

## ON FUZZY TOPOLOGICAL $BRK$ -IDEAL

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**ABSTRACT.** In this article, the notion of fuzzy topological  $BRK$ -ideal of a  $BRK$ -algebra in a topology is introduced. Some theorems and properties of  $f\tau BRKI$  are stated and proved. The epimorphic and into homomorphic inverse images of a  $f\tau BRKI$  is also studied well. Also, we introduced a Cartesian product of a  $f\tau BRKI$  and studied their properties.

### 1. INTRODUCTION

Imai and Iseki [3] subjected two classes of abstract algebras:  $BCK$ -algebras and  $BCI$ -algebras in the year of 1996. In 1983, the notion of a  $BCH$ -algebra was introduced by Hu and Li [2], which is a generalization of  $BCK$  and  $BCI$ -algebras. In 2002, a new notion  $B$ -algebra was introduced by Neggers and Kim [8]. Also a  $BF$ -algebra and  $BG$ -algebra was introduced by Walendziak [11] in 2007 and C. B. Kim and H. S. Kim [5], which is a generalization of  $B$ -algebra. In 2012, R. K. Bandaru [9] introduced  $BRK$ -algebra, which is a generalization of  $BCK/BCI/BCH/Q/QS/BM$ -algebras [4,6,7]. In [1], El-Gendy introduced the notion of fuzzy  $BRK$ -ideal of  $BRK$ -algebra. S. Sivakumar et al. introduced a topology on  $BRK$ -algebra [10] and also studied several concepts. In this present paper we introduce a new notion of  $f\tau BRKI$  of a  $\tau BRK$  Alg. Also study some related properties in a  $f\tau BRKI$ . At last we introduce the Cartesian product of a  $f\tau BRKI$  and their properties.

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## 2. PRELIMINARIES

**Definition 2.1.** [9] A *BRK-algebra* (briefly, *BRK Alg*)  $(I, \star, 0)$  is a non-empty set  $I$  with a constant  $0$  and a binary operation  $\star$  satisfying the following axioms:

$$(2.1) \quad (BRK_1) \quad i_1 \star 0 = i_1,$$

$$(2.2) \quad (BRK_2) \quad (i_1 \star i_2) \star i_1 = 0 \star i_2$$

for any  $i_1, i_2 \in I$ . In a *BRK Alg*  $I$ ,  $\leq$  a partially ordered relation can be defined by  $i_1 \leq i_2$  iff  $i_1 \star i_2 = 0$ .

**Definition 2.2.** [10] Let  $(I, \star, 0)$  be a *BRK Alg* and  $\tau$  a topology on  $I$ . Then  $I = (I, \star, 0, \tau)$  is called a *topological BRK Alg* (briefly,  $\tau$ *BRK Alg*), if “ $\star$ ” is continuous or equivalently, for any  $m, n \in X$  and  $\forall O$  open set of  $m \star n$ ,  $\exists$  two open sets  $M$  and  $N$  respectively, such that  $M \star N$  is a subset of  $O$ .

**Definition 2.3.** [10] Let  $I$  be a  $\tau$ *BRK Alg* and  $D$  be a subset of  $I$ , then  $D$  is called a  $\tau$ *BRK-ideal* (briefly,  $\tau$ *BRK I*) of  $I$ , if for any  $i_{11}, i_{22} \in I$ :

$$(i) \quad 0 \in D,$$

$$(ii) \quad 0 \star (i_{11} \star i_{22}) \in D \text{ and } 0 \star i_{22} \in D \text{ imply } i_{11}, i_{22} \in I.$$

**Definition 2.4.** [1] Let  $I$  be a set. A function  $\mu_I : I \rightarrow [0, 1]$  where  $\mu_I$  a fuzzy set in  $I$ .

**Definition 2.5.** [1] Let  $(I, \star, 0)$  be a *BRK Alg*. A fuzzy set  $\mu_I$  in  $I$  is called a fuzzy *BRK-ideal* (briefly, *fBRKI*) of  $I$  if

$$(BRKFI_1) \quad \mu_I(0) \geq \mu_I(i_1),$$

$$(BRKFI_2) \quad \mu_I(0 \star i_1) \geq \min\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\}, \text{ for all } i_1, i_2 \in I.$$

3. FUZZY  $\tau$ BRK-IDEAL

**Definition 3.1.** Let  $(I, \star, 0, \tau)$  be a  $\tau$ *BRK Alg*. A fuzzy set  $\mu_I$  in  $I$  is called an *fuzzy topological BRK-ideal* (briefly, *f $\tau$ BRKI*) of  $I$  if

$$(3.1) \quad \mu_I(0) \geq \mu_I(i_1),$$

$$(3.2) \quad \mu_I(0 \star i_1) \geq \min\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\}, \text{ for all } i_1, i_2 \in I.$$

**Definition 3.2.** Let  $(I, \star, 0, \tau)$  be a  $\tau$ BRK Alg. A fuzzy set  $\mu_I$  in  $I$  is called an Anti fuzzy topological BRK-ideal (briefly,  $Af\tau$ BRK  $I$ ) of  $I$  if

$$(3.3) \quad \mu_I(0) \leq \mu_I(i_1),$$

$$(3.4) \quad \mu_I(0 \star i_1) \leq \max\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\}, \text{ for all } i_1, i_2 \in I.$$

**Example 1.** Let  $(I = \{0, a_1, b_1, c_1\}, \star, 0)$  be a BRK Alg defined by

$\star$	0	$a_1$	$b_1$	$c_1$
0	0	0	$b_1$	$b_1$
$a_1$	$a_1$	0	$b_1$	$b_1$
$b_1$	$b_1$	$b_1$	0	0
$c_1$	$c_1$	$c_1$	$a_1$	0

Define a topology  $\tau = \{\phi, I, \{b_1\}, \{c_1\}, \{b_1, c_1\}, \{0, a_1\}, \{0, a_1, b_1\}, \{0, a_1, c_1\}\}$  is a  $\tau$ BRK Alg. Now define  $\mu_I : I \rightarrow [0, 1]$  by  $\mu_I(0) = K_1$ ,  $\mu_I(a_1) = \mu_I(b_1) = \mu_I(c_1) = K_2$ , where  $K_1, K_2 \in [0, 1]$  with  $K_1 > K_2$  gives that  $\mu_I$  is an  $f\tau$ BRKI.

**Proposition 3.1.** Let  $\mu_I$  be an  $f\tau$ BRKI of  $\tau$ BRK Alg  $I$  and if  $i_1 \geq i_2$ , then  $\mu_I(0 \star i_1) \geq \mu_I(0 \star i_2)$ ,  $\forall i_1, i_2 \in I$ .

*Proof.* Let  $\mu_I$  be an  $f\tau$ BRKI of a  $\tau$ BRK Alg  $I$ . For any  $i_1, i_2 \in I$  such that  $i_1 \geq i_2$ . Since  $i_1 \geq i_2$ , then  $i_1 \star i_2 = 0$ .

$$\begin{aligned} \mu_I(0 \star i_1) &\geq \min\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\} \\ &= \min\{\mu_I(0 \star 0), \mu_I(0 \star i_2)\} \\ &= \min\{\mu_I(0), \mu_I(0 \star i_2)\} \\ &= \mu_I(0 \star i_2). \end{aligned}$$

Hence  $\mu_I(0 \star i_1) \geq \mu_I(0 \star i_2)$ . □

**Theorem 3.1.** A fuzzy subset  $\mu_I$  of a  $\tau$ BRK Alg  $I$  is a  $Af\tau$ BRK  $I$  of  $I$  iff  $\mu_I^c$  is an  $f\tau$ BRKI of  $I$ .

*Proof.* Let  $\mu_I$  be a  $Af\tau$ BRK  $I$  of a  $\tau$ BRK Alg  $I$ , and let  $i_1, i_2 \in I$ . Then Since  $\mu_I(0) \leq \mu_I(i_1)$  then

$$1 - \mu_I(0) \geq 1 - \mu_I(i_1)$$

$$(3.5) \quad \mu_I^c(0) \geq \mu_I^c(i_1).$$

Further,

$$\begin{aligned}
 \mu_I(0 \star i_1) &\leq \max\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\} \\
 1 - \mu_I(0 \star i_1) &\geq 1 - \max\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\} \\
 \mu_I^c(0 \star i_1) &\geq \min\{1 - \mu_I(0 \star (i_1 \star i_2)), 1 - \mu_I(0 \star i_2)\} \\
 (3.6) \quad \mu_I^c(0 \star i_1) &\geq \min\{\mu_I^c(0 \star (i_1 \star i_2)), \mu_I^c(0 \star i_2)\}
 \end{aligned}$$

So,  $\mu_I^c$  is an  $f\tau BRKI$  of  $I$ .

Now let  $\mu_I^c$  is an  $f\tau BRKI$  of a  $\tau BRK$  Alg  $I$ , and let  $i_3, i_4 \in I$ . Then Since  $\mu_I^c(0) \geq \mu_I^c(i_3)$  then

$$\begin{aligned}
 1 - \mu_I^c(0) &\leq 1 - \mu_I^c(i_3) \\
 (3.7) \quad \mu_I(0) &\leq \mu_I(i_3).
 \end{aligned}$$

So,

$$\begin{aligned}
 \mu_I^c(0 \star i_3) &\geq \min\{\mu_I^c(0 \star (i_3 \star i_4)), \mu_I^c(0 \star i_4)\} \\
 1 - \mu_I^c(0 \star i_3) &\leq 1 - \min\{\mu_I^c(0 \star (i_3 \star i_4)), \mu_I^c(0 \star i_4)\} \\
 \mu_I(0 \star i_3) &\leq \max\{1 - \mu_I^c(0 \star (i_3 \star i_4)), 1 - \mu_I^c(0 \star i_4)\} \\
 (3.8) \quad \mu_I(0 \star i_3) &\leq \max\{\mu_I(0 \star (i_3 \star i_4)), \mu_I(0 \star i_4)\}
 \end{aligned}$$

Therefore,  $\mu_I$  is a  $Af\tau BRK$   $I$  of a  $\tau BRK$  Alg  $I$ . □

**Theorem 3.2.** Let  $\mu_I$  be an  $f\tau BRKI$  of  $\tau BRK$  Alg  $I$ . Then  $I_{\mu_I} = \{i_1 \in I \mid \mu_I(0 \star i_1) = \mu_I(0)\}$  is a  $\tau BRK$   $I$ .

*Proof.* Clearly  $0 \in I_{\mu_I}$ . Let  $i_1, i_2 \in I_{\mu_I}$  be such that  $(0 \star (i_1 \star i_2)) \in I_{\mu_I}$  and  $0 \star i_2 \in I_{\mu_I}$ . Then  $\mu_I(0 \star (i_1 \star i_2)) = \mu_I(0 \star i_2) = \mu_I(0)$ . It follows that

$$\begin{aligned}
 \mu_I(0 \star i_1) &\geq \min\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\} \\
 \mu_I(0 \star i_1) &\geq \min\{\mu_I(0), \mu_I(0)\} \\
 \mu_I(0 \star i_1) &\geq \mu_I(0).
 \end{aligned}$$

So, by combining with Definition 3.1, we get that  $\mu_I(0 \star i_1) = \mu_I(0)$  and hence  $0 \star i_1 \in I_{\mu_I}$ . □

**Definition 3.3.** Let  $(I, \star, 0, \tau)$  and  $(J, \star', 0', \tau)$  be  $\tau$ BRK Algs. A mapping  $h : I \rightarrow J$  is said to be a homomorphism of a  $\tau$ BRK Alg if  $h(i_1 \star i_2) = h(i_1) \star' h(i_2)$ ,  $\forall i_1, i_2 \in I$ .

**Definition 3.4.** Let a map  $h : I \rightarrow J$ . If  $\mu_I^*$  is a fuzzy subset of  $J$ , then the fuzzy subset defined by  $\mu_I^*(h(i_1)) = \mu_I(i_1) \forall i_1 \in I$  is said to be the inverse image of  $\mu_I^*$  under  $h$ .

**Theorem 3.3.** The epimorphic image of an  $f\tau$ BRKI is also an  $f\tau$ BRKI.

*Proof.* Let  $h : I \rightarrow J$  be an epimorphism of  $\tau$ BRK Alg's  $(I, \star, 0, \tau)$  and  $(J, \star', 0', \tau)$ . Consider that  $\beta$  is an  $f\tau$ BRKI of  $I$  and  $\mu_I$  is the image of  $\beta$  under  $h$ . Let  $j_1 \in J$ . Then  $\exists i_1 \in I$  such that  $h(i_1) = j_1$ . Then

$$\mu_I(j_1) = \mu_I(h(i_1)) = \beta(i_1) \leq \beta(0) = \mu_I(h(0)) = \mu_I(0').$$

Let  $i'_1, j'_1 \in J$ . Then  $\exists i_1, j_1 \in I \ni h(i_1) = i'_1$  &  $h(j_1) = j'_1$ . It follows that

$$\begin{aligned} \mu_I(0' \star' i'_1) &= \mu_I(h(0 \star i_1)) \\ &= \beta(0 \star i_1) \geq \min\{\beta(0 \star (i_1 \star j_1)), \beta(0 \star j_1)\} \\ &= \min\{\mu_I(h(0 \star (i_1 \star j_1))), \mu_I(h(0 \star j_1))\} \\ &= \min\{\mu_I(h(0) \star' (h(i_1) \star' h(j_1))), \mu_I(h(0) \star' h(j_1))\} \\ &= \min\{\mu_I(0' \star' (i'_1 \star' j'_1)), \mu_I(0' \star' j'_1)\}. \end{aligned}$$

Hence  $\mu_I$  is an  $f\tau$ BRKI of  $J$ . □

**Theorem 3.4.** The into homomorphic inverse image of an  $f\tau$ BRKI is also an  $f\tau$ BRKI.

*Proof.* Let  $h : I \rightarrow J$  be an into homomorphism of  $\tau$ BRK Alg's  $(I, \star, 0, \tau)$ ,  $(J, \star', 0', \tau)$ . And  $\mu_I^*$  is an  $f\tau$ BRKI of  $J$  and  $\mu_I$  is the inverse image of  $\mu_I^*$  under  $h$ . By definition 3.4 we find that  $\mu_I^*(h(i_1)) = \mu_I(i_1)$ , for all  $i_1 \in I$ , since  $\mu_I^*$  is an  $f\tau$ BRKI of  $J$ , then  $\mu_I^*(0') \geq \mu_I^*(h(i_1)) \forall i_1 \in I$ .

So that (3.7) holds, since  $\mu_I(0) = \mu_I^*(h(0)) = \mu_I^*(0') \geq \mu_I^*(h(i_1)) = \mu_I(i_1)$ . For all  $i_1, i_2 \in I$ , we have

$$\begin{aligned} \mu_I(0 \star i_1) &= \mu_I^*(h(0 \star i_1)) = \mu_I^*(h(0) \star' h(i_1)) \\ &\geq \min\{\mu_I^*(h(0) \star' (h(i_1) \star' h(i_2))), \mu_I^*(h(0) \star' h(i_2))\} \\ &= \min\{\mu_I^*(h(0 \star (i_1 \star i_2))), \mu_I^*(h(0 \star i_2))\} \\ &= \min\{\mu_I(0 \star (i_1 \star i_2)), \mu_I(0 \star i_2)\}. \end{aligned}$$

Hence  $\mu_I(0 \star i_1) = \mu_I^*(h(0 \star i_1)) = (\mu_I^* \circ h)(0 \star i_1)$  is an  $f\tau BRKI$  of  $I$ . The proof is complete.  $\square$

#### 4. CARTESIAN PRODUCT OF $f\tau BRK$ -IDEAL

**Definition 4.1.** A  $\mu_I$  be fuzzy relation on any set  $I$  is a fuzzy subset  $\mu_I : I \times I \rightarrow [0, 1]$ .

**Definition 4.2.** Let  $\mu_I$  and  $\mu_I^*$  be fuzzy subsets of a set  $I$ . The Cartesian product of  $\mu_I$  and  $\mu_I^*$  is defined by  $(\mu_I \times \mu_I^*)(i_1, j_1) = \min\{\mu_I(i_1), \mu_I^*(j_1)\} \forall i_1, j_1 \in I$ .

**Corollary 4.1.** Let  $(I, \star, 0, \tau)$  and  $(J, \star, 0', \tau)$  be  $\tau BRK$  Alg's, we define  $\star$  on  $I \times J$  by for every  $(i_3, i_4), (j_3, j_4) \in I \times J$ ,  $(i_3, i_4) \star (j_3, j_4) = (i_3 \star j_3, i_4 \star j_4)$  then  $(I \times J, \star, (0, 0'), \tau)$  is a  $\tau BRK$  Alg.

*Proof.* Let  $(I, \star, 0, \tau)$  and  $(J, \star, 0', \tau)$  be  $\tau BRK$  Alg's (see Definition 3.1). For all  $(i_3, i_4), (j_3, j_4) \in I \times J$ , then

$$\begin{aligned} (i) - (i_3, i_4) \star (0, 0') &= (i_3 \star 0, i_4 \star 0') = (i_3, i_4) \\ (ii) - ((i_3, i_4) \star (j_3, j_4)) \star (i_3, i_4) &= (i_3 \star j_3, i_4 \star j_4) \star (i_3, i_4) \\ &= ((i_3 \star j_3) \star i_3, (i_4 \star j_4) \star i_4) = (0 \star j_3, 0' \star j_4). \end{aligned}$$

So,  $(I \times J, \star, (0, 0'), \tau)$  is a  $\tau BRK$  Alg.  $\square$

**Theorem 4.1.** If  $\mu_I$  and  $\mu'_I$  are  $f\tau BRKI$ 's of  $\tau BRK$  Alg's  $I$ , then  $\mu_I \times \mu'_I$  is an  $f\tau BRKI$  of  $(I \times I, \star, (0, 0'), \tau)$ .

*Proof.* Let  $i_3, i'_3 \in I \times I$ . Then

$$(\mu_I \times \mu'_I)(0, 0') = \min\{\mu_I(0), \mu'_I(0')\} \geq \min\{\mu_I(i_3), \mu'_I(i'_3)\} = (\mu_I \times \mu'_I)(i_3, i'_3).$$

For any  $(i_3, i'_3), (i_4, i'_4) \in I \times I$  we have

$$\begin{aligned} (\mu_I \times \mu'_I)(0 \star i_3, 0' \star i'_3) &= \min\{\mu_I(0 \star i_3), \mu'_I(0' \star i'_3)\} \\ &= \min\{\min\{\mu_I(0 \star (i_3 \star i_4)), \mu_I(0 \star i_4)\}, \min\{\mu'_I(0' \star (i'_3 \star i'_4)), \mu'_I(0' \star i'_4)\}\} \\ &= \min\{\min\{\mu_I(0 \star (i_3 \star i_4)), \mu'_I(0' \star (i'_3 \star i'_4))\}, \min\{\mu_I(0 \star i_4), \mu'_I(0' \star i'_4)\}\} \\ &= \min\{(\mu_I \times \mu'_I)((0, 0') \star ((i_3, i'_3) \star (i_4, i'_4))), (\mu_I \times \mu'_I)((0, 0') \star (i_4, i'_4))\}. \end{aligned}$$

Hence  $\mu_I \times \mu'_I$  is a  $f\tau BRKI$  of  $(I \times I, \star, (0, 0'), \tau)$ .  $\square$

**Definition 4.3.** If  $\zeta$  is a fuzzy subset of a set  $I$ , the strongest fuzzy relation on  $I$  that is a fuzzy relation on  $\zeta$  is  $\mu_{I_\zeta}$  given by  $\mu_{I_\zeta}(i_1, i_2) = \min\{\zeta(i_1), \zeta(i_2)\} \forall i_1, i_2 \in I$ .

**Proposition 4.1.** For a fuzzy subset  $\zeta$  of a  $\tau BRK$  Alg  $I$ , let  $\mu_{I_\zeta}$  be the strongest fuzzy relation on  $I$ . If  $\mu_{I_\zeta}$  is an  $f\tau BRKI$  of  $(I \times I; \star, (0, 0))$ , then  $\zeta(0) \geq \zeta(i_1)$  for all  $i_1 \in I$ .

*Proof.* Since  $\mu_{I_\zeta}$  is a  $f\tau BRKI$  of  $I \times I$ , it follows from (3.5) that  $\mu_{I_\zeta}(0, 0) \geq \mu_{I_\zeta}(i_1, i_1)$ . So that  $\mu_{I_\zeta}(0, 0) = \min\{\zeta(0), \zeta(0)\} \geq \max\{\zeta(i_1), \zeta(i_1)\} = \mu_{I_\zeta}(i_1, i_1)$ . This implies that  $\zeta(0) \geq \zeta(i_1)$ .  $\square$

**Theorem 4.2.** Let  $\zeta$  be a fuzzy subset of  $\tau BRK$  Alg  $I$  and  $\mu_{I_\zeta}$  be the strongest fuzzy relation on  $I$ . If  $\zeta$  is a  $f\tau BRKI$  of  $I$  then  $\mu_{I_\zeta}$  is a  $f\tau BRKI$  of  $(I \times I; \star, (0, 0'), \tau)$ .

*Proof.* Suppose that,  $\zeta$  is a fuzzy subset of a  $f\tau BRKI$   $I$  and  $\mu_{I_\zeta}$  is the strongest fuzzy relation on  $I$ . Then  $\mu_{I_\zeta}(0, 0') = \min\{\zeta(0), \zeta(0')\} \geq \min\{\zeta(i_1), \beta(j_1)\} = \mu_{I_\zeta}(i_1, j_1) \forall (i_1, j_1) \in I \times I$ .

For all  $(i_1, i'_1), (j_1, j'_1) \in I \times I$ , we get that

$$\begin{aligned} \mu_{I_\zeta}((0, 0') \star (i_1, i'_1)) &= \mu_{I_\zeta}(0 \star i_1, 0' \star i'_1) = \min\{\beta(0 \star i_1), \beta(0' \star i'_1)\} \\ &\geq \min\{\min\{\beta(0 \star (i_1 \star j_1)), \beta(0 \star j_1)\}, \min\{\zeta(0' \star (i'_1 \star j'_1)), \zeta(0' \star j'_1)\}\} \\ &= \min\{\min\{\zeta(0 \star (i_1 \star j_1)), \beta(0' \star (i'_1 \star j'_1))\}, \min\{\beta(0 \star j_1), \beta(0' \star j'_1)\}\} \\ &= \min\{\mu_{I_\zeta}(0 \star (i_1 \star j_1), 0' \star (i'_1 \star j'_1)), \mu_{I_\zeta}(0 \star j_1, 0' \star j'_1)\} \\ &= \min\{\mu_{I_\zeta}((0, 0') \star ((i_1, i'_1) \star (j_1, j'_1))), \mu_{I_\zeta}((0, 0') \star (j_1, j'_1))\}. \end{aligned}$$

Hence  $\mu_{I_\zeta}$  is a  $f\tau BRKI$  of  $(I \times I; \star, (0, 0'), \tau)$ .  $\square$

## 5. CONCLUSION

In this paper, the  $f\tau BRKI$  concept of  $\tau BRK$  Alg was introduced and studied their properties. The epimorphic and into homomorphic inverse images of a  $f\tau BRKI$  are also discussed and studied well. The  $f\tau BRKI$  of a cartesian product was also discussed in this work.

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