

TOPOLOGICAL OPERATORS OVER INTUITIONISTIC FUZZY MULTISSETS OF TYPE II

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ABSTRACT. In this paper, we introduce some new topological operators like Closure operator $C(A)$ and Interior operator $I(A)$ over Intuitionistic Fuzzy Multi Sets of second type. Also, we study some of its properties and their relations.

1. INTRODUCTION

Modern set theory formulated by the German mathematician George Cantor is fundamental for the whole mathematics. In fact, set theory is the language of mathematics, science, logic and philosophy. One issue associated with the notion of set is the concept of vagueness.

Considering the unpredictable factor in decision making Lofti A Zadeh introduced the idea of Fuzzy set [6] which has a membership function that assigns to each element of the universe of discourse, a member from the unit interval $[0, 1]$ to indicate the degree of belongingness to the set under consideration. Atanassov subsequently proposed the concept of Intuitionistic Fuzzy Set by bringing a non-membership function together with the membership function of the fuzzy set. Among the various notions of higher order fuzzy sets, IFS proposed by Atanassov provides a flexible framework to elaborate uncertainty and vagueness.

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As a generalization of fuzzy sets, Yager introduced the concept of Fuzzy Multiset. An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values. Then years after, Shinoj and Sunil made an attempt to combine the concepts and named it Intuitionistic Fuzzy Multi Set. The present authors have introduced Intuitionistic Fuzzy Multisets of Second type which is a further extention of IFMS.

The paper proceeds as follows. In the section 2 we give some basic definition of IFMSST. In section 3, we define the closure operator $C(A)$, Interior operator $I(A)$ respectively. We study various theorems and propositions related to it. The paper is concluded in section 4.

2. PRELIMINARIES

Definition 2.1. [4, 6] Let X be a nonempty set. An Intuitionistic Fuzzy Multiset of second type A denoted by IFMSST drawn from X is characterized by two functions Count membership of A (CM_A) and count non membership of A (CN_A) given respectively by $CM_A : X \rightarrow Q$ and $CN_A : X \rightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0, 1]$ such that for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in which CM_A is denoted by $\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^n(x)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^n(x)$ and the corresponding non membership sequence will be denoted by $\nu_A^1(x), \nu_A^2(x), \dots, \nu_A^n(x)$ such that

$$0 \leq (\mu_A^i(x))^2 + (\nu_A^i(x))^2 \leq 1$$

for $x \in X$ and $i = 1, 2, \dots, n$. An IFMS of second type is denoted by

$$A = \{ \langle x : (\mu_A^1(x), \mu_A^2(x), \dots, \mu_A^n(x)), (\nu_A^1(x), \nu_A^2(x), \dots, \nu_A^n(x)) \rangle / x \in X \}.$$

Definition 2.2. The degree of non determinacy (uncertainty or hesitancy) of an element $x \in X$ in the IFMSST A is defined by

$$\pi_A^i(x) = \sqrt{1 - (\mu_A^i(x))^2 - (\nu_A^i(x))^2}$$

for all $x \in X$ and $i = 1, 2, \dots, n$.

Definition 2.3. Length of an element x in an IFMSST A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq (\mu_A^i(x))^2 + (\nu_A^i(x))^2 \leq 1$ and it is denoted

by $L(x : A)$. That is

$$L(x : A) = |CM_A(x)| = |CN_A(x)|.$$

If A and B are IFMSST drawn from X then

$$L(x : A, B) = \text{Max}\{L(x : A), L(x : B)\}.$$

We can use the notation $L(x)$ for $L(x : A, B)$.

3. TOPOLOGICAL OPERATORS ON IFMSST

Definition 3.1. See [7,8] Let A be any IFMSST then the Closure operator for any A can be defined as

$$C(A) = \{x, K, L/x \in X\},$$

where

$$K = \max_{y \in X} \mu_A^i(y) \quad \text{and} \quad L = \min_{y \in X} \nu_A^i(y).$$

Definition 3.2. Let A be any IFMSST then the Interior operator for any A can be defined as

$$I(A) = \{x, k, l/x \in X\},$$

where

$$k = \min_{y \in X} \mu_A^i(y) \quad \text{and} \quad l = \max_{y \in X} \nu_A^i(y).$$

Example 1. Consider $X = \{x, y, z, w\}$ Let A be any IFMSST defined as $A = \{\langle x : (0.5, 0.2), (0.3, 0.6) \rangle, \langle y : (0.6, 0.5, 0.4, 0.2), (0.1, 0.3, 0.2, 0.5) \rangle\}$. Then the closure and interior of A is given as

$$C(A) = \{\langle x : (0.6, 0.5, 0.4, 0.2), (0.1, 0.3, 0.2, 0.5) \rangle, \langle y : (0.6, 0.5, 0.4, 0.2), (0.1, 0.3, 0.2, 0.5) \rangle\}$$

$$I(A) = \{\langle x : (0.5, 0.2, 0, 0), (0.3, 0.6, 1, 1) \rangle, \langle y : (0.5, 0.2, 0, 0), (0.3, 0.6, 1, 1) \rangle\}.$$

Theorem 3.1. For every IFMSST A , we have $I(A) \subset A \subset C(A)$.

Proof. Since

$$\begin{aligned} \min_{y \in X} \mu_A^i(y) &\leq \mu_A^i(x) \leq \max_{y \in X} \mu_A^i(y) \quad \forall x, y \in X \\ \max_{y \in X} \nu_A^i(y) &\geq \nu_A^i(x) \geq \min_{y \in X} \nu_A^i(y) \quad \forall x, y \in X, \end{aligned}$$

it is clear from the definitions that, $I(A) \subset A \subset C(A)$. □

Theorem 3.2. *For every IFMSST A , we have*

- (i) $C[C(A)] = C(A)$;
- (ii) $C[I(A)] = I(A)$;
- (iii) $I[C(A)] = C(A)$;
- (iv) $I[I(A)] = I(A)$.

Proof.

$$\begin{aligned}
 (i) \quad C[C(A)] &= \\
 &= C[C\{x, \mu_A^i(x), \nu_A^i(x)/x \in X\}] \\
 &= C[\{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y)/x \in X\}] \\
 &= \{x, \max_{y \in X} \max_{y \in X} \mu_A^i(y), \min_{y \in X} \min_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= \{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= C(A)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad C[I(A)] &= \\
 &= C[I\{x, \mu_A^i(x), \nu_A^i(x)/x \in X\}] \\
 &= C[\{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y)/x \in X\}] \\
 &= \{x, \max_{y \in X} \min_{y \in X} \mu_A^i(y), \min_{y \in X} \max_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= \{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= I(A)
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad I[I(A)] &= \\
 &= I[I\{x, \mu_A^i(x), \nu_A^i(x)/x \in X\}] \\
 &= I[\{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y)/x \in X\}] \\
 &= \{x, \min_{y \in X} \min_{y \in X} \mu_A^i(y), \max_{y \in X} \max_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= \{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= I(A)
 \end{aligned}$$

$$(iv) \quad I[C(A)] =$$

$$\begin{aligned}
 &= I[C\{x, \mu_A^i(x), \nu_A^i(x)/x \in X\}] \\
 &= I[\{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y)/x \in X\}] \\
 &= \{x, \min \max_{y \in X} \mu_A^i(y), \max \min_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= \{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y)/x \in X\} \\
 &= C(A)
 \end{aligned}$$

□

Theorem 3.3. For every IFMSST A and B , we have

- (i) $C(A \cup B) = C(A) \cup C(B)$,
- (ii) $C(A \cap B) \subset C(A) \cup C(B)$,
- (iii) $I(A \cup B) \supset I(A) \cup I(B)$,
- (iv) $I(A \cap B) = I(A) \cap I(B)$.

Proof.

$$\begin{aligned}
 (i) \quad C(A \cup B) &= \\
 &= C(\{x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x))/x \in X\}) \\
 &= \{x, \max_{y \in X}(\max(\mu_A^i(y), \mu_B^i(y))), \min_{y \in X}(\min(\nu_A^i(y), \nu_B^i(y)))/x \in X\} \\
 &= \{x, \max_{y \in X}(\max \mu_A^i(y), \max \mu_B^i(y)), \min_{y \in X}(\min \nu_A^i(y), \min \nu_B^i(y))/x \in X\} \\
 &= \{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y)/x \in X\} \cup \{x, \max_{y \in X} \mu_B^i(y), \min_{y \in X} \nu_B^i(y)/x \in X\} \\
 &= C(A) \cup C(B)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad C(A \cap B) &= \\
 &= C(\{x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x))/x \in X\}) \\
 &= \{x, \max_{y \in X}(\min(\mu_A^i(y), \mu_B^i(y))), \min_{y \in X}(\max(\nu_A^i(y), \nu_B^i(y)))/x \in X\} \\
 &= \{x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x))/x \in X\}
 \end{aligned}$$

$$\begin{aligned}
& C(A) \cap C(B) = \\
&= C(\{x, \max_{y \in X} \mu_A^i(y), \min_{y \in X} \nu_A^i(y) / x \in X\}) \\
&\quad \cap (\{x, \max_{y \in X} \mu_B^i(y), \min_{y \in X} \nu_B^i(y) / x \in X\}) \\
&= \{x, \max(\max_{y \in X} \mu_A^i(y), \max_{y \in X} \mu_B^i(y)), \min(\min_{y \in X} \nu_A^i(y), \min_{y \in X} \nu_B^i(y)) / x \in X\} \\
&= \{x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x)) / x \in X\} \\
&= \text{since } \min(\mu_A^i(x), \mu_B^i(x)) \leq \max(\mu_A^i(x), \mu_B^i(x)) \\
&= \max(\nu_A^i(x), \nu_B^i(x)) \geq \min(\nu_A^i(x), \nu_B^i(x)).
\end{aligned}$$

Hence $C(A \cap B) \subset C(A) \cup C(B)$.

$$\begin{aligned}
(iii) \quad I(A \cup B) &= \\
&= I(\{x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x)) / x \in X\}) \\
&= \{x, \min(\max(\mu_A^i(y), \mu_B^i(y))), \max(\min(\nu_A^i(y), \nu_B^i(y))) / x \in X\} \\
&= \{x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x)) / x \in X\}
\end{aligned}$$

$$\begin{aligned}
I(A) \cup I(B) &= \\
&= \{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y) / x \in X\} \cup \{x, \min_{y \in X} \mu_B^i(y), \max_{y \in X} \nu_B^i(y) / x \in X\} \\
&= \{x, \max(\min_{y \in X} \mu_A^i(y), \min_{y \in X} \mu_B^i(y)), \min \max_{y \in X} \nu_A^i(y), \max_{y \in X} \nu_B^i(y) / x \in X\} \\
&= \{x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x)) / x \in X\} \\
&= I(A) \cup I(B).
\end{aligned}$$

Hence $I(A \cup B) \supset I(A) \cup I(B)$.

$$\begin{aligned}
(iv) \quad I(A \cap B) &= \\
&= I(\{x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x)) / x \in X\}) \\
&= \{x, \min(\min_{y \in X} \mu_A^i(y), \min_{y \in X} \mu_B^i(y)), \max_{y \in X}(\max(\nu_A^i(y), \nu_B^i(y))) / x \in X\} \\
&= \{x, \min(\min_{y \in X} \mu_A^i(x), \min_{y \in X} \mu_B^i(x)), \max(\max_{y \in X} \nu_A^i(x), \max_{y \in X} \nu_B^i(x)) / x \in X\} \\
&= \{x, \min_{y \in X} \mu_A^i(y), \max_{y \in X} \nu_A^i(y) / x \in X\} \cap \{x, \min_{y \in X} \mu_B^i(y), \max_{y \in X} \nu_B^i(y) / x \in X\} \\
&= I(A) \cap I(B).
\end{aligned}$$

□

Theorem 3.4. *For every IFMSST A , we have $\overline{C(\overline{A})} = I(A)$.*

$$\begin{aligned} \text{Proof. } \overline{C(\overline{A})} &= \overline{C(\{x, \mu_A^i(x), \nu_A^i(x) / x \in X\})} = \overline{C(\{x, \nu_A^i(x), \mu_A^i(x) / x \in X\})} \\ &= \overline{\{x, \max_{y \in X} \nu_A^i(x), \min_{y \in X} \mu_A^i(x)\}} = \{x, \min_{y \in X} \mu_A^i(x), \max_{y \in X} \nu_A^i(x)\} \\ &= I(A) \end{aligned}$$

□

4. CONCLUSION

In this paper we define some new topological operators like closure operator $C(A)$ and Interior operator $I(A)$. Some of its theorems and properties are also studied. This newly defined operators are unique in its own way as it increases both the degrees of membership and the non membership thereby decreasing the degree of uncertainty. It has its wide applications in the area of inter criteria analysis. There is an excellent opportunity for further research in the topic.

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