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A NEW TYPE OF COVERING BASED ROUGH SETS USING MAXIMAL COMPATIBILITY BLOCKS

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ABSTRACT. In this paper a new type of generalized rough sets based on maximal compatibility blocks generated by a tolerance relation is introduced. The peoperties of new type of rough sets are explored. Analogous to existing different types of covering based rough sets, nine types of rough sets are proposed and comparisons were made.

1. INTRODUCTION

The theory of rough sets is introduced by Pawlak [1] to study the incomplete information systems. The theory is based on the information granules defined by equivalence classes. Later the theory is generalized in several directions where the information granules are defined by tolerance relations, covering of the Universe. J. Mahanta and P.K.Das introduced Covering Based Rough sets [5]. X. Ge and Z.Li have studied the concepts of definability in ten types of covering based rough sets [2]. The properties of these ten types of rough sets are also discussed in [3] and [4].

Chen Wu et al.,[10] studied the concept of rough sets based on Maximal Compatibility Blocks(MCBs) and studied the cores of MCBs. However, the each MCB

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is randomly attached to an element of the block. The accuracy measurements and entropies are studied.

MCBs play a major role in the modeling of behavior of interconnected objects. Computer Networks, Social media networks are some of the examples which can be modeled using graph theory and hence MCBs play a crucial role in the study of such networks. In this direction, in the present work, a covering based rough set approach in which MCBs play the role of information granules is presented and the properties are explored. Also 9 more types of rough sets based on MCBs are defined analogous to the types of rough sets defined in [5]. The structure of rough approximations with respect to the rough inclusion ordering is also explored.

2. Preliminaries

Definition 2.1. Rough Set *: Let R be a equivalance relation on a universe U. A pair of approximation operators* $R_L : 2^U \to 2^U$, $R_U : 2^U \to 2^U$, *are defined as follow: for all* $X \in 2^U$

$$R_L(X) = \cup \{x \in U / [x]_R \subseteq X \}$$
$$R_U(X) = \cup \{x \in U / [x]_R \cap X \neq \phi \}$$

are called the lower approximation operator and the upper approximation operator, respectively. For all $X \in 2^U$ *if* $R_L(X) \neq R_U(X)$ *then the pair* $(R_L(X), R_U(X))$ *is called Rough Set.*

Definition 2.2. Covering *: Let U be a universe, C be a family of subsets of U. C is called a covering of U, if none of subsets in C is empty and $\cup C = U$.*

Definition 2.3. Reducible Element *: Let C be a covering of a universe U and K $\in C$. If K is a uninon of some sets in $C - \{K\}$, we say K is reducible element of C, otherwise K is an irreducible element of C. If every element in C is irreducible then C is called irreducible covering.*

Definition 2.4. Neighborhood *: Let C be a covering of domain U and $x \in U$, $N(x) = \cap \{ K \in C / x \in K\}$ is called the neighborhood of x.*

Definition 2.5. Minimal description *: Let C be a covering of domain U and $x \in U$, $Md(x) = \{K \in C / x \in K \text{ and } \forall S \in C \ (x \in S \text{ and } S \subseteq K \Rightarrow S = K) \}$ is called Minimal description of x.*

Definition 2.6. Definability *: Let (U; C) be a approximation space with approximation operators $\underline{C}$ and $\overline{C}$. A subset X of U is called an inner (resp. outer) definable subset of (U; C) if $\underline{C}$ (X) = X (resp. $\overline{C}$ (X) = X).*

Observation 1. *Let (U; C) be a covering approximation space. Then the ten different types of covering based rough sets have already been stated in [6], those defined as follows:*

1. $\underline{C}^1(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^1(X) = \underline{C}^1(X) \cup (\cup \{\cup Md(x) \,/\, x \in X - \underline{C}^1(X)\})$
2. $\underline{C}^2(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^2(X) = \cup \{K \in C \,/\, K \cap X \neq \phi\}$
3. $\underline{C}^3(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^3(X) = \cup \{\cup Md(x) \,/\, x \in X\}$
4. $\underline{C}^4(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^4(X) = \underline{C}^4(X) \cup (\cup \{\cup K \,/\, K \cap (X - \underline{C}^4(X)) \neq \phi\}$
5. $\underline{C}^5(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^5(X) = \underline{C}^5(X) \cup (\cup \{ N(x) \,/\, x \in (X - \underline{C}^5(X)) \neq \phi\}$
6. $\underline{C}^6(X) = \{x \in U \,/\, N(x) \subset X\}$
   $\overline{C}^6(X) = \{x \in U \,/\, N(x) \cap X \neq \phi\}$
7. $\underline{C}^7(X) = \{x \in U \,/\, \forall K \in C \ (x \in K \Rightarrow K \subset X)\}$
   $\overline{C}^7(X) = \cup \{K \in C_{MCB} \,/\, K \cap X \neq \phi\}$
8. $\underline{C}^8(X) = \cup \{K \in C \,/\, K \subseteq X\}$
   $\overline{C}^8(X) = U - \underline{C}^8(U - X)$
9. $\underline{C}^9(X) = \{x \in U \,/\, \forall u \ (x \in u \Rightarrow N(u) \subset X)\}$
   $\overline{C}^9(X) = \{ N(x) \,/\, x \in U \text{ and } N(x) \cap X \neq \phi \}$
10. $\underline{C}^{10}(X) = \{x \in U \,/\, \forall u \ (x \in N(u) \Rightarrow u \in X)\}$
    $\overline{C}^{10}(X) = \cup \{N(x) \,/\, x \in X\}$

3. THE MAIN WORK

In this section we introduce a new covering with Maximal Compatibility Blocks and compare the new covering with normal covering that induced with tolerance clases.

Definition 3.1. *Let U be a universe, R be a tolerence relation (i.e.R satisfies reflexive and symmetric only) on U. A subset B $\subseteq$ U is said to be a Maximal Compatibility*

Block (MCB) with resepect to R if any elements of B is related to every other element of B with resepect to R and no element of U - B is related to all the elements of B. The set of all MCB's forms a covering and is denoted as C_{MCB} = ∪{B / B is Maximal Compatibiliy Block w.r.t. R}, C_{MCB} is need not be irreduceble is shown in the example-1.

Example 1. *Let U = {1,2,3,4,5,6}, R = Δ ∪ {(1,2) (2,3) (1,3) (2,1) (3,2) (3,1) (2,4) (4,2) (2,5) (5,2) (3,5) (5,3) (3,6) (6,3) (4,5) (5,4) (5,6) (6,5)}} then C_{MCB} = {{1,2,3}, {2,4,5}, {3,5,6}, {2,3,5}}, so that C_{MCB} is reducible.*

Observation 2. *Let U be a universe and C be an irreducible covering on U, define a relation on C by aRb if and only if a,b ∈ K for some K ∈ C then R is a tolerence relation.*

Theorem 3.1. *Let U be a universe and C be covering on U, define a relation defined as the above observation 3.3., if every block of C contains at least one element which is exclusive to the block then C_{MCB} is irreducible and $C = C_{MCB}$.*

Proof. Suppose C_{MCB} is reducible then there exists, $K \in C_{MCB}$ such that $K \subseteq \cup K_i$, and $K_i \in C_{MCB}$ - {K}, then for every $a \in K$, $a \in K_j$ for some j (since $K \subseteq \cup K_i$ and $K_i \in C_{MCB}$ - {K}) hence no element is exclusive to K which is contradiction so that C_{MCB} is irreducible.

Let C = { K_1, K_2, K_3 .... K_n} satisfies the condition that every block contain at least one element which is exclusively to that block. Let $K_1 \in C$ and $x \in K_1$ such that $x \notin K_j \ \forall \ K_j \in C$ (since from the condition), there fore by defintion of R every pair of element in K_1 are related with each other and since x is exclusive to K_1, no out side of element is related to x. Then K_1 is MCB with respect ot R. There fore $C = C_{MCB}$ □

Definition 3.2. *Let U be a universe, C be a covering and C_{MCB} be the covering obtained by MCB with respect to R(defined in definition 3.1) then for every arbitrary set X ⊆ U, define*

$$X_L = \cup\{B \subseteq C_{MCB}/B \subseteq X\}$$
$$X_U = \cup\{B \subseteq C_{MCB}/B \cap X \neq \phi\}$$

with respect to the above definition (U,C_{MCB}) is an approximation space, the computation of lower and upper approximations is shown in the following example-2.

Example 2. *Let U = { 1,2,3,4,5}, if C = {{ 1,2},{1,2,3},{3,4,5},{2,3}} then R = {(1,2)(2,1)(2,3)(3,2)(3,4)(4,3)(1,3)(3,1)(3,5)(5,3)(4,5)(5,4)} and C_{MCB} = {{1,2,3},{3,4,5}}. Let X = {1,2,3,4} then X_L = {1,2,3}, X_U = {1,2,3,4,5} = U.*

Observation 3. *If R is an equivalence Relation then MCB becomes an equivalence class.*

Definition 3.3. *Let (U,C_{MCB}) be an approximation space. For each x ∈ U, N^C(x) = ∩ { B ∈ C_{MCB}/ x ∈ B} is called the neighborhood of x.*

Definition 3.4. *Let (U,C_{MCB}) be an approximation space. For each x ∈ U, the family Md^C(x) = { B ∈ C_{MCB} / x ∈ B and ∀ B^I ∈ C_{MCB} (x ∈ B^I and B^I ⊆ B ⇒ B^I = B) } is called the minimal discription of x.*

Example 3. *From exampe 3.6, $N^C(1)$ = {1,2,3}, $N^C(2)$ = {1,2,3}, $N^C(3)$ = {3}, $N^C(4)$ = {3,4,5}, $N^C(5)$ = {3,4,5} and $Md^C(1)$ = {1,2,3}, $Md^C(2)$ = {1,2,3}, $Md^C(3)$ = {{1,2,3},{ 3,4,5}}, $Md^C(4)$ = {3,4,5}, $Md^C(5)$ = {3,4,5}.*

4. PROPERTIES OF APPROXIMATIONS

Let X_L and X_U are lower and upper approximations defined in definition 3.5, then X_L and X_U satisfiese the following properties.

1. $X_L \subseteq X \subseteq X_U$
2. $\phi_L = \phi = \phi_U$
3. $U_L = U = U_U$
4. If X ⊆ Y then $X_L \subseteq Y_L$ and $X_U \subseteq Y_U$
5. $(X \cup Y)_U = X_U \cup Y_U$
6. $(X \cap Y)_L \subseteq X_L \cap Y_L$
7. $X_L \cup Y_L \subseteq (X \cup Y)_L$
8. $(X \cap Y)_U \subseteq X_U \cap Y_U$
9. $(X_L)_L = X_L$
10. $X_U \subseteq (X_U)_U$

Proof. Some of the above proofs are given and the remaining are straight forward.

6. As a result and by definition of lower approximation, X∩Y ⊆ X and X∩Y ⊆ Y and by (1) $(X \cap Y)_L \subseteq X_L$ and $(X \cap Y)_L \subseteq Y_L$, and hence $(X \cap Y)_L \subseteq X_L \cap Y_L$. Converse is not true. For example U = {1,2,3,4,}, C_{MCB} = {{1,2,3},{3,4}}, let

X = {1,2,3} and Y = {3,4} then X_L = {1,2,3} and Y_L = {3,4} by the definition of 3.5, that implies $X_L \cap Y_L$ = {3} and X ∩ Y = {3} and $(X \cap Y)_L = \phi$.

7. Let $B \in C_{MCB}$ and $B \subseteq X_L \cup Y_L$, by the definition of lower approximation B ⊆ X or $B \subseteq$ Y, and hence $B \subseteq$ X ∪ Y thus $B \subseteq (X \cup Y)_L$. Converse is not true, since from example 3.2, let X = {1,2,3,5,6} and Y = {2,3,4} then X_L = {1,2,3,5,6} and $Y_L = \phi$ by the definition of 3.5, that implies $X_L \cup Y_L$ = {1,2,3,5,6} and X ∪ Y = {1,2,3,4,5,6} and $(X \cup Y)_L$ = {1,2,3,4,5,6}.

8. Let $B \in C_{MCB}$ and $B \subseteq (X \cap Y)_U$, by the definition of upper approximation $B \cap$ (X∩Y) $\neq \phi$, implies $B \cap$ X $\neq \phi$ and $B \cap$ Y $\neq \phi$, again by by the definition of upper approximation $B \subseteq X_U \cap Y_U$. Converse is not true. For example U = {1,2,3,4,}, C_{MCB} = {{1,2,3},{3,4}}, let X = {1,2,3} and Y = {4} then X_U = {1,2,3,4} and Y_U = {3,4} by the definition of 3.5, that implies $X_U \cap Y_U$ = {3,4} and X ∩ Y = ϕ and $(X \cap Y)_U = \phi$. □

Observation 4. *Let U be a universe, C be a covering of U, R be a tolerance relation as defined in 3.1. C and C_{MCB} are the old and new coverings respectively, then it can observed that the lower and upper approximations defined in definition 3.5 are similar to 2nd type of approximation given in observation 2.7.*
(i) $\overline{C}^2(X) = X_U$ (ii) $X_L \subseteq \underline{C}^2(X)$.

Observation 5. *(Definability of X_L and X_U) : In the covering of maximal compatibility blocks, collection of union of MCB are inner definable how ever empty set and the universe set U are only the outer definable.*

5. **10 Types of New Rough sets with Maximal Compatibility Blocks**

In this Section we introduce 10 types of new rough sets (U, C_{MCB}) based on Mximal comapatibility blocks , analogous to that of approsimation spacs given in [6] and those Rough Sets defined as follows. In these rough approximations 2nd type is discussed aboove, defined in Definition 3.2. The relationship among different approximation spaces is explored.

1. $\underline{C}^1_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
   $\overline{C}^1_{MCB}(X) = \underline{C}^1_{MCB}(X)$ ∪ (∪ { ∪ Md^C(x) / x ∈ X - $\underline{C}^1_{MCB}(X)$ })
2. $\underline{C}^2_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
   $\overline{C}^2_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ∩ X ≠ ϕ }
3. $\underline{C}^3_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
   $\overline{C}^3_{MCB}(X)$ = ∪ { ∪ Md^C(x) / x ∈ X }

4. $\underline{C}^4_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
$\overline{C}^4_{MCB}(X)$ = $\underline{C}^4_{MCB}(X)$ ∪ (∪ { ∪ B / B ∩ (X - $\underline{C}^4_{MCB}$(X))≠ϕ })
5. $\underline{C}^5_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
$\overline{C}^5_{MCB}(X)$ = $\underline{C}^5_{MCB}(X)$ ∪ (∪ { N^C(x) / x ∈ (X - $\underline{C}^5_{MCB}(X)$) ≠ ϕ })
6. $\underline{C}^6_{MCB}(X)$ = { x ∈ U /N^C(x) ⊂ X }
$\overline{C}^6_{MCB}(X)$ = { x ∈ U / N^C(x) ∩ X ≠ ϕ }
7. $\underline{C}^7_{MCB}(X)$ = { x ∈ U / ∀ B ∈ C_{MCB} (x ∈ MCB ⇒ B ⊂ X) }
$\overline{C}^7_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ∩ X ≠ ϕ }
8. $\underline{C}^8_{MCB}(X)$ = ∪ { B ∈ C_{MCB} / B ⊆ X }
$\overline{C}^8_{MCB}(X)$ = U - $\underline{C}^8_{MCB}(U - X)$
9. $\underline{C}^9_{MCB}(X)$ = { x ∈ U / ∀ u (x ∈ N^C(u) ⇒ N^C(u) ⊂ X) }
$\overline{C}^9_{MCB}(X)$ = { N^C (x) / x ∈ U and N^C (x) ∩ X ≠ ϕ }
10. $\underline{C}^{10}_{MCB}(X)$ = { x ∈ U / ∀ u (x ∈ N^C(u) ⇒ u ∈ X) }
$\overline{C}^{10}_{MCB}(X)$ = ∪ { N^C(x) / x ∈ X }

Lemma 5.1 (Comparision between all upper approximations)**.** *The following holds for the upper approximations given above*
$\overline{C}^5_{MCB}(X) \subseteq \overline{C}^6_{MCB}(X) \subseteq \overline{C}^{10}_{MCB}(X) \subseteq \overline{C}^9_{MCB}(X) \subseteq \overline{C}^8_{MCB}(X) \subseteq \overline{C}^1_{MCB}(X) \subseteq \overline{C}^2_{MCB}(X) = \overline{C}^7_{MCB}(X) = \overline{C}^3_{MCB}(X) \subseteq \overline{C}^4_{MCB}(X)$.

Proof. Some of the proofs of the above tricky comparisons are given in the following.

$\overline{C}^5_{MCB}(X) \subseteq \overline{C}^6_{MCB}(X)$: suppose x ∈ $\overline{C}^5_{MCB}(X)$.

Case 1: x ∈ $\underline{C}^5_{MCB}(X)$ by the definition of $\underline{C}^5_{MCB}(X)$; for all $B \in C_{MCB}$, x ∈ B and we have $B \subseteq$ X (B is Maximal Compatibility Block) so x ∈ X; since N^C(x) $\subseteq B$ this implies N^C(x) ∩ $X \neq \phi$, by the definition of $\overline{C}^6_{MCB}(X)$, x ∈ $\overline{C}^6_{MCB}(X)$.

Case 2: x ∉ $\underline{C}^5_{MCB}(X)$ by the definition of $\overline{C}^5_{MCB}(X)$; for all x ∈ X, x ∈ N^C(x) so obviously N^C(x) ∩ X ≠ ϕ by the definition of $\overline{C}^6_{MCB}(X)$, x ∈ $\overline{C}^6_{MCB}(X)$ this implies $\overline{C}^5_{MCB}(X) \subseteq \overline{C}^6_{MCB}(X)$.

The proofs of the inequalities $\overline{C}^6_{MCB}(X) \subseteq \overline{C}^{10}_{MCB}(X)$, $\overline{C}^{10}_{MCB}(X) \subseteq \overline{C}^9_{MCB}(X)$, $\overline{C}^9_{MCB}(X) \subseteq \overline{C}^8_{MCB}(X)$, $\overline{C}^8_{MCB}(X) \subseteq \overline{C}^1_{MCB}(X)$ and $\overline{C}^1_{MCB}(X) \subseteq \overline{C}^2_{MCB}(X)$ are trivial.

$\overline{C}^2_{MCB}(X) = \overline{C}^3_{MCB}(X)$: If, there exist $B \in C_{MCB}$ and B ∈ $\overline{C}^2_{MCB}(X)$ by the definition of $\overline{C}^2_{MCB}(X)$, $B \cap$ X ≠ ϕ that is there exist x such that x ∈ $B \cap$ X, since B being Maximal Block containing x, $B \in Md^C$(x) by the definition of $\overline{C}^3_{MCB}(X)$, $B \in \overline{C}^3_{MCB}(X)$ thus $\overline{C}^2_{MCB}(X) \subseteq \overline{C}^3_{MCB}(X)$.

Conversely, let Md^C(x) $\in \overline{C}^3_{MCB}(X)$ that is if x $\in$ X then there exist $B \in C_{MCB}$ and $B \in Md^C$(x), so that x $\in B$ thus x $\in B \cap$ X, the definition of $\overline{C}^2_{MCB}(X)$ satisfied, this implies $\overline{C}^3_{MCB}(X) \subseteq \overline{C}^2_{MCB}(X)$.

$\overline{C}^3_{MCB}(X) \subseteq \overline{C}^4_{MCB}(X)$: If there exist B $\in C_{MCB}$ and $B \in \overline{C}^3_{MCB}(X)$, if there exist x $\in$ X such that x $\in B$.

Case 1: If $\forall$ $B \in \overline{C}^3_{MCB}(X)$ then clearly $B \in \overline{C}^4_{MCB}(X)$ thus $\overline{C}^3_{MCB}(X) \subseteq \overline{C}^4_{MCB}(X)$.

Case 2: If there exist B $\in \overline{C}^3_{MCB}(X)$ and Suppose B $\notin \overline{C}^3_{MCB}(X)$ then B $\notin \underline{C}^4_{MCB}(X)$ and B $\cap$ (X - $\underline{C}^4_{MCB}(X)$) = ϕ which is contradiction; Since x $\in$ MCB $\cap$ X by the definition of $\overline{C}^3_{MCB}(X)$, B $\in \overline{C}^4_{MCB}(X)$. □

Lemma 5.2 (Comparision between lower approximations). *The following holds for the Lower approximations given above. Clearly:*

$$\underline{C}^1_{MCB}(X) = \underline{C}^2_{MCB}(X) = \underline{C}^3_{MCB}(X)$$
$$= \underline{C}^4_{MCB}(X) = \underline{C}^5_{MCB}(X) = \underline{C}^8_{MCB}(X) \rightarrow (1)$$

Since

$$\underline{C}^5_{MCB}(X) \subseteq \underline{C}^6_{MCB}(X) = \underline{C}^9_{MCB}(X) \subseteq \underline{C}^{10}_{MCB}(X) \subseteq \underline{C}^7_{MCB}(X) \rightarrow (2)$$

from relation one and two the following inequlity is hold

$$\underline{C}^1_{MCB}(X) = \underline{C}^2_{MCB}(X) = \underline{C}^3_{MCB}(X) = \underline{C}^4_{MCB}(X)$$
$$= \underline{C}^5_{MCB}(X) = \underline{C}^8_{MCB}(X) \subseteq \underline{C}^6_{MCB}(X)$$
$$= \underline{C}^9_{MCB}(X) \subseteq \underline{C}^{10}_{MCB}(X) \subseteq \underline{C}^7_{MCB}(X).$$

Proof. Some of the proofs of the above tricky comparisons are given in the following.

$\underline{C}^5_{MCB}(X) \subseteq \underline{C}^6_{MCB}(X)$:

Let x $\in \underline{C}^5_{MCB}(X)$ by the definition of $\underline{C}^5_{MCB}(X)$, x $\in$ B $\subseteq$ X; since N^C(x) $\subseteq$ B $\subseteq$ X thus x $\in N^C$(x) $\subseteq$ X by the definition of $\underline{C}^6_{MCB}(X)$, x $\in \underline{C}^6_{MCB}(X)$.

$\underline{C}^6_{MCB}(X) = \underline{C}^9_{MCB}(X)$:

$\underline{C}^6_{MCB}(X) \subseteq \underline{C}^9_{MCB}(X)$ is clear. Suppose x $\in N^C(u)$. Claim: $N^C(u) \subset$ X. Then $\underline{C}^9_{MCB}(X) \subseteq \underline{C}^6_{MCB}(X)$.

Case 1: If N^C(u) = {u} then x = u that impies $N^C(u) = N^C$(x) $\subset X$.

Case 2: If N^C(u) = {u , x} then N^C(x) ={ x} or N^C(x) = {u , x} shows that N^C(x) $\subset X$.

Case 3: If N^C(u) = B then clearly N^C(x) = B this implies N^C(x) $\subset$ X.

$\underline{C}^9_{MCB}(X) \subseteq \underline{C}^{10}_{MCB}(X)$ and $\underline{C}^{10}_{MCB}(X) \subseteq \underline{C}^7_{MCB}(X)$ both are clear from its definition's. □

Observation 6. *Let X be a nonempty set and $C^i(X) = (\underline{C}^i_{MCB}(X), \overline{C}^i_{MCB}(X))$, $C^j(X) = (\underline{C}^j_{MCB}(X), \overline{C}^j_{MCB}(X))$ be two roughsets defined on X, then we say that $C^i(X) \leq C^j(X)$ if and only if $\underline{C}^i(X) \subseteq \underline{C}^j(X)$ and $\overline{C}^i(X) \subseteq \overline{C}^j(X)$ for all i,j = 1,2...10, as per the above comparision lemma 5.1, 5.2 the following Hasse diagram can be drawn on the approximation spaces, in the FIGURE 1 each approximation space indicates with number, for example the approximation $(\underline{C}^1_{MCB}(X), \overline{C}^1_{MCB}(X))$ indicates 1.*

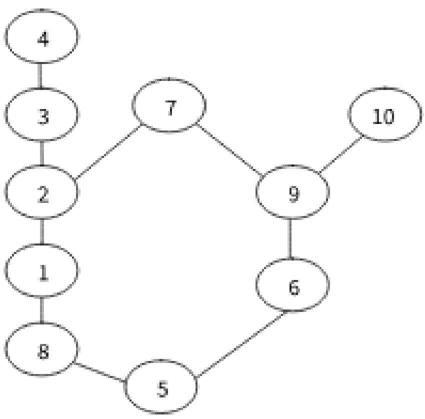

FIGURE 1. Hasse Diagram for the above ten approximation spaces

6. CONCLUSION

In this paper a new rough approximation space is defined in which the maximal compatibility blocks act as granules, the properties of proposed approximation space are studied further a comparision is established among ten approximation spaces defined analogous to the existing ten types of covering based rough sets. The approxiamtion spaces defined in terms of maximal compatibility blocks can be applied in various fields like computer netwoks, information retrival and so on.

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