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SOME RESULTS ON S-METRIC SPACE

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ABSTRACT. The aim of this paper is to establish two common fixed point theorems in S- metric space using semi compatible, weakly semi compatible, weakly compatible and occasionally weakly compatible (OWC) mappings. Further our theorems are also justified with suitable examples.

1. INTRODUCTION

The fixed point theory is one of the most attractive areas of the research in analysis.In the recent past several theorems have been evolved in different platforms of metric space. One of the generalizations of metric space is S-metric space [2],[3],[5],[6] ,[7] and [10]. In 2012, S. Sedhi, N. Shobe and A. Aliouche, [1] developed the notion of S-metric space and proved some fixed point theorems. Junck established the weaker form of compatible mappings as weakly compatible mappings.The idea of semi compatibility in metric space is introduced by Sharma and Sahu [4]. Further A. S. Saluja, Mukesh and Pankaj Kumar Jhade [9] introduced the weaker form of semi compatibility in the form of weak semi compatibility. The occasionally weakly compatible mappings (OWC) is developed by Al-Thagafi and Shahzed [8]which is weaker than weakly compatible mappings.

In this paper we discuss two common fixed point theorems in S-metric space using the new contraction condition along with the weaker form of compatible

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mappings such as semi compatible, weakly semi compatible, weakly compatible mappings and OWC mappings. These results generalize and extend some of the existing theorems in S-metric space. Further some examples are also discussed to carry our outcomes.

2. Preliminaries

Definition 2.1. *A non empty set X defined on a function $S : X^3 \to [0, \infty)$ holding the following conditions:*

(2.1.2) $S(\alpha, \beta, \gamma) \geq 0;$

(2.1.1) $S(\alpha, \beta, \gamma) = 0;$ *if and only if* $\alpha = \beta = \gamma,$

(2.1.3) $S(\alpha, \beta, \gamma) \leq S(\alpha, \alpha, a) + S(\beta, \beta, a) + S(\gamma, \gamma, a),$ *for all* $\alpha, \beta, \gamma, a \in X.$

Then the pair (X, S) is called an ***S-metric space****.*

Remark 2.1. *In an S-metric space, we observe that $S(\alpha, \alpha, \beta) = S(\beta, \beta, \alpha)$.*

Remark 2.2. *In an S-metric space, by triangle inequality we have $S(\alpha, \alpha, \beta) = 2S(\alpha, \alpha, \gamma) + S(\beta, \beta, \gamma)$.*

Remark 2.3. *In an S-metric space, if there exist sequences $\{\alpha_n\}$ and $\{\beta_n\}$ such that $\lim_{k\to\infty} \alpha_k = \alpha$ and $\lim_{k\to\infty} \beta_k = \beta$ then $S(\alpha_k, \alpha_k, \beta_k) = S(\alpha, \alpha, \beta)$.*

Definition 2.2. *Let(X, S) be an S-metric space and $A \subset X$,*

(2.2.1) *the set A is said to be* ***S-bounded*** *if there exists $r > 0$ such that $S(\alpha, \alpha, \beta) < r, \forall \alpha, \beta \in X.$*

(2.2.2) *A sequence $\{\alpha_k\}$ in X* ***converges to x*** *if $S(\alpha_k, \alpha_k, \alpha) \to 0$ as $k \to \infty$, that is for every $\epsilon > 0$ there exists $k_0 \in$ **N** such that $S(\alpha_k, \alpha_k, \alpha) < \epsilon$, for $k \geq k_0$.*

(2.2.3) *A sequence $\{\alpha_k\}$ in X is said to be a* ***Cauchy sequence*** *if for each $\epsilon > 0$ there exists $k_0 \in$ **N** such that $S(\alpha_k, \alpha_k, \alpha_l) < \epsilon$, for all $k, l \geq k_0$.*

(2.2.4) *A* ***complete*** *S-metric space is one in which every Cauchy sequence is convergent .*

Definition 2.3. *Define G and I are two self maps of an S-metric space ,then G and I are said to be* ***commuting*** *if and only if $GI\alpha = IG\alpha$ for all $\alpha \in X$.*

Definition 2.4. *We define mappings G and I of an S-metric space as* ***weakly commuting*** *on X if $S(GI\alpha, GI\alpha, IG\alpha) \leq S(G\alpha, G\alpha, I\alpha)$ for all $\alpha \in X$.*

Definition 2.5. *We define mappings G and I of an S-metric space as **compatible** if $S(GI\alpha_k, GI\alpha_k, IG\alpha_k) = 0$ as $k \to \infty$ whenever there is a sequence $\{\alpha_k\}$ in X such that $G\alpha_k = I\alpha_k = \mu$ as $k \to \infty$ for all $\mu \in X$.*

Definition 2.6. *Suppose G and I are mappings of S-metric space in which $G\mu = I\mu$ for some $\mu \in X$ such that $GI\mu = IG\mu$ holds.Then G and I are known as **weakly compatible mappings**.*

Now we give an example in which the mappings are weakly compatible but not compatible.

Example 1. *Let X=$(0,1)$ be an S-metric space with δ_1 and δ_2 are two metrics on X and $S(\alpha,\beta,\gamma)$=$\delta_1(\alpha,\gamma) + \delta_2(\beta,\gamma)$. Define G and I as*

$$G(\alpha) = \begin{cases} 1-\alpha & \text{if } 0 < \alpha < \frac{2}{3}; \\ \frac{2\alpha+2}{5} & \text{if } \frac{2}{3} \leq \alpha < 1. \end{cases} \text{ and } I(\alpha) = \begin{cases} \frac{3\alpha+1}{3} & \text{if } 0 < \alpha < \frac{2}{3}; \\ \frac{\alpha+2}{4} & \text{if } \frac{2}{3} \leq \alpha < 1. \end{cases}$$

Take a sequence $\{\alpha_k\}$ as $\alpha_k = \frac{1}{3} - \frac{1}{k}$,for $k \geq 0$. Now $G(\alpha_k)$=$G(\frac{1}{3}-\frac{1}{k})$=$1-(\frac{1}{3}-\frac{1}{k})$=$\frac{2}{3}$ and $I(\alpha_k) = I(\frac{1}{3} - \frac{1}{k})$=$\frac{3(\frac{1}{3}-\frac{1}{k})+1}{3}$=$\frac{2}{3}$ as $k \to \infty$.

Therefore $G(\alpha_k) = I(\alpha_k) = \frac{2}{3}$ as $k \to \infty$.

Further $GI(\alpha_k) = GI(\frac{1}{3} - \frac{1}{k})$=$G(\frac{2}{3} - \frac{1}{k}) = \frac{1}{3}$ and $IG(\alpha_k) = IG(\frac{1}{3} - \frac{1}{k})$=$I(\frac{2}{3} + \frac{1}{k}) = (\frac{8}{12} + \frac{1}{4k}) = \frac{2}{3}$ as $k \to \infty$.

Therefore $S(GI\alpha_k, GI\alpha_k, IG\alpha_k) = S(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}) \neq 0$, showing that the mappings G and I are not compatible. But $G(\frac{2}{3}) = I(\frac{2}{3}) = \frac{2}{3}$ and $GI(\frac{2}{3}) = G(\frac{2}{3}) = \frac{2}{3}$ and $IG(\frac{2}{3}) = I(\frac{2}{3}) = \frac{2}{3}$ implies $GI(\frac{2}{3}) = IG(\frac{2}{3})$. This gives the pair (G,I) is weakly compatible.

Definition 2.7. *We define mappings G and I of an S-metric space as **OWC** if there exists a point $\mu \in X$ which is a coincidence point of G and I at which they commute.*

Now we present an example in which the mappings are OWC but not weakly compatible.

Example 2. *Let X=$[0,1]$ be an S-metric ,δ_1 and δ_2 are two metrics on X and $S(\alpha,\beta,\gamma)$=$\delta_1(\alpha,\gamma) + \delta_2(\beta,\gamma)$. Define G and I as*

$$G(\alpha) = \begin{cases} \frac{1+\alpha}{5} & \text{if } 0 \leq \alpha < \frac{1}{2}; \\ 1-\alpha & \text{if } \frac{1}{2} \leq \alpha \leq 1. \end{cases} \text{ and } I(\alpha) = \begin{cases} \frac{2\alpha+1}{5} & \text{if } 0 \leq \alpha < \frac{1}{2}; \\ \frac{2\alpha+1}{4} & \text{if } \frac{1}{2} \leq \alpha \leq 1. \end{cases}$$

Take a sequence $\{\alpha_k\}$ as $\alpha_k = \frac{1}{2} + \frac{1}{k}$,for $k \geq 0$. Now $G(\alpha_k)$=$G(\frac{1}{2}+\frac{1}{k})$=$1-(\frac{1}{2}-\frac{1}{k})$=$\frac{1}{2}$ and $I(\alpha_k) = I(\frac{1}{2} + \frac{1}{k})$=$\frac{2(\frac{1}{2}+\frac{1}{k})+1}{4}$=$\frac{1}{2}$ as $k \to \infty$.

Therefore $G(\alpha_k) = I(\alpha_k) = \frac{1}{2}$ *as* $k \to \infty$.

Further $GI(\alpha_k) = GI(\frac{1}{2} + \frac{1}{k})$=$G(\frac{1}{2} - \frac{1}{2k}) = \frac{1}{2}$ *and* $IG(\alpha_k) = IG(\frac{1}{2} + \frac{1}{k}) = I(\frac{1}{2} - \frac{1}{k}) = \frac{2(\frac{1}{2}+\frac{1}{k})+1}{5} = \frac{2}{5}$ *as* $k \to \infty$.

Therefore $S(GI\alpha_k, GI\alpha_k, IG\alpha_k) = S(\frac{1}{2}, \frac{1}{2}, \frac{2}{5}) \neq 0$*, showing that the mappings G and I are not compatible. Now* $G(0) = I(0)$=$\frac{1}{5}$ *and* $G(\frac{1}{2})$=$I(\frac{1}{2})$=$\frac{1}{2}$.
Thus 0 and $\frac{1}{2}$ *are coincidence points.*

Further GI(0)=G($\frac{1}{5}$*)=*$\frac{6}{25}$ *and IG(0)=I(*$\frac{1}{5}$*)=*$\frac{8}{25}$*, which implies* $GI(0) \neq IG(0)$*. But* $GI(\frac{1}{2})$=$IG(\frac{1}{2})$=($\frac{1}{2}$) *which gives* $GI(\frac{1}{2})$=$IG(\frac{1}{2})$*. Therefore G and I are only OWC but not weakly compatible mappings.*

Definition 2.8. *We define mappings* G *and* I *of an S-metric space as* ***semi-compatible*** *if* $S(GI\alpha_k, GI\alpha_k, I\mu) = 0$ *as* $k \to \infty$ *whenever there is a sequence* $\{\alpha_k\}$ *in* X *such that* $G\alpha_k = I\alpha_k = \mu$ *as* $k \to \infty$ *for all* $\mu \in X$.

Definition 2.9. *We define mappings* G *and* I *of an S-metric space as* ***weakly semi-compatible*** *if* $S(GI\alpha_k, GI\alpha_k, I\mu) = 0$ *or* $S(IG\alpha_k, IG\alpha_k, G\mu) = 0$ *as* $k \to \infty$ *whenever there is a sequence* $\{\alpha_k\}$ *in* X *such that* $G\alpha_k = I\alpha_k = \mu$ *as* $k \to \infty$ *for all* $\mu \in X$.

Now we present an example in which the mappings are only weakly semi compatible but not semi- compatible.

Example 3. *Let X=*$[0, \infty)$ *be an S-metric space ,*δ_1 *and* δ_2 *are two metrics on X and* $S(\alpha, \beta, \gamma)$=$\delta_1(\alpha, \gamma)$+$\delta_2(\beta, \gamma)$*. Define G and I such that* $G(\alpha) = \begin{cases} \frac{\alpha+1}{2} & \text{if } 0 < \alpha \leq \frac{1}{2}; \\ \frac{2\alpha-1}{4} & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$

and $I(\alpha) = \begin{cases} \frac{3\alpha}{2} & \text{if } 0 < \alpha \leq \frac{1}{2}; \\ \frac{\alpha+1}{2} & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$

Take a sequence $\{\alpha_k\}$ *as* $\alpha_k = \frac{1}{2} - \frac{1}{k}$ *for* $k \geq 0$.

Now $G(\alpha_k) = G(\frac{1}{2} - \frac{1}{k}) = \frac{(\frac{1}{2}-\frac{1}{k})+1}{2} = \frac{3}{4} - \frac{1}{2k}$=$\frac{3}{4}$ *, and* $I(\alpha_k) = I(\frac{1}{2} - \frac{1}{k}) = \frac{3(\frac{1}{2}-\frac{1}{k})}{2}$=$\frac{3}{4} - \frac{3}{2k} = \frac{3}{4}$ *as* $k \to \infty$*. Therefore* $G(\alpha_k) = I(\alpha_k) = \frac{3}{4} = \mu$ *(say) as* $k \to \infty$.

Further $GI(\alpha_k)$=$GI(\frac{1}{2} - \frac{1}{k})$=$G(\frac{3}{4} - \frac{3}{2k})$=$\frac{(\frac{3}{4}-\frac{3}{2k}+1)}{2}$ $=\frac{7}{8} - \frac{3}{4k} = \frac{7}{8}$ *as* $k \to \infty$*. Also* $IG(\alpha_k)$=$IG(\frac{1}{2} - \frac{1}{k})$=$I(\frac{3}{4} - \frac{1}{2k})$=$\frac{3(\frac{3}{4}-\frac{2}{k})}{2}$ $=(\frac{9}{8} + \frac{3}{4k}) = \frac{9}{8}$ *as* $k \to \infty$*. This gives* $S(GI\alpha_k, GI\alpha_k, IG\alpha_k)$=$S(\frac{7}{8}, \frac{7}{8}, \frac{9}{8}) \neq 0$ *and this gives the pair (G,I) is not compatible.*

Further $G(\mu) = G(\frac{3}{4}) = \frac{2\frac{3}{4}-1}{4} = \frac{1}{8}$ *and* $I(\mu) = I(\frac{3}{4}) = \frac{\frac{3}{4}+1}{2} = \frac{7}{8}$ *and* $S(IG\alpha_k, IG\alpha_k, G\mu)$=$S(\frac{9}{8}, \frac{9}{8}, \frac{1}{8}) \neq 0$ *as* $k \to \infty$*, this gives the pair (G,I) is not semi-compatible. But* $S(GI\alpha_k, GI\alpha_k, I\mu)$=$S(\frac{7}{8}, \frac{7}{8}, \frac{7}{8}) = 0$ *or* $S(IG\alpha_k, IG\alpha_k, G\mu) = S(\frac{9}{8}, \frac{9}{8}, \frac{9}{8}) = 0$ *as* $k \to \infty$*, this gives the pair (G,I) is weakly semi-compatible.*

Now we proceed for our main theorems.

3. MAIN RESULTS

Theorem 3.1. *Let* (X, S) *be a complete S-metric space and there are four mappings G, H, I and J holding the conditions*

(3.1.1) $G(X) \subseteq J(X)$ *and* $H(X) \subseteq I(X)$

(3.1.2)

$$S(G\alpha, G\alpha, H\beta) \leq \lambda \; max \left\{ S(I\alpha, I\alpha, J\beta), \frac{S(G\alpha, G\alpha, I\alpha)S(H\beta, H\beta, J\beta)}{S(I\alpha, I\alpha, J\beta)}, S(H\beta, H\beta, G\alpha) \right\}$$

for all $\alpha, \beta \in X$, *where* $\lambda \in (0, 1)$

(3.1.3) *one of G or I is continuous on X*

(3.1.4) *the pair* (G, I) *is weakly semi compatible on X*

(3.1.5) *the pair* (H, J) *is weakly compatible on X.*

Then the above mappings will be having unique common fixed point.

Proof. Begin with using the condition (3.1.1), there is a point $\alpha_0 \in X$ such that $G\alpha_0 = J\alpha_1 = \beta_0$ (say) for some $\alpha_1 \in X$.For this point α_1 then $\exists$ a point $\alpha_2 \in X$ such that $H\alpha_1 = I\alpha_2 = \beta_1$(say).

Continuing this process, it is possible to construct a sequence $\{\beta_k\} \in X$ such that β_{2k}=$G\alpha_{2k} = J\alpha_{2k+1}$ and.β_{2k+1}=$H\alpha_{2k+1} = I\alpha_{2k+2}$ for $k \geq 0$.

We now prove $\{\beta_k\}$ is a Cauchy sequence in S-metric space.

Consider $S(\beta_{2k}, \beta_{2k}, \beta_{2k+1}) =$

$$S(G\alpha_{2k}, G\alpha_{2k}, H\alpha_{2k+1}) \leq \lambda \max\left\{ S(I\alpha_{2k}, I\alpha_{2k}, J\alpha_{2k+1}), \frac{S(G\alpha_{2k}, G\alpha_{2k}, I\alpha_{2k})S(H\alpha_{2k+1}, H\alpha_{2k+1}, J\alpha_{2k+1})}{S(I\alpha_{2k}, I\alpha_{2k}, J\alpha_{2k+1})}, S(H\alpha_{2k+1}, H\alpha_{2k+1}, G\alpha_{2k}) \right\}$$

$$S(\beta_{2k},\beta_{2k},\beta_{2k+1}) \leq \lambda \max \left\{ S(\beta_{2k-1},\beta_{2k-1},\beta_{2k}), \frac{S(\beta_{2k},\beta_{2k},\beta_{2k-1})S(\beta_{2k+1},\beta_{2k+1},\beta_{2k})}{S(\beta_{2k-1},\beta_{2k-1},\beta_{2k})}, S(\beta_{2k},\beta_{2k},\beta_{2k+1}) \right\}$$

on simplification

$$S(\beta_{2k},\beta_{2k},\beta_{2k+1}) \leq \lambda S(\beta_{2k-1},\beta_{2k-1},\beta_{2k}). \tag{3.1}$$

By Similar arguments we have

$$S(\beta_{2k-1},\beta_{2k-1},\beta_{2k}) \leq \lambda(S(\beta_{2k-2},\beta_{2k-2},\beta_{2k-1}). \tag{3.2}$$

Now from (3.1) and (3.2) we have

$$S(\beta_k,\beta_k,\beta_{k-1}) \leq \lambda S(\beta_{k-1},\beta_{k-1},\beta_{k-2}), k \geq 2 \text{ where } 0 < \lambda < 1.$$

Therefore in general

$$S(\beta_k,\beta_k,\beta_{k-1}) \leq \lambda^{k-1} S(\beta_1,\beta_1,\beta_0).$$

Hence for $k > l$, on using the multiplicative triangle inequality we get

$$\begin{aligned} S(\beta_k,\beta_k,\beta_l) \leq & 2S(\beta_l,\beta_l,\beta_{l+1}) + 2S(\beta_{l+1},\beta_{l+1},\beta_{l+2}) + 2S(\beta_{l+2},\beta_{l+2},\beta_{l+3}) + \cdots \\ & + 2S(\beta_{k-1},\beta_{k-1},\beta_k). \\ & \leq 2(\lambda^l + \lambda^{l+1} + \lambda^{l+2} + \cdots + \lambda^{k-1}) S(\beta_1,\beta_1,\beta_0) \\ & \leq 2\lambda^l (1 + \lambda + \lambda^2 + \cdots) S(\beta_1,\beta_1,\beta_0) \\ & \leq 2\frac{\lambda^l}{1-\lambda} S(\beta_1,\beta_1,\beta_0) \to 0, asl \to \infty. \end{aligned}$$

This results $\{\beta_k\}$ as a Cauchy sequence in S-metric space.
By the completeness of X, $\{\beta_k\}$ converges to some point in X as $k \to \infty$.
Consequently, the sub sequences $\{G\alpha_{2k}\}$,$\{I\alpha_{2k}\}$,$\{J\alpha_{2k+1}\}$ and $\{H\alpha_{2k+1}\}$ of $\{\beta_k\}$ also converge to the same point $\mu \in X$.
Suppose G is continuous, then

$$\lim_{k\to\infty} GG\alpha_{2k} = \lim_{k\to\infty} GI\alpha_{2k} = G\mu. \tag{3.3}$$

Also the pair (G,I) weakly semi- compatible, then

$$\lim_{k\to\infty} S(GI\alpha_{2k}, GI\alpha_{2k}, I\mu) = 0 \text{ or } \lim_{k\to\infty} S(IG\alpha_{2k}, IG\alpha_{2k}, G\mu) = 0. \tag{3.4}$$

From (3.4)

$$\lim_{k\to\infty} S(IG\alpha_{2k}, IG\alpha_{2k}, G\mu) = 0.$$

Therefore from (3.3) and (3.4)

$$GG\alpha_{2k} = GI\alpha_{2k} = IG\alpha_{2k} = G\mu.$$

Putting $\alpha = G\alpha_{2k}$ and $\beta = \alpha_{2k+1}$ in contraction condition (3.1.2) we have

$$\begin{aligned} S(GG\alpha_{2k}, GG\alpha_{2k}, H\alpha_{2k+1}) \leq \lambda \max\Big\{ & S(IG\alpha_{2k}, IG\alpha_{2k}, J\alpha_{2k+1}), \\ & \frac{S(GG\alpha_{2k}, GG\alpha_{2k}, IG\alpha_{2k}) S(H\alpha_{2k+1}, H\alpha_{2k+1}, J\alpha_{2k+1})}{S(IG\alpha_{2k}, IG\alpha_{2k}, J\alpha_{2k+1})}, \\ & S(H\alpha_{2k+1}, H\alpha_{2k+1}, GG\alpha_{2k})\Big\} \end{aligned}$$

letting $k \to \infty$.

$$\begin{aligned} S(G\mu, G\mu, \mu) \leq & \lambda \max\Big\{ S(G\mu, G\mu, \mu), \frac{S(G\mu, G\mu, G\mu)S(\mu,\mu,\mu)}{S(G\mu, G\mu, \mu)}, S(\mu,\mu,G\mu)\} \\ & \lambda \max\Big\{ S(G\mu, G\mu, \mu), \frac{1}{S(G\mu, G\mu, \mu)}, S(\mu,\mu,G\mu)\} \\ & \leq \lambda S(G\mu, G\mu, \mu) \end{aligned}$$

which gives $G\mu = \mu$.
Putting $\alpha = \mu$ and $\beta = \alpha_{2k+1}$ in contraction condition (3.1.2) we have

$$\begin{aligned} S(G\mu, G\mu, H\alpha_{2k+1}) \leq \lambda \max \Big\{ & S(I\mu, I\mu, J\alpha_{2k+1}), \\ & \frac{S(G\mu, G\mu, J\alpha_{2k+1}) S(H\alpha_{2k+1}, H\alpha_{2k+1}, J\alpha_{2k+1})}{S(I\mu, I\mu, J\alpha_{2k+1})}, \\ & S(H\alpha_{2k+1}, H\alpha_{2k+1}, G\mu)\Big\} \end{aligned}$$

this gives that

$$\begin{aligned} S(\mu,\mu,\mu) \leq & \lambda \max\Big\{ S(I\mu, I\mu, \mu), \frac{S(\mu,\mu,\mu)S(\mu,\mu,\mu)}{S(I\mu, I\mu, \mu)}, S(\mu,\mu,\mu)\Big\} \\ & \leq \lambda \max \Big\{ S(I\mu, I\mu, \mu), \frac{1}{S(I\mu, I\mu, \mu)}, S(\mu,\mu,\mu)\Big\}. \\ & \leq \lambda S(I\mu, I\mu, \mu) \end{aligned}$$

which implies $I\mu = \mu$.
Therefore

$$G\mu = I\mu = \mu. \tag{3.5}$$

Since $G(X) \subseteq J(X)$ implies there exists $u \in X$ such that $G\alpha_{2k} = Ju$,
as $k \to \infty . Ju = \mu$.
Putting $\alpha = \mu$ and $\beta = u$ in contraction condition (3.1.2) we have

$$S(\beta_{2k}, \beta_{2k}, \beta_{2k+1}) \leq \lambda \max\Big\{ S(\beta_{2k-1}, \beta_{2k-1}, \beta_{2k}), \frac{S(\beta_{2k}, \beta_{2k}, \beta_{2k-1}) S(\beta_{2k+1}, \beta_{2k+1}, \beta_{2k})}{S(\beta_{2k-1}, \beta_{2k-1}, \beta_{2k})}, S(\beta_{2k}, \beta_{2k}, \beta_{2k+1}) \Big\}$$

$$S(G\mu, G\mu, Hu \leq \lambda \max \left\{ S(I\mu, I\mu, Ju), \frac{S(G\mu, G\mu, Ju) S(Hu, Hu, Ju)}{S(I\mu, I\mu, Ju)}, S(Hu, Hu, G\mu) \right\}$$

which implies that

$$\begin{aligned} S(\mu, \mu, Hu) \leq& \lambda \max \left\{ S(\mu, \mu, \mu), \frac{S(\mu, \mu, \mu) S(Hu, Hu, \mu)}{S(\mu, \mu, \mu)}, S(Hu, Hu, \mu) \right\} \\ &\leq \lambda S(Hu, Hu, \mu) \end{aligned}$$

which gives $Hu = \mu$.
Therefore $Ju = Hu = \mu$
Again since the pair (H,J) is weakly compatible having u as a coincidence point then we get $HJu = JHu$ which gives $H\mu = J\mu$.
Putting $\alpha = \alpha_{2k}$ and $\beta = \mu$ in contraction condition (3.1.2) we have

$$S(G\alpha_{2k}, G\alpha_{2k}, H\mu) \leq \lambda \max \left\{ S(I\alpha_{2k}, I\alpha_{2k}, J\mu), \frac{S(G\alpha_{2k}, G\alpha_{2k}, I\alpha_{2k}) S(H\mu, H\mu, J\mu)}{S(I\alpha_{2k}, I\alpha_{2k}, J\mu)}, S(H\mu, H\mu, G\alpha_{2k}) \right\}$$

this implies that

$$\begin{aligned} S(\mu, \mu, H\mu) \leq& \lambda \max\left\{ S(\mu, \mu, J\mu), \frac{S(\mu, \mu, \mu) S(H\mu, H\mu, Ju)}{S(\mu, \mu, J\mu)}, S(H\mu, H\mu, \mu) \right\}. \\ &\leq \lambda \max \left\{ S(\mu, \mu, H\mu), \frac{S(\mu, \mu, \mu) S(H\mu, H\mu, H\mu)}{S(\mu, \mu, H\mu)}, S(H\mu, H\mu, \mu) \right\}. \\ &\leq \lambda S(H\mu, H\mu, \mu) \end{aligned}$$

which implies $H\mu = \mu$.

$$H\mu = J\mu = \mu. \tag{3.6}$$

Therefore from (3.5) and (3.6)

$$G\mu = I\mu = H\mu = J\mu = \mu.$$

Hence μ is a common fixed point of G,I,J and H.
Similarly we can prove the result in another case.

For Uniqueness

Consider $\phi(\mu \neq \phi)$ is another common fixed point of G,I,H,and J then $G\phi = J\phi = H\phi = I\phi = \phi$. Substitute $\alpha = \phi$ and $\beta = \mu$ in the inequality (3.1.2) we have

$$S(G\phi, G\phi, H\mu) \leq \lambda\max\ S(I\phi, I\phi, J\mu), \frac{S(G\phi, G\phi, I\mu)S(H\phi, H\phi, J\mu)}{S(I\phi, I\phi, J\mu)}, S(H\mu, H\mu, G\phi)\Bigg\}$$

$$\begin{aligned} S(\phi, \phi, \mu) \leq & \lambda\max\Bigg\{S(\phi, \phi, \mu), \frac{S(\phi, \phi, \mu)S(\phi, \phi, \mu)}{S(\phi, \phi, \mu)}, S(\mu, \mu, \phi)\Bigg\} \\ & \leq \lambda \max\Bigg\{S(\phi, \phi, \mu), (S(\phi, \phi, \mu), (S(\phi, \phi, \mu)\Bigg\} \\ & \leq \lambda S(\phi, \phi, \mu). \end{aligned}$$

Thus $\phi = \mu$.
This assures the uniqueness of the common fixed point. □

Now we substantiate our result with an example.

Example 4. *Suppose $X = (0, \infty)$ in S-metric space, δ_1 and δ_2 are two metrics on X and $S(\alpha, \beta, \gamma)$=$\delta_1(\alpha, \gamma) + \delta_2(\beta, \gamma)$ for all $\alpha, \beta, \gamma \in$ X.*
We define self maps G,H,I and J as follows $G(\alpha) = H(\alpha) = \begin{cases} 1 - 2\alpha & \text{if } 0 < \alpha \leq \frac{1}{3}; \\ \frac{4\alpha+1}{7} & \text{if } \alpha \geq \frac{1}{3}. \end{cases}$
and $I(\alpha) = J(\alpha) = \begin{cases} 2 - 5\alpha & \text{if } 0 < \alpha \leq \frac{1}{3}; \\ 1 - \alpha & \text{if } \alpha \geq \frac{1}{3}. \end{cases}$
Now G(X)=H(X)=$[\frac{1}{3}, 1) \cup (\frac{1}{3})$ *and I(X)=J(X)=* $[\frac{1}{3}, 2) \cup (\frac{2}{3})$. *We have $G(X) \subseteq J(X)$ and $H(X) \subseteq I(X)$ so that the condition (3.1.1) is satisfied.*

Now take a sequence $\{\alpha_k\}$ *as* $\alpha_k = \frac{1}{3} - \frac{1}{k}$ *for k*≥ 0. *Now* $G(\alpha_k) = G(\frac{1}{3} - \frac{1}{k})$=$1 - 2(\frac{1}{3} - \frac{1}{k})$=$(\frac{1}{3} + \frac{2}{k})$=$\frac{1}{3}$ *as* $k \to \infty$ *and* $I(\alpha_k) = I(\frac{1}{3} - \frac{1}{k})$= $2 - 5(\frac{1}{3} - \frac{1}{k}) = (\frac{1}{3} + \frac{5}{k})$=$\frac{1}{3}$ *as* $k \to \infty$. *Therefore* $G(\alpha_k) = I(\alpha_k) = \frac{1}{3}$ *as* $k \to \infty$.

Further $GI(\alpha_k) = GI(\frac{1}{3} - \frac{1}{k})$=$G(2 - 5(\frac{1}{3} - \frac{1}{k}))$=$G(\frac{1}{3} + \frac{5}{k}) = \frac{4(\frac{1}{3}-\frac{1}{k})+1}{7}$ =$\frac{1}{3}$ *and* $IG(\alpha_k) = IG(\frac{1}{3} - \frac{1}{k})$=$I(1 - 2(\frac{1}{3} - \frac{1}{k})$=$I(\frac{1}{3} + \frac{2}{k}) = 1 - (\frac{1}{3} + -\frac{1}{k}) = \frac{2}{3}$ *as* $k \to \infty$. *Thus* $S(GI\alpha_k, GI\alpha_k, IG\alpha_k)$=$S(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}) \neq 0$ *, showing that the pair (G,I) is not compatible.*

Further $H(\frac{1}{3}) = \frac{1}{3}$ *and* $J(\frac{1}{3}) = \frac{1}{3}$ *this gives* $H(\frac{1}{3}) = J(\frac{1}{3})$. *This assures* $HJ(\frac{1}{3}) = H(\frac{1}{3}) = \frac{1}{3}$ *and* $JH(\frac{1}{3}) = J(\frac{1}{3}) = \frac{1}{3}$
and this gives $HJ(\frac{1}{3}) = JH(\frac{1}{3})$. *Showing that the pair (H,J) is weakly compatible mapping. Hence the condition (3.1.5) is satisfied.*

Further $S(GI\alpha_k, GI\alpha_k, I(\frac{1}{3})) = S(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = 0$ *or* $S(IG\alpha_k, IG\alpha_k, G(\frac{1}{3})) = S(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}) \neq 0$ *as* $k \to \infty$, *showing that the pair* (G, I) *is weakly semi compatible. Hence the condition (3.1.4) is satisfied.*

$GG(\alpha_k) = GG(\frac{1}{3} - \frac{1}{k})$=$G(1 - 2(\frac{1}{3} - \frac{1}{k})$=$G(\frac{1}{3} + \frac{2}{k}) = \frac{4(\frac{1}{3}-\frac{1}{k})+1}{7} = (\frac{1}{3} + \frac{4}{7k})$ =$\frac{1}{3}$ *as* $k \to \infty$. *Moreover G is continuous.Hence the condition (3.1.3) is satisfied.*

We now establish that the mappings G,H,I and J satisfy the condition (3.1.2).

Case I *If* $\alpha, \beta \in (0, \frac{1}{3}]$,*then* $S(\alpha, \beta, \gamma) = |\alpha - \gamma| + |\beta - \gamma|$ *on putting* $\alpha = \frac{1}{4}$, $\beta = \frac{1}{3}$ *the inequality (3.1.2) which gives*

$$S(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}) \leq \lambda \, \mathit{max}\left\{S(\frac{3}{4}, \frac{3}{4}, \frac{1}{3}), \frac{S(\frac{1}{2}, \frac{1}{2}, \frac{1}{3})S(\frac{1}{2}, \frac{1}{2}, \frac{1}{3})}{S(\frac{3}{4}, \frac{3}{4}, \frac{1}{3})}, S(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})\right\}$$

$$\begin{aligned} 0.33 \leq & \lambda \, \mathit{max} \, \{0.08, \frac{0.33X0.33}{0.08}, 0.33\} \\ & \leq \lambda \, \mathit{max} \, \{0.08, 1.36, 0.33\} \\ & \leq \lambda(1.36) \end{aligned}$$

thus $\lambda = 0.24 \in (0, 1)$, *so that the inequality (3.1.2) holds.*

Case II *If* $\alpha, \beta \in (\frac{1}{3}, 1)$*,then* $S(\alpha, \beta, \gamma) = |\alpha - \gamma| + |\beta - \gamma|$ *on putting* $\alpha = \frac{3}{4}$, $\beta = \frac{4}{5}$ *the inequality (3.1.2) which gives*

$$S(\frac{4}{7}, \frac{4}{7}, \frac{21}{35}) \leq \lambda \mathit{max}\Big\{ S(\frac{1}{4}, \frac{1}{4}, \frac{1}{5}), \frac{S(\frac{4}{7}, \frac{4}{7}, \frac{1}{5})S(\frac{4}{7}, \frac{4}{7}, \frac{1}{5})}{S(\frac{1}{4}, \frac{1}{4}, \frac{1}{5})}, S(\frac{21}{35}, \frac{21}{35}, \frac{4}{7}) \Big\}$$

$$\begin{aligned} 0.05 \leq & \lambda \ \mathit{max}\{0.01, \frac{0.11X0.74}{0.1}, 0.05\} \\ & \leq \lambda \ \mathit{max}\{0.01, 0.81, 0.05\} \\ & \leq \lambda(0.81) \end{aligned}$$

thus λ=$0.06 \in (0, 1)$*, so that the inequality (3.1.2) holds. Similarly we can prove the other cases.*

It is also the fact to note that $\frac{1}{3}$ *is the unique common fixed point of the four mappings H,G,I and J.*

Now we prove another theorem on OWC mappings.

Theorem 3.2. *Let* (X, S) *be a complete S-metric space and there are four mappings G, H, I and J holding the conditions*

(3.2.1) $G(X) \subseteq J(X)$ *and* $H(X) \subseteq I(X)$

(3.2.2)

$$S(G\alpha, G\alpha, H\beta) \leq \lambda \mathit{max} \Big\{ S(I\alpha, I\alpha, J\beta), \frac{S(G\alpha, G\alpha, I\alpha)S(H\beta, H\beta, J\beta)}{S(I\alpha, I\alpha, J\beta)}, S(H\beta, H\beta, G\alpha) \Big\} \text{for all} \alpha, \beta \in X, \text{where } \lambda \in (0, 1)$$

(3.2.3) *one of G,I is continuous*

(3.2.4) *the pair* (G, I) *is semi compatible,*

(3.2.5) *the pair* (H, J) *is occasionally weakly compatible.*

Then the above mappings will be having unique common fixed point.

Proof. Since X is complete, the sub sequences $G\alpha_{2k}, I\alpha_{2k}, J\alpha_{2k+1}$ and $H\alpha_{2k+1}$ converge to some point as in the Theorem 2.1. Suppose G is continuous then $GG\alpha_{2k} \to G\mu$ and $GI\alpha_{2k} \to G\mu$ as $k \to \infty$.
Since the pair (G,I) is semi compatible mapping, then
$\lim_{k\to\infty} S(GI\alpha_{2k}, GI\alpha_{2k}, I\mu) = 0$. Therefore

$$G\mu = I\mu.$$

Now we claim that $G\mu = \mu$. Put $\alpha = G\alpha_{2k}$ and $\beta = \alpha_{2k+1}$ in (3.2.2), we have

$$\begin{aligned} S(GG\alpha_{2k}, GG\alpha_{2k}, H\alpha_{2k+1}) \leq & \lambda \max \Big\{ S(GI\alpha_{2k}, GI\alpha_{2k}, J\alpha_{2k+1}) \\ & , \frac{S(GG\alpha_{2k}, GG\alpha_{2k}, IG\alpha_{2k}) S(H\alpha_{2k+1}, H\alpha_{2k+1}, J\alpha_{2k+1})}{S(IG\alpha_{2k}, IG\alpha_{2k}, J\alpha_{2k+1})}, \\ & S(H\alpha_{2k+1}, H\alpha_{2k+1}, GG\alpha_{2k}) \Big\}, (letting\ k \to \infty.) \end{aligned}$$

$$\begin{aligned} S(G\mu, G\mu, \mu) \leq & \lambda \max \left\{ (S(G\mu, G\mu, \mu), \frac{S(G\mu, G\mu, \mu) S(\mu, \mu, \mu)}{S(G\mu, G\mu, \mu)}, S(\mu, \mu, G\mu) \right\} \\ & \lambda \max \Big\{ (S(G\mu, G\mu, \mu), S(\mu, \mu, \mu), S(\mu, \mu, G\mu) \Big\} \\ & \leq \lambda S(G\mu, G\mu, \mu) \end{aligned}$$

which implies $G\mu = \mu$. Hence

$$G\mu = I\mu = \mu.$$

Also since $G(X) \subseteq J(X)$ implies that there exists $w \in X$ such that $G\alpha_{2k} = Jw$, as $k \to \infty$ this implies $Jw = \mu$.
Now we claim that $Hw = \mu$. Putting $\alpha = \alpha_{2k}$ and $\beta = w$ in (3.2.2), we get

$$\begin{aligned} S(G\alpha_{2k}, G\alpha_{2k}, Hw) \leq & \lambda \max \Big\{ S(I\alpha_{2k}, I\alpha_{2k}, Jw), \\ & \frac{S(G\alpha_{2k}, G\alpha_{2k}, J\alpha_{2k}) S(Hw, Hw, Jw)}{S(I\alpha_{2k}, I\alpha_{2k}, Jw)}, \\ & S(Hw, Hw, G\alpha_{2k}) \Big\} \text{ letting } k \to \infty. \end{aligned}$$

$$\begin{aligned} S(\mu, \mu, Hw) \leq & \lambda \max \left\{ S(\mu, \mu, \mu), \frac{S(\mu, \mu, \mu) S(Hw, Hw, \mu)}{S(\mu, \mu . \mu)}, S(Hw, Hw, \mu) \right\} \\ & \lambda \max \Big\{ S(\mu, \mu, \mu), S(Hw, Hw, \mu), S(Hw, Hw, \mu) \Big\} \\ & \leq \lambda S(Hw, Hw, \mu), \end{aligned}$$

which implies $Hw = \mu$.

$$Hw = Jw = \mu. \tag{3.7}$$

Again since the pair (H,J) is occasionally weakly compatible and $Hw = Jw$ implies $HJw = JHw$. This gives $H\mu = J\mu$.

Put $\alpha = \mu$ and $\beta = \mu$ in the condition (3.2.2), we get

$$S(G\mu, G\mu, H\mu) \leq \lambda \max\left\{S(I\mu, I\mu, J\mu), \frac{S(G\mu,G\mu,I\mu)S(H\mu,H\mu,J\mu)}{S(I\mu,I\mu,J\mu)}, S(H\mu, H\mu, G\mu)\right\}$$

$$\begin{aligned} S(\mu, \mu, H\mu) &\leq \lambda \max\left\{S(\mu, \mu, H\mu), \frac{S(\mu, \mu, \mu)S(H\mu, H\mu, H\mu)}{S(\mu, \mu, H\mu)}, S(H\mu, H\mu, \mu)\right\} \\ &\lambda \max\left\{S(\mu, \mu, H\mu), \frac{1}{S(\mu, \mu, H\mu)}, S(H\mu, H\mu, \mu)\right\} \\ &\leq \lambda S(H\mu, H\mu, \mu) \end{aligned}$$

which implies $H\mu = \mu$.

Therefore

$$H\mu = J\mu = \mu. \tag{3.8}$$

Hence from (3.7) and (3.8) we have

$$G\mu = I\mu = H\mu = J\mu = \mu,$$

This implies μ is a common fixed point of G,H,I and J.

The Uniqueness can be proved easily, □

Now we present an example to discuss the validity of the above theorem.

Example 5. *Suppose $X = (0, \infty)$ in S-metric space ,δ_1 and δ_2 are two metrics on X, $S(\alpha, \beta, \gamma)$=$\delta_1(\alpha, \gamma) + \delta_2(\beta, \gamma)$ for all $\alpha, \beta, \gamma \in X$.*

$G(\alpha) = H(\alpha) = \begin{cases} \frac{3-\alpha}{5} & \text{if } 0 < \alpha \leq \frac{1}{2}; \\ 1-\alpha & \text{if } \frac{1}{2} < \alpha \leq 1. \end{cases}$ *and* $I(\alpha) = J(\alpha) = \begin{cases} \frac{2-\alpha}{3} & \text{if } 0 < \alpha \leq \frac{1}{2}; \\ 0 & \text{if } \frac{1}{2} < \alpha \leq 1. \end{cases}$

Now G(X)=H(X)=$[\frac{1}{2}, \frac{3}{5}) \cup [0, \frac{1}{2})$ and I(X)=J(X)= $[\frac{1}{2}, \frac{2}{3}) \cup (0)$. Thus $G(X) \subseteq J(X)$and $H(X) \subseteq I(X)$ so that the condition (3.2.1) is satisfied.

Now take a sequence $\{\alpha_k\}$ as $\alpha_k = \frac{1}{2} - \frac{1}{k}$ for$k > 0$. Now $G(\alpha_k) = G(\frac{1}{2} - \frac{1}{k}) = \frac{3-(\frac{1}{2}-\frac{1}{k})}{5} = (\frac{1}{2} + \frac{1}{5k}) = \frac{1}{2}$ as $k \to \infty$ and $I(\alpha_k) = I(\frac{1}{2} - \frac{1}{k}) = \frac{2-(\frac{1}{2}-\frac{1}{k})}{3} = \frac{1}{2} + \frac{1}{3k} = \frac{1}{2}$. Therefore $G(\alpha_k) = I(\alpha_k) = \frac{1}{2} = \mu$ (say) , as $k \to \infty$.

Further $GI(\alpha_k) = GI(\frac{1}{2} - \frac{1}{k})$=$G(\frac{1}{2} + \frac{1}{3k})$=$1 - (\frac{1}{2} - \frac{1}{3k})$=$(\frac{1}{2} + \frac{1}{3k})$ =$\frac{1}{2}$ and $IG(\alpha_k) = IG(\frac{1}{2} - \frac{1}{k}) = I\frac{(3-(\frac{1}{2}-\frac{1}{k})}{5}$=$I(\frac{1}{2} + \frac{1}{5k})$=$0$ as $k \to \infty$. Thus $S(GI\alpha_k, GI\alpha_k, IG\alpha_k)$ $= S(\frac{1}{2}, \frac{1}{2}, 0) \neq 0$, showing that the pair (G , I) is not compatible.

Further $G(\frac{1}{2}) = I(\frac{1}{2})$ and $G(1) = I(1)$. Therefore $\frac{1}{2}$ and 0 are the coincidence points of G and I. But $GI(\frac{1}{2}) = IG(\frac{1}{2})$and $GI(0) \neq IG(0)$. Showing that the *pair (G,I) is occasionally weakly compatible but not weakly compatible as it is not commuting at two coincidence points. Hence the condition (3.2.4) is satisfied.*

Further $S(GI\alpha_k, GI\alpha_k, I\mu)$=$S(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = 0$ *or* $S(IG\alpha_k, IG\alpha_k, G\mu)$=$S(0, 0, \frac{1}{2}) \neq 0$ *as* $k \to \infty$*, this gives the pair (G,I) is weakly semi-compatible. Moreover G is continuous.*

We now establish that the mappings G,H,I and J satisfy the Condition (3.2.2).

Case I *If* $\alpha, \beta \in (0, \frac{1}{2}]$*,then* $S(\alpha, \beta, \gamma) = |\alpha - \gamma| + |\beta - \gamma|$ *on putting* $\alpha = \frac{1}{3}$, $\beta = \frac{1}{4}$ *the inequality (3.2.2) gives*

$$S(\frac{8}{15}, \frac{8}{15}, \frac{11}{20}) \leq \lambda\ max\{S(\frac{5}{9}, \frac{5}{9}, \frac{7}{12}), \frac{S(\frac{8}{15}, \frac{8}{15}, \frac{5}{9})S(\frac{11}{20}, \frac{11}{20}, \frac{7}{12})}{S(\frac{5}{9}, \frac{5}{9}, \frac{7}{12})}, S(\frac{11}{20}, \frac{11}{20}, \frac{8}{15})\}$$

$$\begin{aligned} 0.03 &\leq \lambda max\left(0.05, \frac{0.4X0.06}{0.05}, 0.03\right) \\ &\leq \lambda max\,(0.05, 0.48, 0.03) \\ &\leq \lambda(0.48) \end{aligned}$$

thus λ*=.0625* $\in (0, 1)$ *so that the inequality (3.2.2) holds.*

Case II *If* $\alpha \in (\frac{1}{2}, 1]$ *and*$\beta \in (0, \frac{1}{2}]$ *,then* $S(\alpha, \beta, \gamma) = |\alpha - \gamma| + |\beta - \gamma|$ *on putting* $\alpha = \frac{1}{3}$ *and* $\beta = \frac{2}{3}$ *the inequality (3.2.2) gives*

$$S(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}) \leq \lambda\ max\ \{S(\frac{4}{3}, \frac{4}{3}, \frac{5}{3}), \frac{S(\frac{2}{3}, \frac{2}{3}, \frac{4}{3})S(\frac{1}{3}, \frac{1}{3}\frac{5}{3})}{S(\frac{4}{3}, \frac{4}{3}, \frac{5}{3})}, S(\frac{1}{3}, \frac{1}{3}, \frac{2}{3})\}$$

$$\begin{aligned} 0.66 &\leq \lambda\ max\{.66, \frac{0.66X2.66}{0..66}, 0.66\} \\ &\leq \lambda\ max\,(0.66, 2.66, 0.66) \\ &\leq \lambda(2.66) \end{aligned}$$

thus λ*=0.248* $\in (0, 1)$ *so that the inequality (3.2.2) holds. Similarly we can prove the other cases. It is observed that* $\frac{1}{2}$ *is the unique common fixed point of the four mappings H,G,I and J.*

4. CONCLUSION

In this paper we established two results on S- metric space using the new contraction condition along with weaker form of compatible mappings semi compatible ,weakly semi compatible,occasionally weakly compatible and weakly compatible mappings. Further some examples are also discussed to support our results.

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