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# MINIMIZATION OF TOTAL WAITING TIME OF JOBS IN NX3 SPECIALLY STRUCTURED FLOWSHOP SCHEDULING IN WHICH PROCESSING TIME WITH THEIR RESPECTIVE PROBABILITIES INCLUDING TRANSPORTATION TIME OF JOBS

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ABSTRACT. Adding transportation time to nx3 specially structured flow shop scheduling in which processing time with their respective probabilities is described in this paper. Aimed to be focused at optimizing overall waiting time of 'n' jobs when the number of machines(or processing stations) is '3'.The problem discussed is an extension to an earlier solution. Further addition of transportation time has made this research paper much closer to its real life applicability. Here, the total waiting time is the sum of machine waiting time and the transfer time taken by a job before its processing starts on the successive machine. The solution is discussed using an iterative algorithm. Furthermore, a practical example is discussed step by step to clarify the application of the algorithm.

## 1. INTRODUCTION

A little work has been done in minimizing total waiting time for obtaining on optimal schedule of jobs. The waiting time is to be important for scheduling jobs on the machines. The idea of minimizing the waiting time or cost may be an economical expression from industry / factory manager's point of view However this minimization of total waiting time is more important to economize.One of

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the earliest results in flow shop scheduling theory is an algorithm by S.M. Johson (1954) for scheduling jobs in two machine flow shop to minimizing the time at which all jobs are completed. Maggu and Das (1980) First time Introduced the concept of transportation time in sequencing n-jobs, 2-machine problem and to obtain sequence considering the criteria to minimize total elapsed time. Maggu and Das (1980) originally established a theorem to provide us a decomposition algorithm for determining an optional schedule for the 2-machine n-job flow shop scheduling problem involving transportation time of jobs. n-jobs three machine flow shop problem which have separately found techniques to minimize total impresses inventory time for all jobs was faintly introduced by Gupta D and S. Singh (2020).The underlying idea behind optimality in the given flow shop scheduling problem is termed as minimization of total waiting time of jobs which includes the job idle time before processing and transport time, of which this paper is about. There are some papers available in the literature of scheduling theory dedicating to the transport time in nx2 setup. Here is an extension applied to nx3 setup in an attempt to make it more acceptable.

## 2. LITERATURE REVIEW

The Johnson's algorithm is especially popular among analytical approaches that are used for solving n- jobs, 2- machines sequence problem to minimize the total elapsed time.Ignall and Schrage introduced the permutation flow shop problem with branch and bound algorithms for make span minimization.Lockett A.G. and Muhlemann A.P., Crowin and Esogbue, Maggu and Dass made attempts to extend the study by introducing various parameters. [8]. Yoshida and Hitomi solved two stage production scheduling, the set up time being separated from processing time. Solution methods for flow shop scheduling using heuristics developed by [6] Singh T.P., Rajendran and Chaudhuri. Singh T.P., Gupta D. studied the problem related with group job restrictions in a flow shop which involves independent set- up time and transportation time. [7] Singh V. put his efforts to study three machine flow shop scheduling problems with total rental cost. Further [5] Gupta D. studied minimization of Rental Cost in nx2 Flow Shop Scheduling with job block concept and setup Time was separated from Processing Time and each coupled with probabilities. [1] Gupta D. and et. al. studied optimal two and three stage open shop specially structured scheduling in which the processing times are coupled with probabilities with transportation time to minimize the rental cost.Recently [3] Gupta D. and Goyal B. Considered the concept of reducing waiting time of jobs by considering processing times coupled with probabilities, reducing in the three machine without setup time. The problem conferred here has noteworthy use of hypothetical results in process industries or in the conditions when the purpose is to minimize the total waiting time of jobs. The paper discussed here is an extension made by [4] Gupta D. and Goyal B. in the sense that we have taken into consideration the three machines alienated from processing time. Extending the study three machine specially structured flowshop scheduling with the objective of minimizing total waiting time of jobs.

## 3. PRACTICAL SITUATION

Manufacturing units/industries play a crucial role in the economic progress of a country. Almost on a daily basis, Flow shop scheduling occurs in various offices, repair shops, banks, airports etc. Let us take an example involving transportation times in addition to waiting times of jobs. In a car factory, assume there are three stations, a body shop, a paint shop and an assembling shop. Suppose, chassis has just undergone welding in the body shop and side windows have just been painted in the paint shop. The same robot is assigned for painting all different parts. Now, although the machine (robot) in paint shop is free, it still needs to wait for the chassis to be brought into the paint shop from the body shop. In reality a car manufacturing unit has shops located at some distance from one another. Thus transfer times come into picture which significantly affects the pre planning and scheduling of jobs.

## 4. NOTATIONS

$a_k$: Order made by the algorithm.
$A_i$: Processing time of $i^{th}$ job on machine A.
$A_i'$: Equivalent time for processing of $j^{th}$ job on machine A.
$B_i$: Processing time of $i^{th}$job on machine B.
$B_i'$: Equivalent time for processing of $j^{th}$ job on machine B.
$C_i$: Processing time of $i^{th}$ job on machine C.
$C_i'$: Equivalent time for processing of $j^{th}$ job on machine C.

$P_i$: Processing time for $i^{th}$ job on first fictitious machine
$Q_i$: Processing time for $i^{th}$ job on second fictitious machine
$W_T$: Sum of holding time of all the jobs.
$T_i$: Transportation time $i^{th}$ job from machine A to machine B
$G_i$: Transportation time $i^{th}$ job from machine B to machine C

## 5. PROBLEM FORMULATION

Assume that three machines A,B and C are transforming n jobs in the form ABC, $A_i$ ,$B_i$ and $C_i$ are the respective processing times, with probabilities $a_i$, $b_i$ and $c_i$. $T_i$ Transportation time $i^{th}$ job from machine A to machine B, and $G_i$ Transportation time $i^{th}$ job from machine B to machine C Our aim is to find an optimize oder $a_k$ of jobs optimization the total holding time of all jobs.

TABLE 1

| Jobs | Machine A | | Transportation time T | Machine B | | Tranportation time G | Machine C | |
|---|---|---|---|---|---|---|---|---|
| I | $A_i$ | $a_i$ | $T_i$ | $B_i$ | $b_i$ | $G_i$ | $C_i$ | $c_i$ |
| 1 | $A_1$ | $a_1$ | $T_1$ | $B_1$ | $b_1$ | $G_1$ | $C_1$ | $c_1$ |
| 2 | $A_2$ | $a_2$ | $T_2$ | $B_2$ | $b_2$ | $G_2$ | $C_2$ | $c_2$ |
| 3 | $A_3$ | $a_3$ | $T_3$ | $B_3$ | $b_3$ | $G_3$ | $C_3$ | $c_3$ |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| n | $A_n$ | $a_n$ | $T_n$ | $B_n$ | $b_n$ | $G_n$ | $C_n$ | $c_n$ |

## 6. ASSUMPTIONS

1) There are n figure of jobs (I) and 3 machines (A, B and C).
2) The arrangements of procedure in all machines act by identical.
3) Jobs are not dependent on one another.
4) Once a job is begin on a machine, the processing can't be put to an end as long as the job is finished.

## 7. ALGORITHM

**Step 1:** Equivalent processing times of $i^{th}$ job on machine A,B and C are defined as $A'_i = A_i * a_i$ , $B'_i = B_i * b_i$ ,$C'_i$,$= C_i * c_i$.

TABLE 2

| $A'_i$ | $T_i$ | $B'_i$ | $G_i$ | $C'_i$ |
|---|---|---|---|---|
| $A'_1$ | $T_1$ | $B'_1$ | $G_1$ | $C'_1$ |
| $A'_2$ | $T_2$ | $B'_1$ | $G_2$ | $C'_2$ |
| $A'_3$ | $T_3$ | $B'_3$ | $G_3$ | $C'_3$ |
| . | . | . | . | . |
| . | . | . | . | . |
| $A'_n$ | $T_n$ | $B'_n$ | $G_n$ | $C'_n$ |

**Step 2:** Check the condition either $\max(B'_i + T_i) \leq (\min A'_i + T_i)$ or $\max(B'_i + G_i) \leq (\min C'_i + G_i)$ if higher terms are fulfill that time we substitute the 3 machine by two imaginary machine P and Q along with uniform transaction time defined as $P_i = A'_i + T_i + B'_i + G_i$ and $Q_i = T_i + B'_i + G_i + C'_i$, where $P_i$ and $Q_i$ are the transaction times for $i^{th}$ job on machine P and Q independently. By

TABLE 3

| Jobs | Machine P | Machine Q |
|---|---|---|
| I | $P_i$ | $Q_i$ |
| 1 | $P_1$ | $Q_1$ |
| 2 | $P_2$ | $Q_2$ |
| 3 | $P_3$ | $Q_3$ |
| 4 | $P_4$ | $Q_4$ |
| 5 | $P_5$ | $Q_5$ |

Computing the new transaction times, We consider the preferable flow of the jobs for the machines P and Q in the normal way. Transaction times satisfies structural relationship $\max P_i \leq \min Q_i$.

**Step 3:** Equivalent processing times $P_i$ and $Q_i$ on machine P and Q respectively be calculated is defined in steps.

TABLE 4

| Jobs (J) | Machine P $(P_i)$ | Machine Q $(Q_i)$ | $z_{i_r} = (n-r)x_i$ | | | | |
|---|---|---|---|---|---|---|---|
| | | | $x_i = Q_i - P_i$ | $r = 1$ | $r = 2$ | $\dots$ | $r = (n-1)$ |
| 1 | $P_1$ | $Q_1$ | $x_1$ | $z_{11}$ | $z_{12}$ | ... | $z_{1(n-1)}$ |
| 2 | $P_2$ | $Q_2$ | $x_2$ | $z_{21}$ | $z_{22}$ | ... | $z_{2(n-1)}$ |
| 3 | $P_3$ | $Q_3$ | $x_3$ | $z_{31}$ | $z_{32}$ | ... | $z_{3(n-1)}$ |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. |
| n | $P_n$ | $Q_n$ | $x_n$ | $z_{n_1}$ | $z_{n_2}$ | ... | $z_{n(n-1)}$ |

**Step 4:** Calculate the entries for the following table

**Step 5:** Assemble the jobs in enhancing series of $x_j$. Assuming the sequence found be $(\sigma_1, \sigma_2, \sigma_3, \sigma_m)$.

**Step 6:** Locate $\min P_i$. For the following two possibilities:

$P_{\alpha_1} = \min P_i$ arranged aligning to step 4 is the needed preferable flow

$P_{\alpha_1} \neq \min P_i$ shift at step 6

**Step 7:** Examine the distinct order of jobs $a_1, a_2, a_3, a_m$ .Where $a_1$ is the flow accomplished in step 4, Sequence $a_k$ (k= 2,3,âĂę$m$) can be accomplished by shifting $k^{th}$ job in the flow $a_1$ to the $1^{st}$ spot and stub of the flow staying identical.

**Step 8:** Figure out the total holding time $W_T$ for all the order $a_1, a_2, a_3, a_m$ using the following formula:

$$W_T = nX_i + \sum_{r=1}^{n-1} Z_{a_r} - \sum_{k=1}^{n} X_i$$

$P_i$ = Correspond transforming time of the 1st job on machine P in orders i;
$Z_{ir} = (n-r)x_i$; $a$=$\sigma_1, \sigma_2, \sigma_3, \sigma_n$.
The flow along lesser total holding time is the needed preferable flow.

## 8. NUMERICAL ILLUSTRATION

Considered five jobs $1^{st}$,$2^{nd}$, $3_{rd}$, $4^{th}$, $5^{th}$ along transforming times $A_i$ and $B_i$ and $B_i$ and $B_i$ to be transacted on the machines A, B and C.

TABLE 5

| Jobs | Machine A | | Transportation time T | Machine B | | Tranportation time G | Machine C | |
|---|---|---|---|---|---|---|---|---|
| 1 | 18 | 0.1 | 4 | 3 | 0.1 | 2 | 25 | 0.1 |
| 2 | 6 | 0.2 | 5 | 1 | 0.2 | 1 | 10 | 0.4 |
| 3 | 4 | 0.3 | 5 | 3 | 0.1 | 1 | 15 | 0.2 |
| 4 | 9 | 0.2 | 4 | 5 | 0.1 | 2 | 30 | 0.1 |
| 5 | 7 | 0.2 | 5 | 1 | 0.5 | 1 | 20 | 0.2 |

Our purpose leads to attain preferable flow which lesser the total holding time of the jobs

**Solution.**

TABLE 6

| $A_i'$ | $T_i$ | $B_i'$ | $G_i$ | $C_i'$ |
|---|---|---|---|---|
| 1.8 | 4 | 0.3 | 2 | 2.5 |
| 1.2 | 5 | 0.2 | 1 | 4.0 |
| 1.2 | 5 | 0.3 | 1 | 3.0 |
| 1.8 | 4 | 0.5 | 2 | 3.0 |
| 1.4 | 5 | 0.5 | 1 | 4.0 |

**According to step 1:** After satisfying structured conditions reducing the three machine into two fictitious machine P and Q satisfying $\max(B_i' + T_i)$= 5.5$\leq$ $\min A_i'$+$T_i$=5.8 two imaginary machine G and H along with uniform transaction time given by $P_i = A_i' + B_i'$ and $Q_i = B_i' + C_i'$.

Two imaginary machines G and Hn along with uniform transaction time given by $P_i$= $A_i' + T_i + B_i' + G_i$ and $Q_i$=$T_i + B_i' + G_i + C_i'$,

$$\max P_i = 8.3 \leq \min Q_i = 8.8.$$

TABLE 7

| Jobs | Machine P | Machine Q |
|---|---|---|
| I | $P_i$ | $Q_i$ |
| 1 | 8.1 | 8.8 |
| 2 | 7.4 | 10.2 |
| 3 | 7.5 | 9.3 |
| 4 | 8.3 | 9.5 |
| 5 | 7.9 | 10.5 |

**According to step 2:** Achieving the code for

TABLE 8

| Jobs | Machine P | Machine Q | $z_{ir} = (n-r)x_i$ | | | | |
|---|---|---|---|---|---|---|---|
| (J) | $(P_i)$ | $(Q_i)$ | $x_i = Q_i - P_i$ | $r=1$ | $r=2$ | $r=3$ | $r=4$ |
| 1 | 8.1 | 8.8 | 0.7 | 2.8 | 2.1 | 1.4 | 0.7 |
| 2 | 7.4 | 10.2 | 2.8 | 11.2 | 8.4 | 5.6 | 2.8 |
| 3 | 7.5 | 9.3 | 1.8 | 7.2 | 5.4 | 3.6 | 1.8 |
| 4 | 8.3 | 9.5 | 1.2 | 4.8 | 3.6 | 2.4 | 1.2 |
| 5 | 7.9 | 10.5 | 2.6 | 10.4 | 7.8 | 5.2 | 2.6 |

**According to step 3:**The order so established be $1^{st}$, $4^{th}$, $3^{rd}$ , $5^{th}$ , $2^{nd}$.

**According to step 4:** Min$P_i$=7.4$\neq P_1$

**According to step 5:** Different sequence of jobs can be considered as:
$a_1$: 1,4,3,5,2;
$a_2$ : 4,1,3,5,2;
$a_3$ : 3,1,4,5,2;
$a_4$ :5,1,4,3,2;
$a_5$ : 2,1,4,3,5

**According to step 6:** The total holding time for the order accomplished in step five can be computed.

Present sum of $P_i = 39.2$.

Hence arrangement is $a_3$: 3,1,4,5,2 the needed arrangement with lesser total holding time.

**Table-9**

| $a_i$ | Sequence | Total Waiting Time $W_T$ | |
|---|---|---|---|
| $a_1$ | 1-4-3-5-2 | 13.9 | |
| $a_2$ | 4-1-3-5-2 | 15.4 | |
| $a_3$ | 3-1-4-5-2 | 12.6 | Optimal |
| $a_4$ | 5-1-4-3-2 | 17 | |
| $a_5$ | 2-1-4-3-5 | 15.3 | |

## 9. REMARKS

This edition is worked to conclude the flow shop arrangement problem aimed at lowering the overall waiting time of jobs. Even though additional other costs such as machine idling costs or penalty cost of the jobs may go up, still it proves to be beneficial with respect to the overall cost management. Waiting time reduction is directly linked with more output per unit time. Thus, in practice, it is extremely helpful in servicing time bound consignments. Whenever an order is to be fulfilled in least amount of time, this particular method can be used. Further, additional factors can be brought into picture such as break down interval, transportation time etc. to make the problem much more aligned to the real world situations.

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