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# VERTEX MAGIC LABELING ON $V_4$ FOR SOME CYCLE RELATED GRAPHS

V. L. STELLA ARPUTHA MARY[1] AND S. KAVITHA

ABSTRACT. Let $V_4$ be an abelian group under multiplication. Let $g : E(G) \to V_4 - \{1\}$. The vertex magic labeling on $V_4$ is defined as the vertex labeling $g^* : V(G) \to V_4$ such that $g^*(v) = \prod_u g(uv)$, where the product is taken over all edges $uv$ of $G$ incident at $v$ is a constant . A graph is said to be $V_4$-magic if it admits a vertex magic labeling on $V_4$. In this paper we prove that Rafflesia graph , Cycle Flower graph and $S^{'}(C_n)$ graphs are $V_4$-magic graphs.

## 1. INTRODUCTION

In 1963, Sedlack introduced Magic labelings. Later Kong, Lee and Sun used the term magic labeling for edge labeling with non negative integers such that for each vertex, the sum of the labels of all edges incident at any vertex $v$ is the same for all the vertices. For a non trivial Abelian group $V_4$ under multiplication a graph $G$ is said to be $V_4$-magic graph if there exists a labeling $g$ of the edges of $G$ with non zero elements of $V_4$ such that the vertex labeling $g^*$ defined as $g^*(v) = \prod_u g(uv)$ taken over all edges $uv$ incident at $v$ is a constant.

Let $V_4 = \{i, -i, -1, 1\}$ we prove that Rafflesia graph, Cycle flower graph and splitting graph are $V_4$-magic graphs. For further references see [1,2].

[1]*corresponding author*

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## 2. Basic Definitions

**Definition 2.1.** *A cycle is a non-trivial path whose first and last vertices are the same, but no other vertex is repeated.*

**Definition 2.2.** *A graph which is obtained by joining each pendent vertex of the Helm to the apex of the helm is called a flower graph.*

**Definition 2.3.** *Let $G$ be a graph, for each point $v$ of $G$, take a new point $v^{'}$. Join $v^{'}$ to the adjacent points of $v$. A graph thus obtained is called the splitting graph of $G^{''}$. It is denoted as $S^{'}(G)$.*

## 3. Main Results

**Definition 3.1.** ***(Rafflesia Graph)*** *Consider a cycle of $n$ vertices $n \geq 3$. For every edge of the cycle, introduce a vertex outside the cycle. Now join the outer vertex to the respective edge end vertices. Also join all the outer vertices to form a cycle. This graph is called as Rafflesia graph and is denoted as $RF_n$.*

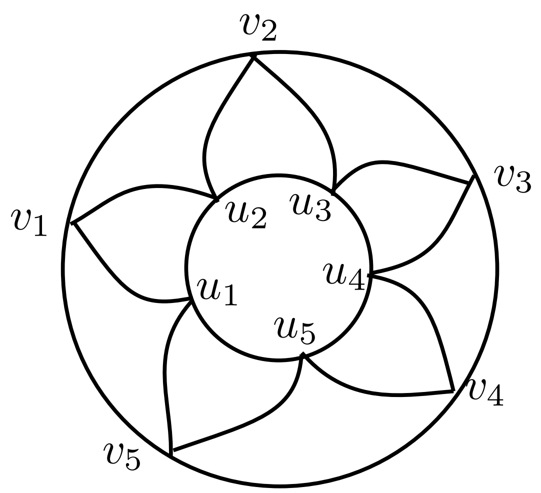



Figure 1

**Definition 3.2.** *Consider a cycle of length $m, m \geq 3$. Each edge of this cycle is covered by a cycle of length $n, n \geq 3$, with $n-2$ vertices. Such a graph is called as Cycle Flower Graph and is denoted by $CF(m,n)$.*

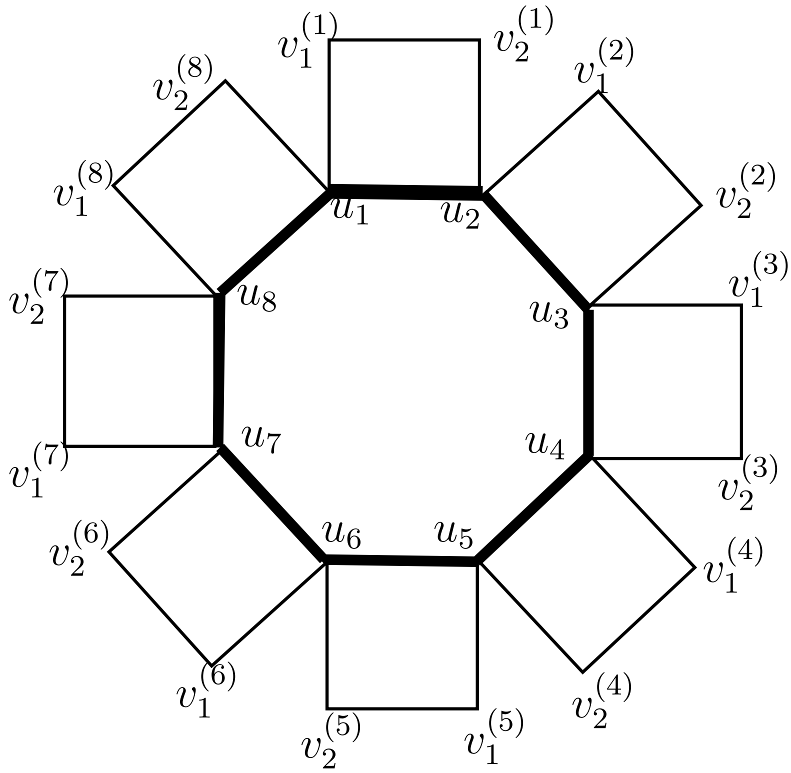



FIGURE 2

**Theorem 3.1.** *Rafflesia Graph $RF_n$ is $V_4-$ Magic Graph,$n \geq 3$.*

*Proof.* Let $V(RF_n) = \{u_j/1 \leq j \leq n\} \cup \{v_j/1 \leq j \leq n\}$ and

$$\begin{aligned} E(RF_n) &= \{u_j u_{j+1}/1 \leq j \leq n\} \cup \{u_j v_j \cup v_j u_{j+1}/1 \leq j \leq n\} \\ &\quad \cup \{v_j v_{j+1}/1 \leq j \leq n\} \end{aligned}$$

$$[u_{n+1} = u_1, u_0 = u_n, v_{n+1} = v_1, v_0 = v_n]$$

Define $g : E(RF_n) \to \{i, -i, -1\}$ as

$$g(u_j u_{j+1}) = i, \ \forall\, j = 1, 2, \ldots, n$$

$$g(u_j v_j) = g(v_j u_{j+1}) = -i, \ \forall\, j = 1, 2, \ldots, n$$

$$g(v_j v_{j+1}) = i, \ \forall\, j = 1, 2, \ldots, n$$

Now the mapping $g^* : V(RF_n) \to V_4$ is given by

$$g^*(u_j) = g(u_j u_{j+1}) * g(u_j u_{j-1}) * g(u_j v_j) * g(u_j v_{j-1}), \ \forall\, j = 1, 2, \ldots, n$$

$$g^*(v_j) = g(v_j u_j) * g(v_j u_{j+1}) * g(v_j v_{j+1}) * g(v_j v_{j-1}), \forall\, j = 1, 2, \ldots, n$$

Clearly

$$g^*(u_j) = 1, \forall\ j = 1, 2, \ldots, n$$
$$g^*(v_j) = 1, \forall\ j = 1, 2, \ldots, n$$

So $RF_n$ becomes a $V_4$- magic graph by admitting $V_4$-magic labelling. □

**Example 1.** *$RF_5$ is a $V_4$-magic graph.*

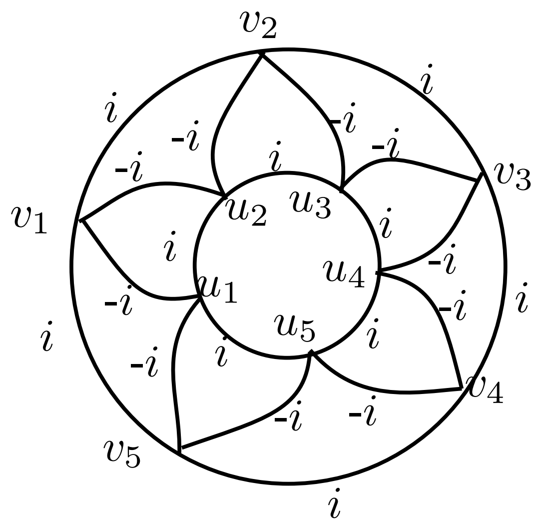



FIGURE 3

**Theorem 3.2.** *The cycle flower graph $CF(m, n), m, n \geq 3$ is a $V_4$-magic graph.*

*Proof.*

**Case (i):** Let $m$ be an even number.

Let $n$ be either even or odd number.

Let $V(CF(m, n)) = \{u_j/1 \leq j \leq m\} \cup \{v_k^{(t)}/1 \leq k \leq n-2, 1 \leq t \leq m\}$

$$\begin{aligned} E(CF(m,n)) &= \{u_j u_{j+1}/1 \leq j \leq m\} \cup \\ &\cup \{u_j v_1^{(t)}/1 \leq j \leq m, 1 \leq t \leq m\} \\ &\cup \{v_k^{(t)} v_{k+1}^{(t)}/1 \leq k \leq n-3, 1 \leq t \leq m\} \\ &\cup \{v_{n-2}^{(t)} u_j/1 \leq t \leq m, 1 \leq j \leq m\} \end{aligned}$$

$$[u_{m+1} = u_1, u_0 = u_m, v_k^{(0)} = v_k^{(m)}, v_{n-1}^{(j)} = u_{j+1}, v_0^{(j)} = u_j]$$

Define $g : E(CF(m, n)) \to \{i, -i, -1\}$ as

When $j$ is odd, $g(u_j u_{j+1}) = i, \forall\ j = 1, 2, \ldots, m$

When $j$ is even, $g(u_j u_{j+1}) = -i, \forall\ j = 1, 2, \ldots, m$

$$g(u_j v_k^{(j)}) = -1, \forall\ j = 1, 2, \ldots, m, k = 1, \forall\ t = 1, 2, \ldots, j-1$$
$$g(u_j v_k^{(j-1)}) = -1, \forall\ j = 1, 2, \ldots, m \ and\ k = n-2,$$
$$g(v_k^{(t)} v_{k+1}^{(t)}) = -1, \forall\ t = 1, 2, \ldots, m, \forall\ k = 1, 2, \ldots, n-2$$

The mapping $g^* : V(CF(m,n)) \to V_4$ is given by
$g^*(u_j) = g(u_j v_j^{(j)}) * g(u_j u_{j+1}) * g(u_j v_k^{(j-1)}) * g(u_j u_{j-1})$,when $j = 1$, $\forall\ k = 1, 2, \ldots, n-2$
$g^*(u_j) = g(u_j u_{j+1}) * g(u_j u_{j-1}) * g(u_j v_k^{(j)}) * g(u_j v_{n-2}^{(j-1)}), \forall\ j = 2, 3, \ldots m$, and $k = 1$.
$g^*(v_k^{(t)}) = g(u_j v_k^{(t)}) * g(v_k^{(t)} v_{k+1}^{(t)}), \forall\ j = 1, 2, \ldots, m, t = 1, 2, \ldots, m$, $j = t, k = 1$
$g^*(v_k^{(t)}) = g(u_j v_k^{(t)}) * g(v_{k-1}^{(t)} v_k^{(t)})$,when $j = t+1, \forall\ t = 1, 2, \ldots, m, k = n-2$
$g^*(v_k^{(t)}) = g(v_{k-1}^{(t)} v_k^{(t)}) * g(v_k^{(t)} v_{k+1}^{(t)}), \forall\ t = 1, 2, \ldots, m$ and $1 < k < n-2$
Clearly
$$g^*(u_j) = 1\ \forall\ j = 1, 2, \ldots, m$$
$$g^*(v_k^{(t)}) = 1\ \forall\ k = 1, 2, \ldots, n-2$$

From this case, we can clearly say that $CF(m,n)$ admits vertex magic labelling on $V_4$.
Hence $CF(m,n)$ is a $V_4$- magic graph.
**Case (ii):** Let $m$ be an odd number.
**Subcase (i)** Let $n$ be an odd number.
Let $V(CF(m,n)) = \{u_j / 1 \le j \le m\} \cup \{v_k^{(t)} / 1 \le k \le n-2, 1 \le t \le m\}$

$$\begin{aligned} E(CF(m,n)) &= \{u_j u_{j+1} / 1 \le j \le m\} \cup \\ &\cup \{u_j v_1^{(t)} / 1 \le j \le m, 1 \le t \le m\} \\ &\cup \{v_k^{(t)} v_{k+1}^{(t)} / 1 \le k \le n-3, 1 \le t \le m\} \\ &\cup \{v_{n-2}^{(t)} u_j / 1 \le t \le m, 1 \le j \le m\} \end{aligned}$$
$$[u_{m+1} = u_1, u_0 = u_m, v_k^{(0)} = v_k^{(m)}, v_{n-1}^{(j)} = u_{j+1}, v_0^{(j)} = u_j]$$

Define $g : E(CF(m,n)) \to \{i, -i, -1\}$ as

$$g(u_j u_{j+1}) = -1, \forall\ j = 1, 2, \ldots, m$$
$$g(u_j v_1^{(j)}) = i, \forall\ j = 1, 2, \ldots, m$$

$g(u_j v_k^{(j-1)}) = -i, \forall\ j = 1, 2, \ldots, m$ and $k = n-2$,
$g(v_k^{(t)} v_{k+1}^{(t)}) = -i, \forall\ t = 1, 2, \ldots, m, \forall\ k = 1, 2, \ldots, n-2$ and $k$ is odd

$g(v_k^{(t)}v_{k+1}^{(t)}) = i, \forall\, t = 1, 2, \ldots, m, \forall\, k = 1, 2, \ldots, n-2$ and $k$ is even
The mapping $g^* : V(CF(m,n)) \to V_4$ is given by
$g^*(u_j) = g(u_jv_j^{(j)}) * g(u_ju_{j+1}) * g(u_jv_k^{(j-1)}) * g(u_ju_{j-1})$,when $j = 1$, $\forall\, k = 1, 2, \ldots, n-2$
$g^*(u_j) = g(u_ju_{j+1}) * g(u_ju_{j-1}) * g(u_jv_k^{(j)}) * g(u_jv_{n-2}^{(j-1)}), \forall$ æ $= 2, 3, \ldots, m$ and $k = 1$
$g^*(v_k^{(t)}) = g(u_jv_k^{(t)}) * g(v_k^{(t)}v_{k+1}^{(t)}), \forall\, j = 1, 2, \ldots, m, \forall\, t = 1, 2, \ldots, m$,$j = t, k = 1$
$g^*(v_k^{(t)}) = g(u_jv_k^{(t)}) * g(v_{k-1}^{(t)}v_k^{(t)})$,when $j = t+1, \forall\, t = 1, 2, \ldots, m$,$k = n-2$
$g^*(v_k^{(t)}) = g(v_{k-1}^{(t)}v_k^{(t)}) * g(v_k^{(t)}v_{k+1}^{(t)}), \forall\, t = 1, 2, \ldots, m$ and $1 < k < n-2$
Clearly

$$g^*(u_j) = 1\ \forall\, j = 1, 2, \ldots, m$$
$$g^*(v_k^{(t)}) = 1\ \forall\, k = 1, 2, \ldots, n-2$$

From this case, we can clearly say that $CF(m,n)$ admits vertex magic labelling on $V_4$ when $m, n$ are both odd numbers.
Hence $CF(m,n)$ is a $V_4$- magic graph.
**Subcase (ii):** Let $n$ be an even number.
Let $V(CF(m,n)) = \{u_j/1 \le j \le m\} \cup \{v_k^{(t)}/1 \le k \le n-2, 1 \le t \le m\}$

$$\begin{aligned} E(CF(m,n)) &= \{u_ju_{j+1}/1 \le j \le m\} \cup \\ &\quad \cup\{u_jv_1^{(t)}/1 \le j \le m, 1 \le t \le m, j = t\} \cup \\ &\quad \cup\{v_k^{(t)}v_{k+1}^{(t)}/1 \le k \le n-3, 1 \le t \le m\} \cup \\ &\quad \cup\{v_k^{(t)}u_j/1 \le t \le m, 1 \le j \le m, k = n-2\} \end{aligned}$$

$$[u_{m+1} = u_1, u_0 = u_m, v_k^{(0)} = v_k^{(m)}, v_{n-1}^{(j)} = u_{j+1}, v_0^{(j)} = u_j]$$

Define $g : E(CF(m,n)) \to \{i, -i, -1\}$ as

$$g(u_ju_{j+1}) = -1, \forall\, j = 1, 2, \ldots, m$$
$$g(u_jv_1^{(j)}) = -1, \forall\, j = 1, 2, \ldots, m$$

$g(u_jv_k^{(j-1)}) = -1, \forall\, j = 1, 2, \ldots, m$ and $k = n-2$,
$g(v_k^{(t)}v_{k+1}^{(t)}) = -1, \forall\, t = 1, 2, \ldots, m, \forall\, k = 1, 2, \ldots, n-2$
The mapping $g^* : V(CF(m,n)) \to V_4$ is given by
$g^*(u_j) = g(u_jv_j^{(j)}) * g(u_ju_{j+1}) * g(u_jv_k^{(j-1)}) * g(u_ju_{j-1})$,when $j = 1$,
$\forall\, k = 1, 2, \ldots n-2$
$g^*(u_j) = g(u_ju_{j+1}) * g(u_ju_{j-1}) * g(u_jv_k^{(j)}) * g(u_jv_{n-2}^{(j-1)}), \forall\, j = 2, 3, \ldots, m$ ,and $k = 1$
$g^*(v_k^{(t)}) = g(u_jv_k^{(t)}) * g(v_k^{(t)}v_{k+1}^{(t)}), \forall\, j = 1, 2, \ldots, m, \forall\, t = 1, 2, \ldots, m, j = t, k = 1$
$g^*(v_k^{(t)}) = g(u_jv_k^{(t)}) * g(v_{k-1}^{(t)}v_k^{(t)})$,when $j = t+1, \forall\, t = 1, 2, \ldots m, k = n-2$

$g^*(v_k^{(t)}) = g(v_{k-1}^{(t)} v_k^{(t)}) * g(v_k^{(t)} v_{k+1}^{(t)}), \forall\, t = 1, 2, \dots, m$ and $1 < k < n-2$

Clearly

$$g^*(u_j) = 1 \;\forall\; j = 1, 2, \dots, m$$

$$g^*(v_k^{(t)}) = 1 \;\forall\; k = 1, 2, \dots, n-2$$

From this case, we can clearly say that $CF(m,n)$ admits vertex magic labelling on $V_4$, when $m$ is odd and $n$ is an even number.

Hence $CF(m,n)$ is a $V_4$- magic graph.

From all three cases, we conclude that the Cycle flower graph $CF(m,n)$ is a $V_4$-magic graph. □

**Example 2.** *$CF(8,4)$ is a $V_4$- magic graph.*

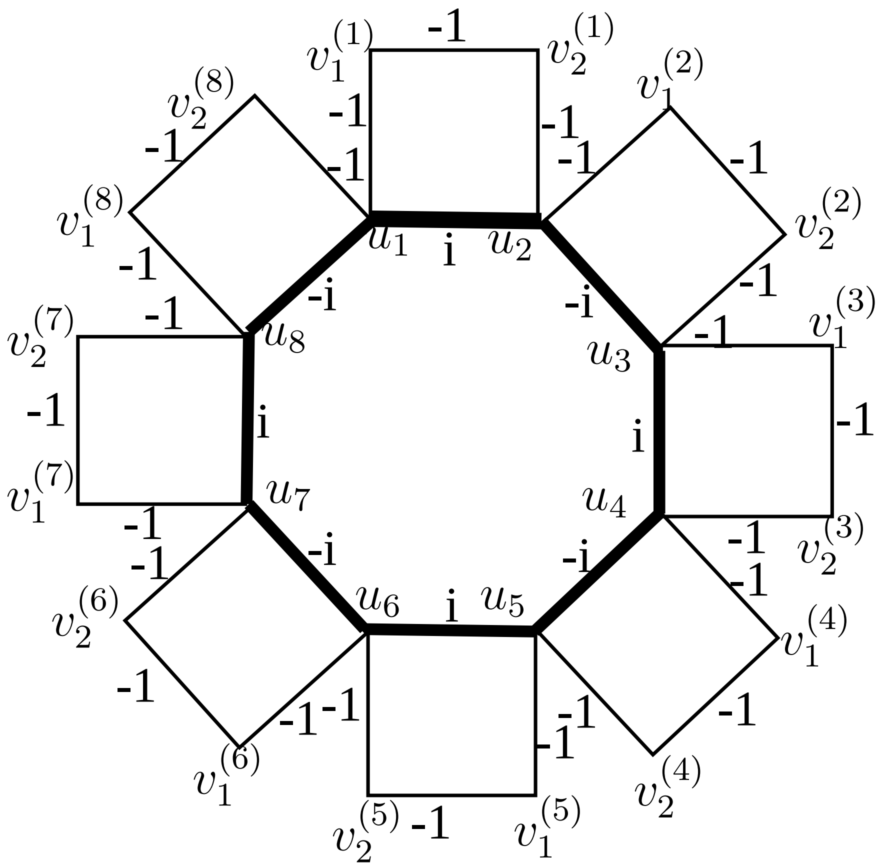



FIGURE 4

**Example 3.** *$CF(9,3)$ is a $V_4$- magic graph.*

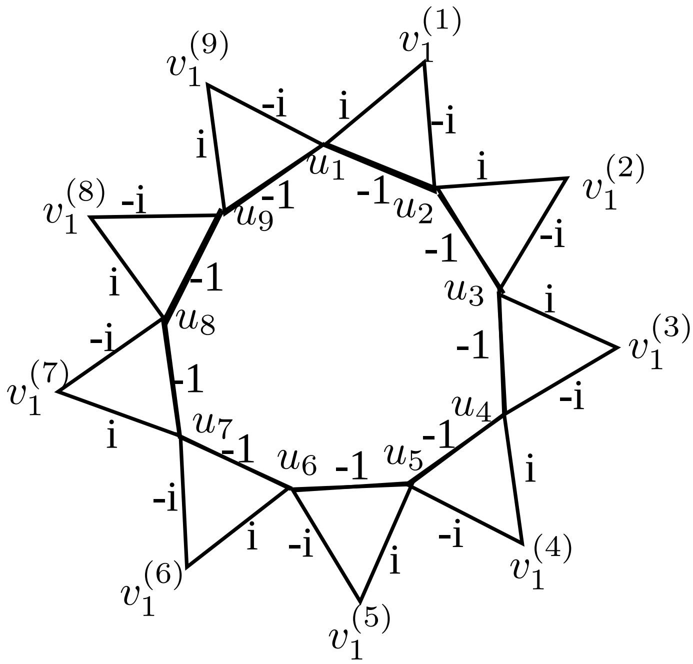



FIGURE 5

**Example 4.** *$CF(5,4)$ is a $V_4$-magic graph.*

**Theorem 3.3.** *The Splitting graph $S^{'}(C_n)$ is a $V_4$-Magic graph with $n \geq 3$.*

*Proof.* Let $V(S^{'}(C_n)) = \{v_j/1 \leq j \leq n\} \cup \{v_j^{'}/1 \leq j \leq n\}$ and

$$\begin{aligned} E(S^{'}(C_n)) &= \{v_jv_{j+1}/1 \leq j \leq n\} \cup \{(v_{j-1}v_j^{'}/1 \leq j \leq n\} \\ &\quad \cup\{v_j^{\prime}v_{j+1}^{'}/1 \leq j \leq n\} \end{aligned}$$

$$[v_{n+1} = v_1 \ \& \ v_0 = v_n]$$

**Case(i):** When $n$ is odd.
Let us define $g : E(S^{'}(C_n)) \to \{i, -i, -1\}$ as

$$g(v_jv_{j+1}) = -1, 1 \leq j \leq n$$

$$g(v_{j-1}v_j^{'}) = -1, 1 \leq j \leq n$$

$$g(v_j^{'}v_{j+1}) = -1, 1 \leq j \leq n$$

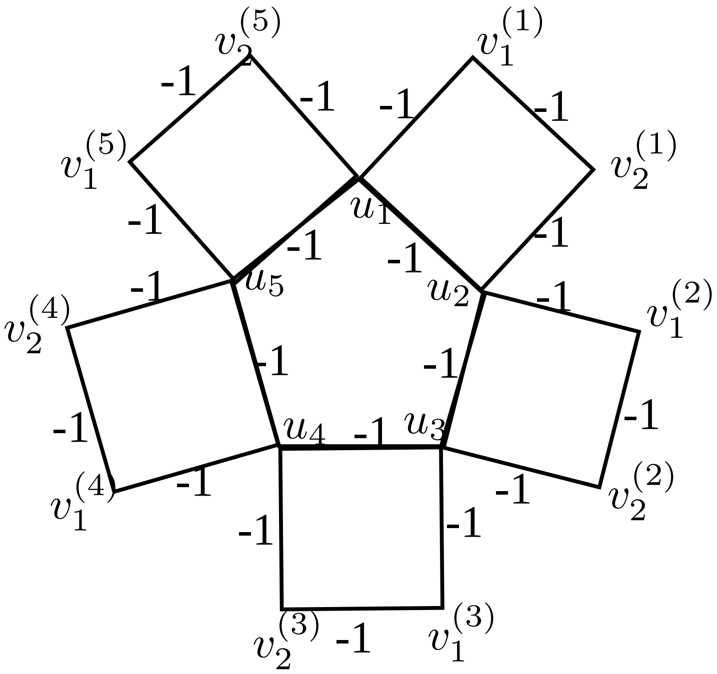



FIGURE 6

Now $g^* : V(S^{'}(C_n)) \to \{i, -i, -1\}$

$$\begin{aligned} g^*(v_j) &= g(v_{j-1}v_j) * g(v_jv_{j+1}) * g(v^{'}_{j-1}v_j) * g(v_jv^{'}_{j+1}), 1 \le j \le n \\ &= (-1) * (-1) * (-1) * (-1) \\ &= 1, 1 \le j \le n \\ g^*(v^{'}_j) &= g(v_{j-1}v^{'}_j) * g(v^{'}_jv_{j+1}), 1 \le j \le n \\ &= (-1) * (-1) \\ &= 1, 1 \le j \le n \end{aligned}$$

Here $g^*(v_j)$ and $g^*(v^{'}_j)$ are constant and equals one for all $v_j, v^{'}_j \in V(S^{'}(C_n)), 1 \le j \le n$

Hence $S^{'}(C_n)$ admits vertex magic labelling on $V_4$, when $n$ is odd.

**Case(ii):** When $n$ is even.

Let us define $g : E(S^{'}(C_n)) \to \{i, -i, -1\}$ as

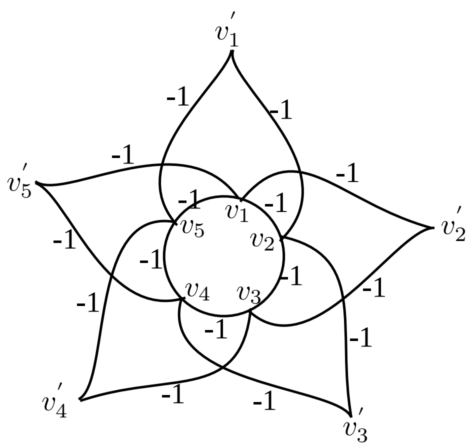
$g(v_jv_{j+1}) = i, 1 \le j \le n$, when $j$ is odd,

$g(v_jv_{j+1}) = -i, 1 \le j \le n$, when $j$ is even,

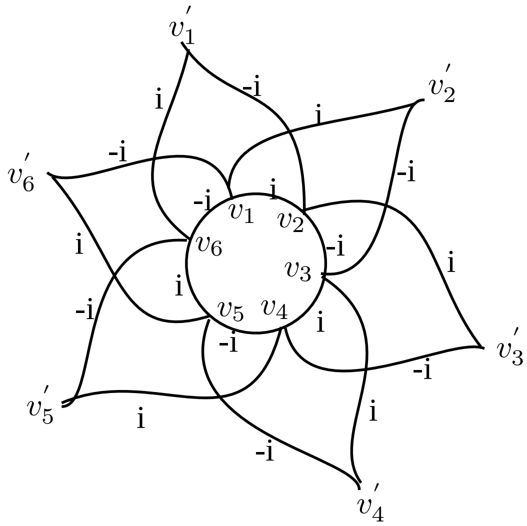
$g(v_{j-1}v^{'}_j) = i, 1 \le j \le n,$

$g(v_j^{'}v_{j+1}) = -i, 1 \leq j \leq n$,
Now $g^* : V(S^{'}(C_n)) \rightarrow \{i, -i, -1\}$
$g^*(v_j) = g(v_{j-1}v_j) * g(v_jv_{j+1}) * g(v_{j-1}^{'}v_j) * g(v_jv_{j+1}^{'}), j$ is odd $1 \leq j \leq n$
$= (-i) * (i) * (-i) * (i)$
$= (-i^2) * (-i^2)$
$= (1) * (1)$
$= 1, j$ is odd,$1 \leq j \leq n$
$g^*(v_j) = g(v_{j-1}v_j) * g(v_jv_{j+1}) * g(v_{j-1}^{'}v_j) * g(v_jv_{j+1}^{'}), j$ is even,$1 \leq j \leq n$
$= (i) * (-i) * (i) * (-i)$
$= (-i^2) * (-i^2)$
$= (1) * (1)$
$= 1, j$ is even,$1 \leq j \leq n$
$g^*(v_j^{'}) = g(v_{j-1}v_j^{'}) * g(v_j^{'}v_{j+1}), 1 \leq j \leq n$
$= (i) * (-i)$
$= (-i^2)$
$= 1, 1 \leq j \leq n$
Here too $g^*(v_j)$ and $g^*(v_j^{'})$ are constant and equals one for all $v_j, v_j^{'} \in V(S^{'}(C_n)), 1 \leq j \leq n$
Hence from both cases we conclude that $S^{'}(C_n)$ admits Vertex magic labelling on $V_4$.
Hence $S^{'}(C_n)$ is a $V_4$- Magic graph. □

**Example 5.** *$S^{'}(C_5)$ is a $V_4$-Magic graph.*




**Example 6.** *$S^{'}(C_6)$ is a $V_4$-Magic graph*

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DEPARTMENT OF MATHEMATICS
ST.MARY'S COLLEGE (AUTONOMOUS), THOOTHUKUDI
*Email address*: `prisstell@yahoo.com`

REGISTER NUMBER 19212212092007
ST.MARY'S COLLEGE (AUTONOMOUS), THOOTHUKUDI,
MANONMANIAM SUNDARANAR UNIVERSITY
ABISHEKAPATTI, TIRUNELVELI, INDIA
*Email address*: `kavithavikunth@gmail.com`