15-16)(28;1-3-4-14-15-16)(39;1-3-11-13-14-15-16)	1-4-14-15-16
24	(36;1-4-14-15-24) (24;1-4-14-15-16-24)	1-4-14-15-16-24

Now $S_5 = \{19, 22\}$.

TABLE 5. Shortest path from the vertex 1 to S_5

Ending Vertex	Number of paths from the vertex 1	Shortest path
19	(31;1-2-6-8-18-19) (28;1-3-11-12-17-19) (28;1-3-11-13-12-17-19) (37;1-3-11-13-12-17-19)	1-3-11-12-17-19
22	(54;1-4-14-15-16-22) (60;1-3-4-14-15-16-22) (71;1-3-11-13-14-15-16-22)	1-4-14-15-16-22

Now $S_6 = \{20\}$.

TABLE 6. Shortest path from the vertex 1 to S_6

Ending Vertex	Number of paths from the vertex 1	Shortest path
20	(30;1-2-6-8-17-19-20) (29;1-3-11-12-17-19-20) (32;1-2-6-8-18-19-20) (39;1-3-11-13-12-17-19-20)	1-3-11-12-17-19-20

Now $S_7 = \{21\}$.

TABLE 7. Shortest path from the vertex 1 to S_7

Ending Vertex	Number of paths from the vertex 1	Shortest path
21	(33;1-2-6-8-18-17-19-20-21)(32;1-3-11-12-17-19-20-21)(35;1-2-6-8-18-19-20-21) (42;1-3-11-13-12-17-19-20-21)	1-3-11-12-17-19-20-21

Shortest path of the given transportation Network model:

TABLE 8. Shortest paths of the given model

Vertex	Shortest path	Weight of the Shortest path
2	1-2	2
3	1-3	7
4	1-4	3
5	1-2-5	5
6	1-2-6	13
7	1-2-6-7	19
8	1-2-6-8	16
9	1-2-5-9	11
10	1-2-5-10	17
11	1-3-11	8
12	1-3-11-12	15
13	1-4-3-11-13	14
14	1-4-14	4
15	1-4-14-15	15
16	1-4-14-15-16	22
17	1-3-11-12-17	23
18	1-2-6-8-18	21
19	1-3-11-12-17-19	28
20	1-3-11-12-17-19-20	29
21	1-4-14-15-16-22-21	32
22	1-4-14-15-16-22	54
23	1-2-5-9-10-23	20
24	1-4-14-15-16-24	24

Matrix form of the transportation problem is given as tabular form in the Figure 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	2	7	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12
2	0	0	0	0	3	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14
3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3
5	0	0	0	0	0	0	0	0	6	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18
6	0	0	0	0	0	0	6	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	6
9	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	0	7
11	0	0	0	0	0	0	0	0	0	0	0	7	8	0	0	0	0	0	0	0	0	0	0	0	15
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2
13	0	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	8
14	0	0	0	0	0	0	0	0	0	0	0	0	5	0	11	0	0	0	0	0	0	0	0	0	16
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0	21	28
16	0	0	0	0	0	0	0	0	0	.0	0	0	0	0	0	0	0	0	0	0	0	32	0	2	34
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	5
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	10	0	0	0	0	0	0	12
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	6	0	0	9
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	6
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	5
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	2	9	3	3	11	7	3	7	14	1	15	13	1	11	7	4	15	5	0	9	38	13	40	231

FIGURE 2. Matrix form in a table of Transportation problem

Matrix form of the transportation problems optimal path is given as table in the Figure 3.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11	0	0	0	0	0	0	0	0	0	11
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0	0	7
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	3	0	0	0	0	0	0	0	0	0	1	11	7	0	0	0	0	0	0	0	2	24

FIGURE 3. Matrix form in a table of optimal path

The optimal path is represented in a network in Figure 4.

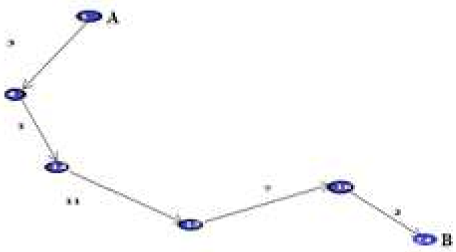

FIGURE 4. Shortest path of the network model

Comparison: Cost obtained from the path labeling algorithm, Dijsktra algorithm and Kruskal algorithm are presented in Table 9.

TABLE 9. Cost comparison with algorithms

Algorithm	cost of the optimal path
Dijkstra algorithm	24
Kruskal Algorithm	24
Path labeling algorithm	24

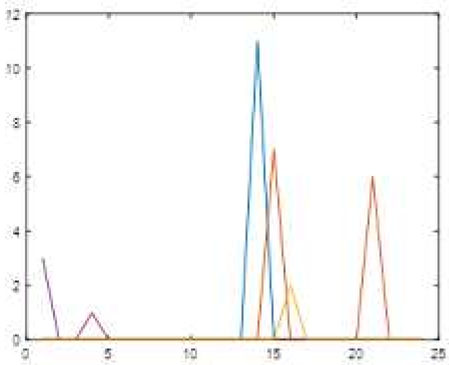

FIGURE 5. Graph of the optimal path by MATLAB

3. CONCLUSION

This paper provides the results of three algorithms such as Dijkstras, Kruskals and Path labeling algorithm. The transportation problem which has been solved by Dijkstra and Kruskals algorithm, has been solved using the Path labeling algorithm which provides the possible shortest paths to reach all the vertices from the initial vertices and also the shortest path to reach vertex B from vertex A in which the problem is considered as a digraph. The user can take any vertex as an initial vertex and can reach any vertex as a destination. Path labeling algorithm provides that kind of possible paths.

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