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STABILITY OF T-S FUZZY MIXED DELAYED NEURAL NETWORKS WITH MARKOVIAN JUMPING PARAMETERS THROUGH RELIABLE SAMPLED DATA CONTROL

D. AJAY[1] AND R. SUGUMAR

ABSTRACT. The point of this paper is to nourish of the issue of Takagi-Sugeno (T-S) fuzzy neural systems with jumping parameters. By employing the reliable sampled-data control along with suitable Lyapunov-Krasovskii functional, bottomless stipulation are inferred to demonstrate that the tended to neural systems is stable. The obtained conditions are surrounded as LMIs. The controller gain matrix is reaped by understanding the LMIs utilizing the notable numerical MATLAB programming. Terminally, numerical example is introduced to exhibit the viability of the conceptual outcomes.

1. INTRODUCTION

It is well known that, neural systems have gotten substantially more consideration because of a wide applications, for example, signal handling, target following, picture preparing, cooperative memory, design acknowledgment, static picture preparing, advancement issues, power frameworks, money, equal processing, mechanics of structures, materials, keen reception apparatus clusters and other logical regions [1–3]. In any case, in numerous physical and organic wonders the pace of variety in the framework state relies upon the antecedent states. This trademark is known as a deferral (or a time delay) and along these

[1]*corresponding author*

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lines a framework with a time delay is known as a time delay framework. As of late, the stability of neural systems has gotten a lot of consideration regarding diminishing the time delays in both hypothetical and useful applications. In this manner, extensive endeavor has been committed to dissecting the stability of neural systems with time delays can be grouped into two types namely delay dependent and delay independent. Thusly delay-dependent stability criteria are more commonly, less traditionalist than delay independent ones particularly when it comes to scale of the deferral is little. The stability of neural networks with delay are examined by numerous researchers in [4–6].

Fuzzy systems as the (T-S) model have pulled in rapidly creating eagerness for late years. T-S fuzzy systems are nonlinear structures delineated by a great deal of IF-THEN rules. It has been demonstrated that the T-S model procedure can give an effective strategy to address complex nonlinear structures by some clear close by direct one of a kind system with their phonetic delineation. Some nonlinear powerful frameworks can be approximated by the general fuzzy direct T-S models with the ultimate objective of trustworthiness examination. Initially, Tanaka and his accomplice have given a satisfactory condition to the quadric stability in the T-S fuzzy frameworks in the felling of Lyapunov in by considering a Lyapunov capacity of sub-fuzzy frameworks. In the light of the foregoing, several creators have stretch out the standard fuzzy models to conceded the deferred neural systems with time fluctuating delays and have decided unfaltering quality guidelines [7–9].

From another perspective, steadfastness and execution of different dynamical structures are tremendously influenced by the unexpected variation occurs in the systems model and its boundaries. Because of the occasion of this variation, dynamical structure model will give a couple of difficulties in their display like any faltering, abrupt condition variation, slow and dreary appearing interconnection dissatisfactions and besides may incite the dissimilarity in conspicuous cases. To stand up to these conditions, Markovian jump structures is used and it accord a suitable out-turn for the system model which are impacted by arbitrary trading conduct. The most basic assessment on Markovian jump structure is to expect that the data on variation probabilities is all around familiar. Considering, in different authentic structures, the variation probability of Markov bouncing may not be quantifiable precisely, or possibly just barely any bit of the variation probabilities is attainable. Along these lines, it is integral to center such wide Markovian jump structures having genuinely well known variation probabilities.

The most significant properties of control frameworks is solidness, by ethicalness of there is no functional applications for unsteady frameworks. On a very basic level, that each control framework is certainly steady and afterward different properties can be examined. By righteousness of the developing innovation of computerized control frameworks, a few flaws are entered that can be from actuators and sensors. A reliable control framework secures the ability to hold the framework disappointments normally and keeps up that the shut circle framework accomplishes dependability. Steadiness investigation of reliable control for neural network with time differing delays is inspected in [10–14]. Because of the quick development of innovation, controllers turned out to be generally, by and sampled data system turns into an examined information framework in which the control signals are in consistent at the hour of testing period and afterward it will change at the inspecting time which prompts the control signals are in stepwise, which implies discontinuities exists and it gathers the dynamical frameworks in entangled position. Besides, dependability and adjustment of framework with sampled data control is explored in [15–17].

Motivated by the ahead deliberations, utilization of both reliable and sampled data control for the stability of mixed delay neural networks is not yet fully studied. So this encourage me to focus on the stability of mixed delay neural networks using reliable control with actuators failures and sampled data control. The main highlights of this paper are given below:

- The main novelty of this paper is to discuss the stability analysis of mixed delayed neural network with T-S fuzzy concept.
- Reliable sampled data control is designed to achieve stability.
- Suitable LKF with integral terms are employed and it's derived by known integral inequalities.
- Finally numerical example is given to analyze the surety of the obtained results.

This paper is organised as follows: The problem description and some useful assumptions, definitions and lemmas are given in Section 2. In Section 3, the main part of this paper i.e. the main results are given through Theorem. A numerical example is fathomed to exhibit the sureness of the theoretical upshot in

Section 4. Eventually, a conclusion part is given in Section 5.

Notations: The notations mentioned below are used throughout the paper.

$\mathbb{R}^n$	n dimensional Euclidean space
$\mathbb{R}^{n\times m}$	set of $n \times m$ matrices
$*$	symmetric term
A^T	transpose of A
A^{-1}	inverse of A
I	identity matrix
$X > 0$	real symmetric positive definite matrix
$\|\| \cdot \|\|$	Euclidean norm

2. Problem description and preliminaries

Consider the following neural networks with mixed delays and Markovian jumping parameters,

$$\begin{aligned}\dot{\text{þ}}(\text{ŧ}) &= -\textsc{a}(\eta(\text{ŧ}))\text{þ}(\text{ŧ}) + \textsc{b}(\eta(\text{ŧ}))f(\text{þ}(\text{ŧ})) \\ &\quad + \text{Ꞓ}(\eta(\text{ŧ}))f(\text{þ}(\text{ŧ} - \text{ƛ}(\text{ŧ}))) + \text{ɑ}(\eta(\text{ŧ}))\int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f(\text{þ}(s))ds + u^F(\text{ŧ}),\end{aligned}$$

where $\text{þ}(\text{ŧ}) \in \mathbb{R}^m$ is state vector of network at time ŧ, $f(\text{þ}(\text{ŧ}))$ is the neuron activation function at time ŧ, $u^F(\text{ŧ})$ is the control input with actuator failures, $\textsc{a}(\eta(\text{ŧ})),\ \textsc{b}(\eta(\text{ŧ}),\ \text{Ꞓ}(\eta(\text{ŧ}))),\ \text{ɑ}(\eta(\text{ŧ}))$ are known constant matrices with appropriate dimensions.

$$0 \leq \text{ƛ}(\text{ŧ}) \leq \text{ƛ},\ \dot{\text{ƛ}}(\text{ŧ}) \leq \rho$$

where ƛ and ρ are constants.
$\eta(\text{ŧ})(\text{ŧ} \geq 0)$ is Markovian process from $\mathbb{S}$ with $\Pi \triangleq \Pi_{pq}$ given by

$$Pr(\eta(\text{ŧ} + \Delta(\text{ŧ})) = q|\eta(\text{ŧ}) = p) = \begin{cases} \Pi_{pq}\Delta(\text{ŧ}) + o(\Delta(\text{ŧ})),\ if\ q \neq p, \\ 1 + \Pi_{pq}\Delta(\text{ŧ}) + o(\Delta(\text{ŧ})),\ if\ q = p, \end{cases}$$

where $\Delta(\text{ŧ}) > 0,\ lim_{\Delta(\text{ŧ})\to 0}\frac{o(\Delta(\text{ŧ}))}{\Delta(\text{ŧ})} = 0$ and $\Pi_{pq} \geq 0$ from mode p at time ŧ to mode q at time $\text{ŧ} + \Delta(\text{ŧ})$ if $p \neq q$ and $\Pi_{pp} = -\sum_{q=1,q\neq p}^{s}\Pi_{pq}, \forall p \in \mathbb{S}, \mathbb{S} =\ \{1, 2, \ldots, s\}$.

Plant rule: IF $\omega_j(\text{ŧ})$ is N_{ij} for $j =, 2, ..., p$ THEN

$$\dot{\text{þ}}(\text{ŧ}) = -\mathrm{A}_i(\eta(\text{ŧ}))\text{þ}(\text{ŧ}) + \mathrm{B}_i(\eta(\text{ŧ}))f(\text{þ}(\text{ŧ})) + \mathrm{C}_i(\eta(\text{ŧ}))$$

$$f(\text{þ}(\text{ŧ} - \text{ƛ}(\text{ŧ}))) + \alpha_i(\eta(\text{ŧ}))\int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f(\text{þ}(s))ds + u^F(\text{ŧ}), i = 1, 2, ...k \tag{2.1}$$

Using fuzzy inference method, the system (2.1) can be written as

$$\dot{\text{þ}}(\text{ŧ}) = \sum_{i=1}^{k} h_i\omega(\text{ŧ})[-\mathrm{A}_i(\eta(\text{ŧ}))\text{þ}(\text{ŧ}) + \mathrm{B}_i(\eta(\text{ŧ}))f(\text{þ}(\text{ŧ})) + \mathrm{C}_i(\eta(\text{ŧ}))$$

$$f(\text{þ}(\text{ŧ} - \text{ƛ}(\text{ŧ}))) + \alpha_i(\eta(\text{ŧ}))\int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f(\text{þ}(s))ds + u^F(\text{ŧ})], \tag{2.2}$$

where $\omega(\text{ŧ}) = [\omega_1(\text{ŧ}), \omega_2(\text{ŧ}), ..., \omega_p(\text{ŧ})]$ and $h_i\omega(\text{ŧ}) = \frac{\mu_i\omega(\text{ŧ})}{\sum_{i=1}^{k}\mu_i\omega(\text{ŧ})}$, $\mu_i\omega(\text{ŧ}) = \Pi_{j=1}^{p}N_{ij}\omega_j(\text{ŧ})$. $N_{ij}\omega_j(\text{ŧ})$ is degree of the membership of $\omega_j(\text{ŧ})$ in N_{ij}.

In this paper, we assume that $\mu_i\omega(\text{ŧ}) \geq 0$, $\sum_{i=1}^{k}\mu_i\omega(\text{ŧ}) > 0$, $\forall \text{ŧ}$, $h_i\omega(\text{ŧ}) \geq 0$, for $i = 1, 2, .., k$ and $\sum_{i=1}^{k}\mu_i\omega(\text{ŧ}) > 0$, $\forall \text{ŧ}$, therefore $h_i\omega(\text{ŧ}) \geq 0$ for i and $\sum_{i=1}^{k} h_i\omega(\text{ŧ}) = 1$.

Control rule: If $\omega_j(\text{ŧ})$ is N_{ij} for $j = 1, 2, ..., p$ then

$$u(\text{ŧ}) = \mathcal{K}_i(\eta(\text{ŧ}))\text{þ}(\text{ŧ})$$

where $\mathcal{K}_i$ $(i = 1, 2, ..., k)$ denote the control gain matrix.

Nowadays most of the practical systems are controlled by digital

$$u(\text{ŧ}) = u(\text{ŧ}_k) = \mathcal{K}_i(\eta(\text{ŧ}))\text{þ}(\text{ŧ}_k)$$

where ŧ_k is the upper limit of k-th sample. Further, by denoting $h(\text{ŧ}) = \text{ŧ} - \text{ŧ}_k$ and $\text{ŧ}_k \leq \bar{h}$ can be written as

$$u(\text{ŧ}) = u(\text{ŧ}_k) = \mathcal{K}_i(\eta(\text{ŧ}))\text{þ}(\text{ŧ} - h(\text{ŧ})).$$

Moreover it is assume that $0 \leq h(\text{ŧ}) \leq \bar{h}$ with $\dot{h}(\text{ŧ}) = 1$ for $\text{ŧ} \neq \text{ŧ}_k$. Also we choose the reliable control input in the following form

$$u^F(\text{ŧ}) = \mathcal{G}u(\text{ŧ})$$

Then the output of the fuzzy reliable sampled data controller expressed in the following form:

$$u^F(ŧ) = \sum_{j=1}^{x} h_j(\omega(ŧ))\mathcal{G}\mathcal{K}_j\eta(ŧ)þ(ŧ - h(ŧ)) \tag{2.3}$$

Combining (2.2) with (2.3) we obtain the following closed loop $T - S$ fuzzy system as:

$$\dot{þ}(ŧ) = \sum_{i=1}^{k} h_i\omega(ŧ)[-\mathrm{A}_i(\eta(ŧ))þ(ŧ) + \mathrm{B}_i(\eta(ŧ))f(þ(ŧ)) + \mathrm{C}_i(\eta(ŧ))$$
$$f(þ(ŧ - ƛ(ŧ))) + ɑ_i(\eta(ŧ))\int_{ŧ-ƛ(ŧ)}^{ŧ} f(þ(s))ds + \mathcal{G}\mathcal{K}_j(\eta(ŧ))þ(ŧ - h(ŧ))].$$

Simply we mention $\mathrm{A}(\eta(ŧ))$ as A_p, $\mathrm{B}(\eta(ŧ))$ as B_p, $\mathrm{C}\eta(ŧ)$ as C_p and $ɑ(\eta(ŧ))$ as $ɑ_p$.

$$\dot{þ}(ŧ) = \sum_{i=1}^{k} h_i\omega(ŧ)[-\mathrm{A}_{ip}þ(ŧ) + \mathrm{B}_{ip}f(þ(ŧ)) + \mathrm{C}_{ip}$$
$$f(þ(ŧ - ƛ(ŧ))) + ɑ_{ip}\int_{ŧ-ƛ(t)}^{t} f(þ(s))ds \quad + \mathcal{G}\mathcal{K}_{jp}þ(ŧ - h(ŧ))] \tag{2.4}$$

Assumption 2.1. *[18] For any $j \in 1, 2, ..., n,\ f_j(0) = 0$ and there exist constants $\mathcal{F}_j^-$ and $\mathcal{F}_j^+$ such that*

$$\mathcal{F}^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq \mathcal{F}^+ \quad \forall \ \ \alpha_1 \neq \alpha_2.$$

Definition 2.1. *[19] The system is said to be asymptotically stable if it is stable, and for any $ŧ_0 \in \mathbb{R}^n$ and any $\epsilon < 0$, there exists a $\delta_a = \delta_a(ŧ_0, \epsilon) > 0$ such that $||x_{ŧ_0}||_c < \delta_a$ implies $lim_{ŧ\to\infty}x(ŧ) = 0$.*

Lemma 2.1. *(Jensen's Inequality) [20] Let $M \in \mathbb{R}^{n\times n}$, $M^T = M > 0$, be a constant matrix, scalars α and β with $\alpha > \beta$ and vector $ƀ : [\beta, \alpha] \to \mathbb{R}^n$, then:*

$$-(\alpha - \beta)\int_{\beta}^{\alpha} ƀ^T(s)Mƀ(s)ds \leq -\left(\int_{\beta}^{\alpha} ƀ(s)ds\right)^T M\left(\int_{\beta}^{\alpha} ƀ(s)ds\right),$$
$$-\frac{(\alpha - \beta)^2}{2}\int_{\beta}^{\alpha}\int_{u}^{\alpha} ƀ^T(s)Mƀ(s)dsdu \leq -\left(\int_{\beta}^{\alpha}\int_{u}^{\alpha} ƀ(s)dsdu\right)^T$$
$$\cdot M\left(\int_{\beta}^{\alpha}\int_{u}^{\alpha} ƀ(s)dsdu\right).$$

3. Main results

In this section, the stability criteria for neural networks with mixed time delay using reliable sampled data control is presented through the following theorem.

Theorem 3.1. *Under Assumption 2.1, for given positive scalars $\rho,\ \text{ƛ},\ h$ the fuzzy neural network* (2.4) *is asymptotically stable if there exist positive definite matrices $\mathcal{P}_1,\ \mathcal{Q}_1,\ \mathcal{Q}_2, \mathcal{Q}_3,\ \mathcal{R}_1,\ \mathcal{R}_2,\ \mathcal{S}_1,\ \mathcal{T}_1,\ \mathcal{T}_2$, any matrix $\mathcal{U}$ with appropriate dimensions such that the following linear matrix inequality hold:*

$$[\Omega]_{11\times 11} < 0,$$

where

$$\Omega_{1,1} = \mathcal{Q}_1 + \mathcal{Q}_2 + \text{ƛ}^2\mathcal{Q}_3 + \mathcal{T}_1 - \mathcal{T}_2 + \frac{\text{ƛ}^4}{4}\mathcal{S}_1 - \mathcal{F}_1\mathcal{L}, \qquad \Omega_{1,2} = \mathcal{P}_1 - \mathcal{A}\mathcal{U},$$

$$\Omega_{1,4} = \mathcal{T}_2, \qquad \Omega_{1,7} = \mathcal{F}_2\mathcal{L}, \qquad \Omega_{2,2} = h^2\mathcal{T}_2 - \mathcal{U} - \mathcal{U}^T, \qquad \Omega_{2,4} = \mathcal{G}\mathcal{Y},$$

$$\Omega_{2,7} = \mathcal{U}\mathrm{B}, \qquad \Omega_{2,8} = \mathcal{U}\mathrm{C}, \qquad \Omega_{2,10} = \mathcal{U}\text{ɑ}, \qquad \Omega_{3,3} = -\mathcal{F}_1\mathcal{S} - \mathcal{Q}_1,$$

$$\Omega_{3,8} = \mathcal{F}_2\mathcal{S}, \qquad \Omega_{4,4} = -2\mathcal{T}_2, \qquad \Omega_{4,6} = \mathcal{T}_2, \qquad \Omega_{5,5} = -\mathcal{Q}_2,$$

$$\Omega_{6,6} = -\mathcal{T}_2 - \mathcal{T}_1, \qquad \Omega_{7,7} = \mathcal{R}_1 + \text{ƛ}^2\mathcal{R}_2 - \mathcal{L}, \qquad \Omega_{8,8} = -(1-\rho)\mathcal{R}_1 - \mathcal{S},$$

$$\Omega_{9,9} = -\mathcal{Q}_3, \qquad \Omega_{10,10} = -\mathcal{R}_2, \qquad \Omega_{11,11} = -\mathcal{S}_1$$

and the values for other entities are zero. Control gain matrix is given by $\mathcal{K} = \mathcal{U}^{-1}\mathcal{Y}$.

Proof. Consider a LKF as follows:

$$V(\text{þ}(\text{ŧ})) = \sum_{r=1}^{5} V_r(\text{þ}(\text{ŧ}))$$

where:

$$V_1(\text{þ}(\text{ŧ})) = \text{þ}^T(\text{ŧ})\mathcal{P}_1\text{þ}(\text{ŧ}),$$

$$V_2(\text{þ}(\text{ŧ})) = \int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_1\text{þ}(s)ds + \int_{\text{ŧ}-\text{ƛ}}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_2\text{þ}(s)ds$$

$$+\,\text{ƛ}\int_{-\text{ƛ}}^{0}\int_{\text{ŧ}+u}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_3\text{þ}(s)dsdu,$$

$$V_3(\text{þ}(\text{ŧ})) = \int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f^T(\text{þ}(s))\mathcal{R}_1 f(\text{þ}(s))ds + \text{ƛ}\int_{-\text{ƛ}}^{0}\int_{\text{ŧ}+u}^{\text{ŧ}} f^T(\text{þ}(s))\mathcal{R}_2 f(\text{þ}(s))dsdu,$$

$$V_4(x(ŧ)) = \frac{ƛ^2}{2}\int_{-ƛ}^{0}\int_{\theta}^{0}\int_{ŧ+u}^{ŧ} þ^T(s)\mathcal{S}_1 þ(s)dsdud\theta,$$

$$V_5(þ(ŧ)) = \int_{ŧ-h}^{ŧ} þ^T(s)\mathcal{T}_1 þ(s)ds + h\int_{-h}^{0}\int_{ŧ+u}^{ŧ} \dot{þ}^T(s)\mathcal{T}_2\dot{þ}(s)dsdu.$$

By derivating the above Lyapunov Krasovskii-functional we get,

$$\dot{V}_1(þ(ŧ)) = 2þ^T(ŧ)\mathcal{P}_1\dot{þ}(ŧ) + þ^T(t)\sum_{q=1}^{N}\Pi_{pq}\mathcal{P}_q þ(t), \tag{3.1}$$

$$\begin{aligned}\dot{V}_2(þ(ŧ)) &= þ^T(ŧ)[\mathcal{Q}_1 + \mathcal{Q}_2 + ƛ^2\mathcal{Q}_3]þ(ŧ) - (1-\rho)þ^T(ŧ - ƛ(ŧ))\\ &\quad \mathcal{Q}_1 þ(ŧ - ƛ(ŧ)) - þ^T(ŧ - ƛ)\mathcal{Q}_2 þ(ŧ - ƛ) - ƛ\int_{ŧ-ƛ}^{ŧ} þ^T(s)\mathcal{Q}_3 þ(s)ds,\end{aligned} \tag{3.2}$$

$$\begin{aligned}\dot{V}_3(þ(ŧ)) &= f^T(þ(ŧ))[\mathcal{R}_1 + ƛ^2\mathcal{R}_2]f(þ(ŧ)) - (1-\rho)f^T(þ(ŧ\\ &\quad - ƛ(ŧ)))\mathcal{R}_1 f(þ(ŧ - ƛ(ŧ))) - ƛ\int_{ŧ-ƛ}^{ŧ} f^T(þ(s))\mathcal{R}_2 f(þ(s))ds,\end{aligned} \tag{3.3}$$

$$\dot{V}_4(þ(ŧ)) = \frac{ƛ^4}{4}x^T(ŧ)\mathcal{S}_1 x(ŧ) - \frac{ƛ^2}{2}\int_{-ƛ}^{0}\int_{ŧ+\theta}^{ŧ} þ^T(s)\mathcal{S}_1 þ(s)dsd\theta, \tag{3.4}$$

$$\begin{aligned}\dot{V}_5(þ(ŧ)) &= þ^T(ŧ)\mathcal{T}_1 þ(ŧ) - þ^T(ŧ - h)\mathcal{T}_1 þ(ŧ - h)\\ &\quad + h^2\dot{þ}(ŧ)\mathcal{T}_2\dot{þ}(ŧ) - h\int_{ŧ-h}^{ŧ} \dot{þ}^T(s)\mathcal{T}_2\dot{þ}(s)ds.\end{aligned} \tag{3.5}$$

By Lemma 2.1, we get:

(3.6)
$$-ƛ\int_{ŧ-ƛ}^{ŧ} þ^T(s)\mathcal{Q}_3 þ(s)ds \leq -\Big(\int_{ŧ-ƛ(ŧ)}^{ŧ} þ(s)ds\Big)^T\mathcal{Q}_3\Big(\int_{ŧ-ƛ(ŧ)}^{t} þ(s)ds\Big),$$

(3.7)
$$-ƛ\int_{ŧ-ƛ}^{ŧ} f^T(þ(s))\mathcal{R}_2 f(þ(s))ds \leq -\Big(\int_{ŧ-ƛ(ŧ)}^{ŧ} f(þ(s))ds\Big)^T\mathcal{R}_2\Big(\int_{ŧ-ƛ(ŧ)}^{ŧ} f(þ(s))ds\Big),$$

(3.8)
$$-\frac{ƛ^2}{2}\int_{-ƛ}^{0}\int_{ŧ+\theta}^{ŧ} þ^T(s)\mathcal{S}_1 þ(s)dsd\theta \leq -\Big(\int_{-ƛ}^{0}\int_{ŧ+\theta}^{ŧ} þ(s)dsd\theta\Big)^T\mathcal{S}_1\Big(\int_{-ƛ}^{0}\int_{ŧ+\theta}^{ŧ} þ(s)dsd\theta\Big).$$

Previous inequality can be written as

(3.9)
$$-h\int_{ŧ-h}^{ŧ}\dot{þ}^{T}(s)\mathcal{T}_2\dot{þ}(s)ds \le -h\int_{ŧ-h}^{ŧ-h(ŧ)}\dot{þ}(s)\mathcal{T}_2\dot{þ}(s)ds - h\int_{ŧ-h(ŧ)}^{ŧ}\dot{þ}(s)\mathcal{T}_2\dot{þ}(s)ds.$$

By applying Jensen's inequality and then simplifying we get:

$$-h\int_{ŧ-h}^{ŧ-h(ŧ)}\dot{þ}(s)\mathcal{T}_2\dot{þ}(s)ds \le -\Big(\int_{ŧ-h}^{ŧ-h(ŧ)}\dot{þ}(s)ds\Big)^{T}\mathcal{T}_2\Big(\int_{ŧ-h}^{ŧ-h(ŧ)}\dot{þ}(s)ds\Big)$$
$$\le -[þ(ŧ-h(ŧ))-þ(ŧ-h)]^{T}\mathcal{T}_2[þ(ŧ-h(ŧ))-þ(ŧ-h)] \tag{3.10}$$

$$-h\int_{ŧ-h(ŧ)}^{ŧ}\dot{þ}(s)\mathcal{T}_2\dot{þ}(s)ds \le -\Big(\int_{ŧ-h(ŧ)}^{ŧ}\dot{þ}(s)ds\Big)^{T}\mathcal{T}_2\Big(\int_{ŧ-h(ŧ)}^{ŧ}\dot{x}(s)ds\Big),$$
$$\le -[þ(ŧ)-þ(ŧ-h(ŧ))]^{T}\mathcal{T}_2[þ(ŧ)-þ(ŧ-h(ŧ))] \tag{3.11}$$

For any diagonal matrices $\mathcal{L},\mathcal{S}$, from Assumption 2.1, we get the following inequalities:

$$0 \le \begin{bmatrix} þ(ŧ) \\ f(þ(ŧ)) \end{bmatrix}^{T}\begin{bmatrix} -\mathcal{F}_1\mathcal{L} & \mathcal{F}_2\mathcal{L} \\ * & -\mathcal{L} \end{bmatrix}\begin{bmatrix} þ(ŧ) \\ f(þ(ŧ)) \end{bmatrix}, \tag{3.12}$$

$$0 \le \begin{bmatrix} þ(ŧ-ƛ(ŧ)) \\ f(þ(ŧ-ƛ(ŧ))) \end{bmatrix}^{T}\begin{bmatrix} -\mathcal{F}_1\mathcal{S} & \mathcal{F}_2\mathcal{S} \\ * & -\mathcal{S} \end{bmatrix}\begin{bmatrix} þ(ŧ-ƛ(ŧ)) \\ f(þ(ŧ-ƛ(ŧ))) \end{bmatrix}. \tag{3.13}$$

For any matrix $\mathcal{U}$ with appropriate dimensions, the following equation holds:

$$2\dot{þ}^{T}(ŧ)\mathcal{U}[-\dot{þ}^{T}(ŧ)+\sum_{i=1}^{k}h_i\omega(t)[-\mathcal{A}_{ip}þ(ŧ)+\mathrm{B}_{ip}f(þ(ŧ))+\mathrm{C}_{ip}$$
$$f(þ(ŧ-ƛ(ŧ)))+\mathrm{a}_{ip}\int_{ŧ-ƛ(ŧ)}^{ŧ}f(þ(s))ds+\mathcal{G}\mathcal{K}_{jp}þ(ŧ-h(ŧ))]]=0. \tag{3.14}$$

From the relations (3.1) - (3.14) we conclude

$$\dot{V}(þ(ŧ)) \le \zeta^{T}(ŧ)\Omega\zeta(ŧ),$$

where $\zeta^{T}(ŧ) = [þ^{T}(ŧ)\ \ \dot{þ}^{T}(ŧ)\ \ þ^{T}(ŧ-ƛ(ŧ))\ \ þ^{T}(ŧ-h(ŧ))\ \ þ^{T}(ŧ-ƛ)\ \ þ^{T}(ŧ-h)\ \ f^{T}(þ(ŧ))\ \ f^{T}(þ(ŧ-ƛ(ŧ)))\ \ \int_{ŧ-ƛ(ŧ)}^{ŧ}þ^{T}(s)ds\ \ \int_{ŧ-ƛ(ŧ)}^{ŧ}f^{T}(þ(s))ds\ \int_{-ƛ}^{0}\int_{ŧ+\theta}^{ŧ}þ^{T}(s)dsd\theta]$. and $\dot{V}(þ(ŧ))<0$. Therefore by Definition 2.1 we get that the mixed delayed neural network is asymptotically stable. This completes the proof. □

Remark 3.1. *Now consider there is no distributed delay, then equation* (2.4) *becomes:*

(3.15)
$$\dot{P}(ŧ) = \sum_{i=1}^{k} h_i\omega(ŧ)[-\mathcal{A}_{ip}P(ŧ) + \mathrm{B}_{ip}f(P(ŧ)) + Ϲ_{ip}f(P(ŧ - ƛ(ŧ))) + \mathcal{G}\mathcal{K}_{jp}P(ŧ - h(ŧ))]$$

For the mixed delay neural networks (3.15)*, we can derive the stability conditions from Theorem 3.1, then we have the following Corollary.*

Corollary 3.1. *Under Assumption 2.1, for given positive scalars $\rho,\ ƛ,\ h$ the system* (3.15) *is asymptotically stable if there exist positive definite matrices $\mathcal{P}_1,\ \mathcal{Q}_1,\ \mathcal{Q}_2, \mathcal{Q}_3$, $\mathcal{S}_1,\ \mathcal{T}_1,\ \mathcal{T}_2$, any matrix $\mathcal{U}$ with appropriate dimensions such that the following linear matrix inequality hold:*

$$[\bar{\Omega}]_{10\times 10} < 0, \tag{3.16}$$

where

$$\begin{aligned}
&\bar{\Omega}_{1,1} = \mathcal{Q}_1 + \mathcal{Q}_2 + ƛ^2\mathcal{Q}_3 + \mathcal{T}_1 - \mathcal{T}_2 + \frac{ƛ^4}{4}\mathcal{S}_1 - \mathcal{F}_1\mathcal{L}, \qquad \bar{\Omega}_{1,2} = \mathcal{P}_1 - \mathcal{A}\mathcal{U},\\
&\bar{\Omega}_{1,4} = \mathcal{T}_2, \qquad \bar{\Omega}_{1,7} = \mathcal{F}_2\mathcal{L}, \qquad \bar{\Omega}_{2,2} = h^2\mathcal{T}_2 - \mathcal{U} - \mathcal{U}^T, \qquad \bar{\Omega}_{2,4} = \mathcal{G}\mathcal{Y},\\
&\bar{\Omega}_{2,7} = \mathcal{U}\mathrm{B}, \qquad \bar{\Omega}_{2,8} = \mathcal{U}Ϲ, \qquad \bar{\Omega}_{3,3} = -\mathcal{F}_1\mathcal{S} - \mathcal{Q}_1, \qquad \bar{\Omega}_{3,8} = \mathcal{F}_2\mathcal{S},\\
&\bar{\Omega}_{4,4} = -2\mathcal{T}_2, \qquad \bar{\Omega}_{4,6} = \mathcal{T}_2, \qquad \bar{\Omega}_{5,5} = -\mathcal{Q}_2, \qquad \bar{\Omega}_{6,6} = -\mathcal{T}_2 - \mathcal{T}_1,\\
&\bar{\Omega}_{7,7} = \mathcal{R}_1 + ƛ^2\mathcal{R}_2 - \mathcal{L}, \qquad \bar{\Omega}_{8,8} = -(1-\rho)\mathcal{R}_1 - \mathcal{S}, \qquad \bar{\Omega}_{9,9} = -\mathcal{Q}_3,\\
&\bar{\Omega}_{10,10} = -\mathcal{S}_1.
\end{aligned}$$

and the values for other terms are zero. Control gain matrix is given by $\mathcal{K} = \mathcal{U}^{-1}\mathcal{Y}$.

Proof. Consider a Lyapunov Krasovskii functional:

$$\begin{aligned}
V_1(þ(ŧ)) &= þ^T(ŧ)\mathcal{P}_1þ(ŧ),\\
V_2(þ(ŧ)) &= \int_{ŧ-ƛ(ŧ)}^{ŧ} þ^T(s)\mathcal{Q}_1þ(s)ds + \int_{ŧ-ƛ}^{ŧ} þ^T(s)\mathcal{Q}_2þ(s)ds\\
&\quad + ƛ\int_{-ƛ}^{0}\int_{ŧ+u}^{ŧ} þ^T(s)\mathcal{Q}_3x(s)dsdu,\\
V_3(þ(ŧ)) &= \frac{ƛ^2}{2}\int_{-ƛ}^{0}\int_{\theta}^{0}\int_{ŧ+u}^{ŧ} þ^T(s)\mathcal{S}_1þ(s)dsdud\theta,
\end{aligned}$$

$$V_4(\text{þ}(t)) = \int_{t-h}^{t} \text{þ}^T(s)\mathcal{T}_1\text{þ}(s)ds + h\int_{-h}^{0}\int_{t+u}^{t} \dot{\text{þ}}^T(s)\mathcal{T}_2\dot{\text{þ}}(s)dsdu.$$

Using the same procedure as in Theorem 3.1, we get equation (3.16). This completes the proof. □

Remark 3.2. *There is no distributed delay and control, then equation* (3.15) *becomes:*

$$\dot{P}(t) = \sum_{i=1}^{k} h_i\omega(t)[-\mathcal{A}_{ip}P(t) + \text{B}_{ip}f(P(t)) + \mathcal{C}_{ip}f(P(t - \lambda(t)))] \tag{3.17}$$

For the mixed delay neural networks (3.17)*, we can derive the stability conditions from Theorem 3.1, then we have the following Corollary.*

Corollary 3.2. *Under Assumption 2.1 for given positive scalars* $\rho,\ \lambda,\ h$ *the system* (3.17) *is asymptotically stable if there exist positive definite matrices* $\mathcal{P}_1,\ \mathcal{Q}_1,\ \mathcal{Q}_2, \mathcal{Q}_3,$ $\mathcal{R}_1,\ \mathcal{R}_2,\ \mathcal{S}_1,\ \mathcal{T}_1,\ \mathcal{T}_2, \mathcal{U}$ *with appropriate dimensions such that the following LMI hold:*

$$[\hat{\Omega}]_{8\times 8} < 0, \tag{3.18}$$

where

$$\hat{\Omega}_{1,1} = \mathcal{Q}_1 + \mathcal{Q}_2 + \lambda^2\mathcal{Q}_3 + \frac{\lambda^4}{4}\mathcal{S}_1 - \mathcal{F}_1\mathcal{L}, \qquad \hat{\Omega}_{1,2} = \mathcal{P}_1 - \mathcal{A}\mathcal{U},$$
$$\hat{\Omega}_{1,5} = \mathcal{F}_2\mathcal{L}, \qquad \hat{\Omega}_{2,2} = -\mathcal{U} - \mathcal{U}^T, \qquad \hat{\Omega}_{2,5} = \mathcal{U}\text{B}, \qquad \hat{\Omega}_{2,6} = \mathcal{U}\mathcal{C},$$
$$\hat{\Omega}_{3,3} = -\mathcal{F}_1\mathcal{S} - \mathcal{Q}_1, \qquad \hat{\Omega}_{3,6} = \mathcal{F}_2\mathcal{S}, \qquad \hat{\Omega}_{4,4} = -\mathcal{Q}_2, \qquad \hat{\Omega}_{5,5} = -\mathcal{L},$$
$$\hat{\Omega}_{6,6} = -\mathcal{S}, \qquad \hat{\Omega}_{7,7} = -\mathcal{Q}_3, \qquad \hat{\Omega}_{8,8} = -\mathcal{S}_1.$$

and the values for other terms are zero.

Proof. Consider a Lyapunov Krasovskii functional:

$$V_1(\text{þ}(t)) = \text{þ}^T(t)\mathcal{P}_1\text{þ}(t),$$

$$
\begin{aligned}
V_2(\text{þ}(\text{ŧ})) &= \int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_1\text{þ}(s)ds + \int_{\text{ŧ}-\text{ƛ}}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_2\text{þ}(s)ds \\
&\quad + \text{ƛ}\int_{-\text{ƛ}}^{0}\int_{\text{ŧ}+u}^{\text{ŧ}} \text{þ}^T(s)\mathcal{Q}_3\text{þ}(s)dsdu, \\
V_3(\text{þ}(\text{ŧ})) &= \frac{\text{ƛ}^2}{2}\int_{-\text{ƛ}}^{0}\int_{\theta}^{0}\int_{\text{ŧ}+u}^{\text{ŧ}} \text{þ}^T(s)\mathcal{S}_1\text{þ}(s)dsdud\theta.
\end{aligned}
$$

Using the same procedure as in Theorem 3.1, we get equation (3.18). This completes the proof. □

4. NUMERICAL EXAMPLES

In this section, a numerical example is provided to illustrate the efficacy of the theoretical results. Consider a mixed delay neural network as:

Fuzzy Rule 1: If ω_1 is N_{11} and... and ω_p is N_{1p} **THEN**

$$
\begin{aligned}
\dot{\text{þ}}(\text{ŧ}) &= -\mathcal{A}_{1p}\text{þ}(\text{ŧ}) + \mathrm{B}_{1p}f(\text{þ}(\text{ŧ})) + \text{Ƈ}_{1p}f(\text{þ}(\text{ŧ}-\text{ƛ}(\text{ŧ}))) + \alpha_{1p}\int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f(\text{þ}(s))ds \\
&\quad + \mathcal{G}\mathcal{K}_{1p}\text{þ}(\text{ŧ}-h(\text{ŧ}))
\end{aligned}
$$

Fuzzy Rule 2: If ω_1 is N_{21} and... and ω_p is N_{2p} **THEN**

$$
\begin{aligned}
\dot{\text{þ}}(\text{ŧ}) &= -\mathcal{A}_{2p}\text{þ}(\text{ŧ}) + \mathrm{B}_{2p}f(\text{þ}(\text{ŧ})) + \text{Ƈ}_{2p}f(\text{þ}(\text{ŧ}-\text{ƛ}(\text{ŧ}))) + \alpha_{2p}\int_{\text{ŧ}-\text{ƛ}(\text{ŧ})}^{\text{ŧ}} f(\text{þ}(s))ds \\
&\quad + \mathcal{G}\mathcal{K}_{2p}\text{þ}(\text{ŧ}-h(\text{ŧ}))
\end{aligned}
$$

By solving the LMIs in Theorem 3.1 using MATLAB tool box, the controller gain matrix is obtained as:

$$
\mathcal{K}_{11} = \begin{bmatrix} -2.0793 & -0.080 \\ -0.0381 & -1.3074 \end{bmatrix},
$$

$$
\mathcal{K}_{21} = \begin{bmatrix} -5.0793 & -0.0690 \\ -0.0451 & -7.3074 \end{bmatrix}.
$$

5. Conclusion

In this paper, the reliable sampled data control problem for mixed delayed fuzzy neural network is studied. By employing the integral inequality technique, and Lyapunov approach, a new set of conditions are obtained which ensures that the fuzzy neural network is asymptotically stable for all possible actuator failures. Particularly, the reliable sampled data control law is designed in terms of the solution of certain linear matrix inequalities. The solvability of the concerned problem has been expressed as the feasibility of a set of LMI. Finally, a numerical example is given to validate the effectiveness of the proposed techniques.

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PG AND RESEARCH DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE(AUTONOMOUS)
TIRUPATTUR-635601, TAMILNADU, INDIA
Email address: dajaypravin@gmail.com

PG AND RESEARCH DEPARTMENT OF MATHEMATICS
C. ABDUL HAKEEM COLLEGE (AUTONOMOUS)
MELVISHARAM-632509, TAMILNADU, INDIA.
Email address: karpagam.sugumar@gmail.com