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TIME INDEPEDENT BEHAVIOUR OF BULK BISERIAL QUEUING SUBSYSTEMS CONNECTED TO A COMMON SERVER

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ABSTRACT. In the present paper we study a network queue model comprising of two hetrogeneous biserial queuing subsystems centrally connected with a single server. Arrival takes place in batches of fixed size according to poission process the and service is provided individualy under first come first served rule. Queue characteristics for the model have been computed using generating function technique and laws of calculus. The model is well demonstrated with the help of an numerical example.

1. INTRODUCTION

The models equipped with bulk input or bulk service proffers the influential structure for evaluating the performance measures of the system and are of great significance in diverse areas. Maggu [1] studied the queues with two servers in biseries. Biserial queues with batch arrival were examined by Mohammad and others [2]. Kumar et al [3] extended the work with parallel queues and analysed the transient behaviour. Further many other analysts worked on these models and gave the significant results. Gupta et al [4, 5] developed waiting line models characterised with both parallel and biserial queues connected with another server. Serial and non serial parallel servers with balking and reneging effect were discussed by Gupta M. et al [6]. Gupta et al [7] investigated biserial servers under fuzzy environment. Threshold effect on fuzzy queue having arrival in

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batches was explored by Mittal et al [8]. Also the biserial bulk queue model was designed by Mittal et al [9]. Agrawal et al [10] used biserial tri-cum model to compute various queue characteristics. Parallel queue network model with static batch arrival was developed by Gupta et al[11]. The study of Gupta et al [12] has been protracted in the present article by replacing parallel queue system with that of biserial queuing subsystem.

2. Model Formulation

The queue network model under consideration comprises of two queuing subsystems S1 and S3 each being centrally connected with a single server S2. The subsystem S1 consists of two biserial servers S11 & S12 and the subsystem S3 comprises of biserial servers S31 & S32.

2.1. **Assumptions and Notations.** The arriving unit demanding services join the servers S_{11} & S_{12} in batches of fixed size β_1 & β_2 with mean arrival rates λ_1 & λ_2 respectively under poisson assumptions and join the queues Q_1 & Q_2 formed in front of servers S_{11} & S_{12}. The mean service rates at the service channels S_{11}, S_{12}, S_2, S_{31}, S_{32} for convenience have been assumed as μ_1, μ_2, μ_3, μ_4 and μ_5 respectively.

2.2. **Description.**

- Customers arriving with mean arrival rate λ_1 are served at S_{11} and thereafter customer may either join S_{12} with the transition probability τ_{12} or may join S_3 with the probability τ_{13} such that $\tau_{12} + \tau_{13} = 1$.
- Those approaching at S_{12} with mean rate λ_2 after being served there, either will move to S_{11} with the probability τ_{21} or join S_3 with the probability τ_{23} such that $\tau_{21} + \tau_{23} = 1$.
- After approaching the service at S_3, the customer will either join S_{31} with the probability τ_{34} or join S_{32} with the probability τ_{35} in accordance with the requirement of the services in such a way that that the total probability i.e. $\tau_{34} + \tau_{35} = 1$.
- Thereafter being served at S_{31}, the customer may either moves to S_{32} with probability τ_{45} or may exit from the system with probability τ_{44} such that $\tau_{44} + \tau_{45} = 1$.

- Similarly the customers after availing services at S_{32} may either move to S_{31} with transition probability τ_{54} or may leave the system with probability τ_{55} such that $\tau_{54} + \tau_{55} = 1$.

The formulation of the model with nomenclature, all the transition states and corresponding probabilities has been well demonstrated pictorially in figure 1.

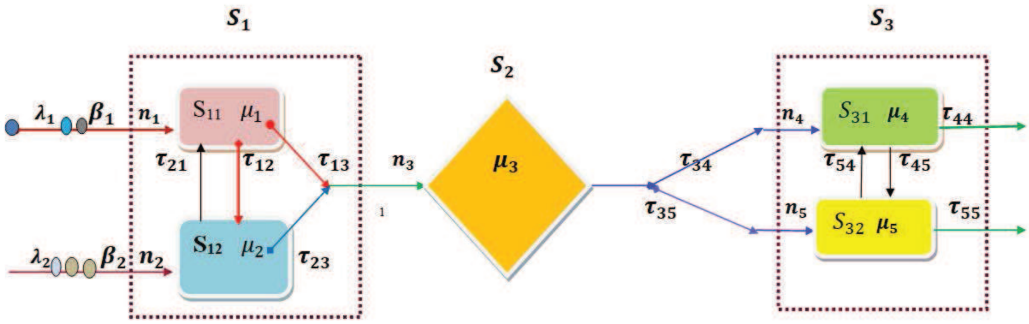

FIGURE 1. BISERIAL QUEUE NETWORK MODEL

3. PRACTICAL ASPECT

The queue network model under consideration is applicable in diverse area of physical word like administrative setup, office management, games club, banking system, computer networks, and many other similar situations. For example, consider a supermarket having three different sections namely food section, auditorium and beverages section, where auditorium is centrally linked to both food section and beverages section. The food section consists of two sections, Indian and Chinese food section. Similarly beverages section is having soft drink and cold drink sections. Customers approaching to food section after taking Indian food may directly go to auditorium to watch movie or may take Chinese food and then enters into the auditorium. Similarly, the customer taking Chinese food may take Indian food and then move to the auditorium or may directly move to auditorium without having Indian food. After watching movie, a customer either go to soft drink section or wish to go hot drink section. After having soft drink, a customer either may have hot drink and then leaves the mall or may directly exit from the mall and vice versa.

4. EQUATIONS FORMATION

Define $P_{n_1,n_2,n_3,n_4,n_5}(\mathrm{t})$ = The probability that there are $\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3, \mathrm{n}_4,$ and n_5 units at any time t waiting in queues, where $\mathrm{n}_1 > \beta_1, \mathrm{n}_2 > \beta_2, \mathrm{n}_3, \mathrm{n}_4, \mathrm{n}_5 > 0$. The difference equations in Steady- State form obtained are as follows:

$$When \quad \mathbf{n}_1 > \boldsymbol{\beta}_1, \mathbf{n}_2 > \boldsymbol{\beta}_2, \mathbf{n}_3, \mathbf{n}_4, \mathbf{n}_5 > 0$$

$$\begin{aligned}
&(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)\,\mathrm{P}_{n_1,n_2,n_3,n_4,n_5} \\
&= \lambda_1 P_{n_1-\beta_1,n_2,n_3,n_4,n_5} \\
&\quad + \lambda_2 P_{n_1,n_2-\beta_2,n_3,n_4,n_5} + \mu_1\tau_{12}P_{n_1+1,n_2-1,n_3,n_4,n_5} \\
&\quad + \mu_1\tau_{13}P_{n_1+1,n_2,n_3-1,n_4,n_5} + \mu_2\tau_{21}P_{n_1-1,n_2+1,n_3,n_4,n_5} \\
&\quad + \mu_2\tau_{23}P_{n_1,n_2+1,n_3-1,n_4,n_5} + \mu_3\tau_{34}P_{n_1,n_2,n_3+1,n_4-1,n_5} \\
&\quad + \mu_3\tau_{35}P_{n_1,n_2,n_3+1,n_4,n_5-1} + \mu_4\tau_{44}P_{n_1,n_2,n_3,n_4+1,n_5} \\
&\quad + \mu_4\tau_{45}P_{n_1,n_2,n_3,n_4+1,n_5-1} + \mu_5\tau_{54}P_{n_1,n_2,n_3,n_4-1,n_5+1} \\
&\quad + \mu_5\tau_{55}P_{n_1,n_2,n_3,n_4,n_5+1}.
\end{aligned}$$

Similarly considering all other possible combinations formed by utilizing the conditions imposed on n_1, n_2, n_3, n_4 and n_5; we obtain in total 72 difference equations in steady state form and the system of equations is solved by applying generating function technique. Generating function is defined as:

$$G(x,y,z,s,t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5}\left(x^{n_1}y^{n_2}z^{n_3}s^{n_4}t^{n_5}\right).$$

Also partial generating function is defined as:

$$G_{n_2,n_3,n_4,n_5}(x) = \sum_{n_1=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5}x^{n_1}$$

$$G_{n_3,n_4,n_5}(x,y) = \sum_{n_2=0}^{\infty} G_{n_2,n_3,n_4,n_5}(x)y^{n_2}$$

$$G_{n_4,n_5}(x,y,z) = \sum_{n_3=0}^{\infty} G_{n_3,n_4,n_5}(x,y)z^{n_3}$$

$$G_{n_5}(x,y,z,s) = \sum_{n_4=0}^{\infty} G_{n_4,n_5}(x,y,z)s^{n_4}$$

$$G(x,y,z,s,t) = \sum_{n_5=0}^{\infty} G_{n_5}(x,y,z,s)t^{n_5}.$$

On solving the difference equations with the help of (2) and (3); the finally obtained equation is as follows:

$$\begin{aligned}
&(\lambda_1+\lambda_2+\mu_1+\mu_2\ +\mu_3+\mu_4+\mu_5)\,G(x,y,z,s,t)\\
&\quad-\mu_1\mathrm{G}(y,z,s,t)-\mu_2\mathrm{G}(x,z,s,t)\\
&\quad-\mu_3\mathrm{G}(x,y,s,t)-\mu_4\mathrm{G}(x,y,z,t)-\mu_5\mathrm{G}(x,y,z,s)\\
&=\lambda_1x^{\beta_1}\mathrm{G}(x,y,z,s,t)+\lambda_2y^{\beta_2}\mathrm{G}(x,y,z,s,t)\\
&\quad+\frac{\mu_1\tau_{12}}{x}y[G(x,y,z,s,t)]-G(y,z,s,t)\Big]\\
&\quad+\frac{\mu_1\tau_{13}}{x}z[G(x,y,z,s,t)-G(y,z,s,t)]\\
&\quad+\frac{\mu_2\tau_{21}}{y}x[G(x,y,z,s,t)-G(x,z,s,t)]\\
&\quad+\frac{\mu_2\tau_{23}}{y}z[G(x,y,z,s,t)-G(x,z,s,t)]\\
&\quad+\frac{\mu_3\tau_{34}s}{z}[G(x,y,z,s,t)-G(x,y,s,t)]\\
&\quad+\frac{\mu_3\tau_{35}t}{z}[G(x,y,z,s,t)-G(x,y,s,t)]\\
&\quad+\frac{\mu_4\tau_{44}}{s}[G(x,y,z,s,t)-G(x,y,z,t)]\\
&\quad+\frac{\mu_4\tau_{45}t}{s}[G(x,y,z,s,t)-G(x,y,z,t)]\\
&\quad+\frac{\mu_5\tau_{54}s}{t}[G(x,y,z,s,t)-G(x,y,z,s)]\\
&\quad+\frac{\mu_5\tau_{55}}{t}[G(x,y,z,s,t)-G(x,y,z,s)].
\end{aligned}$$

Considering $G(y,z,s,t)=G_1; G(x,z,s,t)=G_2; G(x,y,s,t)=G_3; G(x,y,z,t)=G_4; G(x,y,z,s)=G_5$ and further simplifying, we get:

$$G(x,y,z,s,t)$$

$$=\frac{\begin{array}{c}\mu_1G_1\left\{1-\frac{\tau_{12}y}{x}-\frac{\tau_{13}z}{\mathrm{x}}\right\}+\mu_2G_2\left\{1-\frac{\tau_{21}x}{y}-\frac{\tau_{23}z}{\mathrm{y}}\right\}+\mu_3G_3\left\{1-\frac{\tau_{34}s}{z}-\frac{\tau_{35}t}{\mathrm{z}}\right\}\\ +\mu_4G_4\left\{1-\frac{\tau_{44}}{s}-\frac{\tau_{45}t}{\mathrm{s}}\right\}+\mu_5G_5\left\{1-\frac{\tau_{54}s}{t}-\frac{\tau_{55}}{\mathrm{t}}\right\}\end{array}}{\begin{array}{c}\lambda_1\left(1-x^{\beta_1}\right)+\lambda_2\left(1-\mathrm{y}^{\beta_2}\right)+\mu_1\left\{1-\frac{\tau_{12}y}{x}-\frac{\tau_{13}z}{\mathrm{x}}\right\}+\mu_2\left\{1-\frac{\tau_{21}x}{y}-\frac{\tau_{23}z}{\mathrm{y}}\right\}\\ +\mu_3\left\{1-\frac{\tau_{34}s}{z}-\frac{\tau_{35}t}{\mathrm{z}}\right\}+\mu_4\left\{1-\frac{\tau_{44}}{s}-\frac{\tau_{45}t}{\mathrm{s}}\right\}+\mu_5\left\{1-\frac{\tau_{54}s}{t}-\frac{\tau_{55}}{\mathrm{t}}\right\}.\end{array}}$$

4.1. **The Solution.** For $x=y=z=s=t=1$, the equation reduces to $\left(\frac{0}{0}\right)$ indeterminate form. Taking limit $x\to1, y\to1, z\to1, s\to1, t\to1$; one by one keeping the values of other variables fixed as one under the assumptions:

$$G(x,y,z,s,t,)=\begin{cases} 1; n_1,n_2,n_3,n_4,n_5\neq 0 \\ 0;\ \text{otherwise}\end{cases}$$
$$\tau_{12}+\tau_{13}=\tau_{21}+\tau_{23}=1$$
$$\tau_{34}+\tau_{35}=\tau_{44}+\tau_{45}=\tau_{54}+\tau_{55}=1.$$

The following results are obtained:

$$-\lambda_1\beta_1+\mu_1-\mu_2\tau_{21}=\mu_1G_1-\mu_2\tau_{21}G_2$$
$$-\lambda_2\beta_2-\mu_1\tau_{12}+\mu_2=-\mu_1\tau_{12}G_1+\mu_2G_2$$
$$-\mu_1\tau_{13}-\mu_2\tau_{23}+\mu_3=-\mu_1\tau_{13}G_1-\mu_2\tau_{23}G_2+\mu_3G_3$$
$$-\mu_3\tau_{34}+\mu_4-\mu_5\tau_{54}=-\mu_3\tau_{34}G_3+\mu_4G_4-\mu_5\tau_{54}G_5$$
$$-\mu_3\tau_{35}-\mu_4\tau_{45}+\mu_5=-\mu_3\tau_{35}G_3-\mu_4\tau_{45}G_4+\mu_5G_5.$$

Eliminating G_1,G_2,G_3,G_4,G_5, we obtain:

$$G_1=1-\frac{\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}}{\mu_1\left(1-\tau_{12}\tau_{12}\right)}$$
$$G_2=1-\frac{\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2}{\mu_2\left(1-\tau_{12}\tau_{21}\right)}$$
$$G_3=1-\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\mu_3\left(1-\tau_{12}\tau_{11}\right)}$$
$$G_4=1-\left[\frac{\tau_{34}+\tau_{35}\tau_{54}}{\mu_4\left(1-\tau_{45}\tau_{54}\right)}\right]\left[\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\left(1-\tau_{12}\tau_{21}\right)}\right]$$
$$G_5=1-\left[\frac{\tau_{34}\tau_{45}+\tau_{35}}{\mu_5\left(1-\tau_{45}\tau_{54}\right)}\right]\left[\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{11}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\left(1-\tau_{12}\tau_{21}\right)}\right].$$

The solution of the model in the steady state form by using the results obtained above is given as:

$$P_{n_1,n_2,n_3,n_4,n_5}=\left(1-\rho_1\right)\left(1-\rho_2\right)\left(1-\rho_3\right)\left(1-\rho_4\right)\left(1-\rho_5\right)\rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}\rho_5^{n_5}.$$

Here,

$$\rho_1=1-\mathrm{G}_1=\frac{\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}}{\mu_1(1-\tau_{12}\tau_{21})}$$
$$\rho_2=1-G_2=\frac{\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2}{\mu_2(1-\tau_{12}\tau_{21})}$$
$$\rho_3=1-G_3=\frac{\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)}{\mu_3(1-\tau_{12}\tau_{21})}$$
$$\rho_4=1-G_4=\left[\frac{\tau_{34}+\tau_{35}\tau_{54}}{\mu_4(1-\tau_{45}\tau_{54})}\right]\left[\frac{\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_2+\lambda_2)}{(1-\tau_{12}\tau_{21})}\right]$$
$$\rho_5=1-G_5=\left[\frac{\tau_{34}\tau_{45}+\tau_{35}}{\mu_5(1-\tau_{45}\tau_{54})}\right]\left[\frac{\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1+\lambda_2\beta_2)}{(1-\tau_{12}\tau_{21})}\right]$$

The necessary conditions for the existence of the solution in steady state is that $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5 < 1$.

5. PERFORMANCE MEASURES

5.1. **Mean Queue Length.** The average number of customers waiting in the queue at any time t in the system given by mean queue length is:

$$L = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}(n_1+n_2+n_3+n_4+n_5)\,P_{n_1,n_2,n_3,n_4,n_5}$$

$$\int_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_1 P_{n_1,n_2,n_3,n_4,n_5}$$

$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_2 P_{n_1,n_2,n_3,n_4,n_5}$$

$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_3 P_{n_1,n_2,n_3,n_4,n_5}$$

$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_4 P_{n_1,n_2,n_3,n_4,n_5}$$

$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty} n_5 P_{n_1,n_2,n_3,n_4,n_5}$$

Substituting the values and on further simplification, we get:

$L = L_1 + L_2 + L_3 + L_4 + L_5$

$L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} + \frac{\rho_4}{1-\rho_4} + \frac{\rho_5}{1-\rho_5}$

$$L = \begin{cases} \frac{\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}}{\mu_1(1-\tau_{12}\tau_{21})-(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})} + \frac{\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2}{\mu_1(1-\tau_{12}\tau_{21})-(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)} \\ +\frac{\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)}{\mu_3(1-\tau_{12}\tau_{21})-\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)} \\ +\frac{[\tau_{34}+\tau_{35}\tau_{54}][\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)]}{\mu_4(1-\tau_{45}\tau_{54})(1-\tau_{12}\tau_{21})-[\tau_{34}+\tau_{35}\tau_{54}][\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)]} \\ +\frac{[\tau_{34}\tau_{45}+\tau_{35}][\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)]}{\mu_5(1-\tau_{45}\tau_{54})(1-\tau_{12}\tau_{21})-[\tau_{34}\tau_{45}+\tau_{35}][\tau_{13}(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21})+\tau_{23}(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2)]} \end{cases}$$

5.2. **Expected Waiting time.**

$$W = \frac{L}{\lambda}, \text{ where } \lambda = \lambda_1 + \lambda_2$$

5.3. **Variance of Queue.**

$$V\left(n_1+n_2+n_3+n_4+n_5\right)$$
$$=\frac{\rho_1}{\left(1-\rho_1\right)^2}+\frac{\rho_2}{\left(1-\rho_2\right)^2}+\frac{\rho_3}{\left(1-\rho_3\right)^2}+\frac{\rho_4}{\left(1-\rho_4\right)^2}+\frac{\rho_5}{\left(1-\rho_5\right)^2}$$

6. ALGORITHM

The step by step procedure to determine the various performance measures of the model.

(1) Obtain the values of mean arrival rates λ_1, λ_2.
(2) Obtain the values of mean service rates $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$.
(3) Obtain the values of batch sizes β_1, β_2.
(4) Obtain the values of the transition probabilities

$$\tau_{12}, \tau_{13}, \tau_{21}, \tau_{23}, \tau_{34}, \tau_{35}, \tau_{44}, \tau_{45}, \tau_{54}, \tau_{55}.$$

(5) Calculate the values of

$$G_1=1-\frac{\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}}{\mu_1\left(1-\tau_{12}\tau_{21}\right)},$$
$$G_2=1-\frac{\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2}{\mu_2\left(1-\tau_{12}\tau_{21}\right)}$$
$$G_3=1-\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\mu_3\left(1-\tau_{12}\tau_{21}\right)}$$
$$G_4=1-\left[\frac{\tau_{34}+\tau_{35}\tau_{54}}{\mu_4\left(1-\tau_{45}\tau_{54}\right)}\right]\left[\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\left(1-\tau_{12}\tau_{21}\right)}\right]$$
$$G_5=1-\left[\frac{\tau_{34}\tau_{45}+\tau_{35}}{\mu_5\left(1-\tau_{45}\tau_{54}\right)}\right]\left[\frac{\tau_{13}\left(\lambda_1\beta_1+\lambda_2\beta_2\tau_{21}\right)+\tau_{23}\left(\lambda_1\beta_1\tau_{12}+\lambda_2\beta_2\right)}{\left(1-\tau_{12}\tau_{21}\right)}\right]$$

(6) Calculate $\rho_1=1-G_1, \rho_2=1-G_2, \rho_3=1-G_3, \rho_4=1-G_4, \rho_5=1-G_5$.
(7) Check if $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5<1$. If yes, then go to step 8. Else write steady state conditions are not satisfied.
(8) The solution in steady state form is:

$$P_{n_1,n_2,n_3,n_4,n_5}=\left(1-\rho_1\right)\left(1-\rho_2\right)\left(1-\rho_3\right)\left(1-\rho_4\right)\left(1-\rho_5\right)\rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}\rho_5^{n_5}.$$

(9) Calculate mean queue length

$$L=\frac{\rho_1}{1-\rho_1}+\frac{\rho_2}{1-\rho_2}+\frac{\rho_3}{1-\rho_3}+\frac{\rho_4}{1-\rho_4}+\frac{\rho_5}{1-\rho_5}$$

(10) Calculate mean arrival rate of the system $\lambda=\lambda_1+\lambda_2$.
(11) Calculate the average waiting time of customers, $W=\frac{L}{\lambda}$.

(12) Calculate the variance of queue length

$$Variance = \frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2}$$

7. NUMERICAL ILLUSTRATION

Given the number of customers, mean arrival rate, mean service rate, batch sizes and transition probabilities in table 1. Calculate the various performance measures.

TABLE 1. Numerical Values

Mean Arrival Rate	**Mean Service Rate**	**Batch Size**	**Transition Probabilities**
$\lambda_1 = 2$	$\mu_1 = 25$	$\beta_1 = 3$	$\tau_{12} = 0.4, \tau_{13} = 0.6$
$\lambda_1 = 3$	$\mu_2 = 27$	$\beta_2 = 4$	$\tau_{21} = 0.7, \tau_{23} = 0.3$
	$\mu_3 = 19$		$\tau_{34} = 0.8, \tau_{35} = 0.2$
	$\mu_4 = 20$		$\tau_{45} = 0.5, \tau_{44} = 0.5$
	$\mu_5 = 18$		$\tau_{54} = 0.2, \tau_{55} = 0.8$

Using the values from table 1, we have:

$$\rho_1 = \frac{\lambda_1\beta_1 + \lambda_2\beta_2\tau_{21}}{\mu_1(1-\tau_{12}\tau_{21})} = \frac{14.4}{25(0.72)} = 0.8 < 1$$

$$\rho_2 = \frac{\lambda_1\beta_1\tau_{12} + \lambda_2\beta_2}{\mu_2(1-\tau_{12}\tau_{21})} = \frac{14.4}{18.9} = 0.76 < 1$$

$$\rho_3 = \frac{(\lambda_1\beta_1 + \lambda_2\beta_2\tau_{21})\tau_{13} + (\lambda_1\beta_1\tau_{12} + \lambda_2\beta_2)\tau_{23}}{\mu_3(1-\tau_{12}\tau_{21})}$$

$$= \frac{14.4 \times 0.6 + 14.4 \times 0.3}{19 \times (0.72)} = \frac{12.96}{13.68} = 0.95 < 1$$

$$\rho_4 = \left[\frac{\tau_{34} + \tau_{35}\tau_{54}}{\mu_4(1-\tau_{45}\tau_{54})}\right]\left[\frac{\tau_{13}(\lambda_1\beta_1 + \lambda_2\beta_2\tau_{21}) + \tau_{23}(\lambda_1\beta_1\tau_{12} + \lambda_2\beta_2)}{(1-\tau_{12}\tau_{21})}\right]$$

$$= \frac{10.88}{12.96} = 0.841 < 1$$

$$\rho_5 = \left[\frac{\tau_{34}\tau_{45} + \tau_{35}}{\mu_5(1-\tau_{45}\tau_{54})}\right]\left[\frac{\tau_{13}(\lambda_1\beta_1 + \lambda_2\beta_2\tau_{21}) + \tau_{23}(\lambda_1\beta_1\tau_{12} + \lambda_2\beta_2)}{(1-\tau_{12}\tau_{21})}\right]$$

$$= \frac{7.776}{11.664} = 0.66 < 1$$

7.1. Mean Queue Length.

$$L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} + \frac{\rho_4}{1-\rho_4} + \frac{\rho_5}{1-\rho_5}$$

$$L = \frac{0.8}{0.2} + \frac{0.76}{0.24} + \frac{0.95}{0.05} + \frac{0.84}{0.19} + \frac{0.66}{0.34} = 32.52$$

7.2. Expected Waiting Time.

$$W = \frac{L}{\lambda} = \frac{32.52}{5} = 6.504$$

where $\lambda = \lambda_1 + \lambda_2$

7.3. Variance.

$$\begin{aligned} V &= \frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2} \\ &= \frac{0.8}{(0.2)^2} + \frac{0.76}{(0.24)^2} + \frac{0.95}{(0.05)^2} + \frac{0.84}{(0.19)^2} + \frac{0.66}{(0.34)^2} \\ &= 62.12 \end{aligned}$$

8. Particular Cases

- If individual arrival is taken into consideration instead of bulk arrival, the results obtained bears resemblance with that of Gupta et al [5].
- If the queuing subsystem S_3 is taken parallel instead of biserial, then the results obtained in the present manuscript tally with that of Gupta et al [12].
- If only first biserial queuing subsystem S_1 is connected with the single server S_2, and second biserial queueing subsystem is not considered and the customer is allowed to leave the system from the single server, then results of the current manuscript resembles with that of Mittal et al [9].

Conclusion

The current manuscript analyses a queue model having two heterogeneous biserial queuing subsystems each being allied to a common server. The steady state behaviour of the model has been discussed and queue characteristics have been computed using generating function technique. Numerical illustration and particular cases accommodate the validity of the solutions obtained. The model provides an effective mechanism to the system analysts in order to improve the efficiency of the system.

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