Advances in Mathematics: Scientific Journal **9** (2020), no.9, 6447–6456
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.2

ON STRICT-HONEST AND SUPER-HONEST N-SUBGROUPS

NAVALAKHI HAZARIKA[1] AND NARAYAN NAYAK

Abstract. Extending the notion of honesty, complete-honest and strict-honest N-subgroups are introduced in near-ring groups and various characteristics of these N-subgroups are investigated. The torsion and complete-closure exhibit the super-honesty character of certain N-subgroups of E. Also the relation between strict-honest, complete-honest, complete-closed and super-honest characters of N-subgroups are established.

1. Introduction

The concept of honest subgroups was introduced by Abian and Rinehart in [1]. Further the concepts honest submodules, isolated submodules are studied by Fay and Joebert [8]. Further honest submodules were conferred by Jara[6] with respect to a collcection of submodules. Again the notion of super-honest submodules was studied by Joubert and Schoeman [7] and Cheng [2]. Saikia and Hazarika[3] extended the concept of super-honesty in modules to nearing groups. The honest and super-honest character provides a new ore domain. In this paper we define the structures complete-closed, complete-honest and strict-honest structures in near ring and investigate their various characteristics. Torsion and closed character of an N-subgroup exhibit the relation with strict-honest, complete-honest and super-honest N-subgroups.

[1] *corresponding author*

2010 *Mathematics Subject Classification.* 16Y30.

Key words and phrases. near-ring groups, super-honest, strict-honest, c-closed, c-torsion N-subgroups.

2. Preliminaries

All basic concepts used in this paper are available in Pilz [4]. Throughout the paper we consider N a zero symmetric right near-ring with unity 1 and E a left N-group.

Definition 2.1. *N-subgroups A, B of E are such that $A \subseteq B$ then A is essential in B when any non-zero N-subgroup C of E contained in B has a nonzero intersection with A. In such cases B is an essential extension of A. If $B = E$ we say that A is an essential N-subgroup of E.*

Definition 2.2. *An N-subgroup M of E is called essentially closed if whenever C is an N-subgroup of E such that M is essential in C then $C = M$.*

Definition 2.3. *The set $(B : a)$ is defined as $(B : a) = \{n \in N| \quad na \in B\}$.*

Proposition 2.1. *If B is an N-subgroup of E and $a \in E$ then $(B : a)$ is an N-subgroup of N.*

Corollary 2.1. *If B is an N-subgroup of N and $a \in N$ then $(B : a)$ is an N-subgroup of N.*

Definition 2.4. *N-subgroup B of N-group E is called c-closed or complete-closed N-subgroup of E, if for any $x \in E, (B : x) = N$, then $x \in B$.*

Example 1. *We consider the near-ring $N = \{0, 1, 2, 3, 4, 5\}$ under the addition module 6 and multiplication defined in the following table:*

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	4	4	0	4	1
2	0	2	2	0	2	2
3	0	0	0	0	0	3
4	0	4	4	0	4	4
5	0	2	2	0	2	5

Then $(N, +, .)$ is a near-ring. If $A = \{0, 3\}$ then A is N-subgroup of $_NN$. Since $(A : e) = N$ and for $e = 0$ and 3. So A is c-closed.

Definition 2.5. *Complete-Torsion or c-torsion of N-group E is the subset $\{e \in E \quad | \quad (0 : e) = N\}$ and is denoted by $^CT(E)$.*

Definition 2.6. *If* ${}^{C}T(E) = 0$*,* E *is called c-Torsion free* N*-group.*

Example 2. *We consider the near-ring* $N = \{0, 1, 2, 3, 4, 5, 6, 7\}$ *under the addition module 7 and multiplication defined in the following table:*

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7

Here $(0 : e) = N$ only for $e = 0$. So ${}^{C}T(E) = 0$, E is c-torsion free N-group.

Proposition 2.2. *If* N *is distributively generated, then* ${}^{C}T(E)$ *is* N*-subgroup of* E*.*

Proof. Let $e_1, e_2 \in {}^{C}T(E) \Rightarrow (0 : e_i) = N$ for $i = 1, 2$ i.e $ne_1 = 0, ne_2 = 0$ for all $n \in N$. Since N is distributively generated, $n(e_1 - e_2) = 0$. So $e_1 - e_2 \in {}^{C}T(E)$. Again let $e \in {}^{C}T(E)$, so $ne = 0$ for all $n \in N$. Now for $a \in N$, $na \in N$ and we get $(na)e = 0$ for all $n \in N \Rightarrow n(ae) = 0$ for all $n \in N$. So for $a \in N, ae \in {}^{C}T(E)$. Thus ${}^{C}T(E)$ is N-subgroup of E. □

Definition 2.7. *Complete-closure of* M *in* E *is the subset* $\{e \in E| \quad (M : e) = N\}$ *and we denote it by* ${}^{C}Cl(M)$*. So* M *is c-closed if* ${}^{C}Cl(M) = M$*.*

Lemma 2.1. *If* N *is distributively generated then for any left* N*-group* E *and any* N*-subgroup* $M \subseteq E$ *we have* ${}^{C}Cl(M)$ *is an* N*-subgroup of* E*.*

Proof. Let $e_1, e_2 \in {}^{C}Cl(M)$, then $(M : e_1) = N$ and $(M : e_2) = N$ i.e $ne_1, ne_2 \in M$ for all $n \in N$. As N is distributively generated, $n(e_1 + e_2) \in M$. Then $(e_1 + e_2) \in {}^{C}Cl(M)$. Let $x \in {}^{C}Cl(M)$ and $n_1 \in N$, then for all $n \in N, nx \in M$. We have $n(n_1x) = (nn_1)x \in M$, so we have $n_1x \in {}^{C}Cl(M)$ for any $n_1 \in N$. Thus ${}^{C}Cl(M)$ is an N-subgroup of E. □

Definition 2.8. *The set of torsion elements of* E *,* $T_N(E) = \{e \in E| \quad (0, e) \neq 0\}$

Definition 2.9. *If $T_N(E) = E$, E is called torsion N-group.*

Example 3. $N = \{0, a, b, c\}$ *is the Klein's four group with multiplication*

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	0
c	a	a	a	a

Then $(N, +, .)$ is a near-ring. In ${}_NN$, $T_N({}_NN) = {}_NN$. So ${}_NN$ is torsion N-group.

Definition 2.10. *If $T_N(E) = 0$, E is called torsion free N-group.*

Example 4. $N = \{0, a, b, c\}$ *is the Klein's four group with multiplication*

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c

Then $(N, +, .)$ is a near-ring. In ${}_NN$, $T_N({}_NN) = 0$. So ${}_NN$ is torsion free N-group.

Definition 2.11. [5] *An N-subgroup (ideal) M is called super-honest in E if $x \in E \setminus M$ for $n \in N, nx \in M \Rightarrow n = 0$. If B is an N-subgroup (ideal) of N then B is called a super-honest N-subgroup (ideal) of N if B is super-honest N-subgroup (ideal) of N considered N as N-group ${}_NN$.*

3. Strict-honest and c-honest N-subgroups and their characterstics

In this section we characterize complete-honest, strict-honest N-subgroups with complete-closed and torsion structures.

Definition 3.1. *Let $K \subseteq E$ be an N-subgroup of an N-group E. We say K is complete-honest or c-honest N-subgroup of E, if for some $x \in E$, $\forall\, n \in N, nx \in K$, and $\exists\, n \in N$, such that $nx \neq 0$, then $x \in K$.*

Example 5. *Let $N = Z_6$ is a set with operations '+' as addition modulo 6 and '.' defined by following table:*

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	5	5	3	1	1
2	0	4	4	0	2	2
3	3	3	3	3	3	3
4	0	2	2	0	4	4
5	3	1	1	3	5	5

Then (Z_6, +, .) is a near-ring. If $A = \{0, 3\}$ then A is N-subgroup of $_NN$. For $e =$ 0 and 3, $\forall n \in N, ne \in A$ and $ne \neq 0$ for $n =$ 1, 3, 5. So A is c-honest.

Definition 3.2. *Let $K \subseteq E$ be an N-subgroup of an N-group E. We say K is strict-honest N-subgroup of E, if for any $x \in E, n \in N, \quad nx(\neq 0) \in K$ then $x \in K$.*

Remark 3.1. *K is c-closed in E implies K is c-honest in E. But if K is c-honest in E, then K may not be c-closed in E. And K is strict-honest in E implies K is c-honest in E. But if K is c-honest in E, then K may not be strict-honest in E.*

Example 6. *We consider the near-ring $N = \{0, a, b, c, x, y\}$ under the addition and multiplication defined as Example 1.1 [9]. If $A = \{0, a\}$ then A is N-subgroup of $_NN$. For any $n \in N, \quad ne(\neq 0) \in A$ for $e = a$. So A is strict-honest. Similarly $\{0, b\}$ and $\{0, c\}$ are also strict-honest N-subgroups of $_NN$.*

Lemma 3.1. *For N-subgroups H and M of E, we get the following statements.*

(a) *If H is strict-honest in M and M is strict-honest in E then H is strict-honest in E.*

(b) *M is strict-honest in E if and only if M/H is strict-honest in E/H, where H is ideal of E.*

(c) *If H is ideal of E and H, M/H are strict-honest in E/H, then M is strict-honest in E.*

Proof. (a) We consider $e \in E, n \in N$ such that $ne(\neq 0) \in H$. Then $ne \in M$ [since $H \subseteq M$] hence $e \in M$ as M is strict-honest in E. But $e \in M$, $ne \in H \Rightarrow e \in H$ as H is strict-honest in M. So $e \in E$, $n \in N$ such that $ne \in H \Rightarrow e \in H$ gives H is strict-honest in E.
(b) We consider $e \in E$, $n \in N$ such that $n(e+H)(\neq \bar{0}) \in M/H$. Then $ne(\neq 0) \in M$, hence $e \in M$. We get $e + H \in M/H$. So M/H is strict-honest in E/H.
(c) Let $e \in E$, $n \in N$ be such that $ne(\neq 0) \in M$. If $ne \in H$ then $e \in H \subseteq M$. If $ne \neq H$, then $(\bar{0} \neq)n(e + H) \in M/H$, hence $(e + H) \in M/H$ and we get e $\in M$. ☐

Remark 3.2. *The same results follow for c-closed N-subgroups also.*

Lemma 3.2. *Let $M \subseteq E$ be an N-subgroup then For $m \in ({}^{C}Cl(M) \setminus M)$ if have $Nm \cap M = 0$ we get M is c-honest in E.*

Proof. For any $m \in ({}^{C}Cl(M) \setminus M)$ and $Nm \cap M = 0$, to show M is c-honest in E. Let for all $n \in N$ and $e \in E$, $ne \in M$ and for some $n \in N$, $nx(\neq 0)$. To show $e \in M$. If possible $e \notin M$, but $e \in {}^{C}Cl(M)$. Then $Ne \cap M = 0$ is a contradiction, since for all $n \in N$, $ne \in M$. So $e \in M$, which implies M is c-honest in E. ☐

Lemma 3.3. *Let $M \subseteq E$ be an N-subgroup then For $m \in ({}^{C}Cl(M) \setminus M)$, if M is strict-honest in E, we have $Nm \cap M = 0$.*

Proof. We consider M is strict-honest in E and $m \in ({}^{C}Cl(M) \setminus M)$. If possible $P(\neq 0) \in Nm \cap M \Rightarrow P \in Nm$ and $p \in M \Rightarrow P = n_1 m$ for some $n_1 \in N$ and $p \in M$. As $m \in {}^{C}Cl(M)$ for all $n \in N$, $nm \in M$, so $n_1 m \in M$. As M is strict-honest in E and $n_1 m \in M$, $m \notin M$ we must get $n_1 m = 0$. i.e. $P = 0$, a contradiction. ☐

Lemma 3.4. *Let $M \subseteq E$ be an N-subgroup. If M is strict-honest in E then for $m \in ({}^{C}Cl(M) \setminus M)$ we get $Ann(m) = N$.*

Proof. Let $x \in {}^{C}Cl(M)$. So for all $n \in N, nx \in M$, then $nx = 0$ [since M is strict-honest in E, $x \notin M$] i.e. $n \in Ann(x)$ i.e. $N = Ann(x)$. ☐

Lemma 3.5. *Let $M \subseteq E$ be an strict-honest N-subgroup, then ${}^{C}Cl(M) = M \cup {}^{C}T(E)$.*

Proof. Let $x \in {}^{C}Cl(M)$. Since $M \subseteq {}^{C}Cl(M)$ As ${}^{C}Cl(M) = \{e \in E| \quad (M : e) = N\}$ and M is N-subgroup of E . So if $x \in M$ done. If $x \notin M$ as $x \in {}^{C}Cl(M)$, for all $n \in N$, $0 = nx \in M$ [Since M is strict-honest in E] $\Rightarrow x \in {}^{C}T(E)$. Thus $x \in M \cup {}^{C}T(E)$.

Conversely, let $x \in M \cup {}^{C}T(E) \Rightarrow$ if $x \in M$ then $x \in {}^{C}Cl(M)$ is obvious. $x \in {}^{C}T(E) \Rightarrow$ for all $n \in N, nx = 0 \in M \Rightarrow x \in {}^{C}Cl(M)$. So ${}^{C}Cl(M) = M \cup {}^{C}T(E)$ □

Corollary 3.1. *Let $M \subseteq E$ be an strict-honest N-subgroup such that ${}^{C}T(E) \subseteq M$, then it is c-closed. In particular, if E is torsion free then an N-subgroup $M \subseteq E$ is c-closed if it is strict-honest.*

Proof. By Lemma 3.5 if $M \subseteq E$ is strict-honest then ${}^{C}Cl(M) = M \cup {}^{C}T(E) = M$ [as ${}^{C}T(E) \subseteq M$]. So M is c-closed. In particular E is torsion free $\Rightarrow {}^{C}T(E) = 0$. Then we get $M \subseteq E$ is strict-honest which implies ${}^{C}Cl(M) = M \cup {}^{C}T(E) = M$. Thus M is c-closed in E. □

Corollary 3.2. *Let $M \subseteq E$ be an N-subgroup such that ${}^{C}T(E) \subseteq M$, then M is c-honest N-subgroup iff it is c-closed. In particular, if E is torsion free then an N-subgroup $M \subseteq E$ is c-honest iff it is c-closed.*

Proof. Since M strict-honest$\Rightarrow$ M c-honest and M is c-closed $\Rightarrow$ M c-honest. □

Corollary 3.3. *Any strict-honest N-subgroup $M \subseteq E$ satisfies either $M \subseteq {}^{C}T(E)$ or ${}^{C}T(E) \subseteq M$, if N is distributively generated.*

Proof. Since M is strict-honest in E, so ${}^{C}Cl(M) = M \cup {}^{C}T(E)$. ${}^{C}Cl(M)$ and ${}^{C}T(E)$ are subgroups. Hence either $M \subseteq {}^{C}T(E)$ or ${}^{C}T(E) \subseteq M$, as union of two subgroups is subgroup if one contain the other. □

Corollary 3.4. $(\neq 0)M \subseteq E$ *is strict-honest and if M is c-torsion free then E is c-torsion free and* $(\neq 0)M \subseteq E$ *is c-closed where N is distributively generated.*

Proof. Since $(\neq 0)M \subseteq E$ is strict-honest so by corollary 3.3 either $M \subseteq {}^{C}T(E)$ or ${}^{C}T(E) \subseteq M$. First to show ${}^{C}T(E) = 0$, if M is c-torsion free. Let $x(\neq 0) \in {}^{C}T(E)$ $\Rightarrow$for all $n \in N$, $nx = 0$. Now if $x \in M$, for all $n \in N$, $nx = 0$ only for $x = 0$ [since M is c-torsion free], a contradiction. Again let $(\neq 0)x \notin M$, so we get $M \subseteq {}^{C}T(E)$. For this condition for any $y \in M \Rightarrow y \in {}^{C}T(E) \Rightarrow ny = 0$ for all $n \in N$. But as M is c- torsionfree, $ny = 0$ for all $n \in N$ only for $y = 0$, a contradiction. So $x = 0$ which implies E is c-torsion free. Next to show $M \subseteq E$ is c-closed. Let $x \in E$, such that for all $n \in N$, $nx \in M$. Now E is c-torsion-free, so $(0 : x) = N$ only for $x = 0$, which gives for all $n \in N$, $nx = 0 \in M$ for $x = 0 \in M$. Again M is strict-honest in E so M is c-honest in E , so for all $n \in N$, $nx(\neq 0) \in M \Rightarrow x \in M \Rightarrow M \subseteq E$ is c-closed. □

Lemma 3.6. *Let $\{M_\lambda : \lambda \in \Lambda\}$ be a family of strict-honest N-subgroups of E, then $\cap_\lambda M_\lambda$ is strict-honest.*

Proof. Let $e \in E$, $n \in N$ be such that $ne(\neq 0) \in \cap_\lambda M_\lambda$, Therefore $ne \in M_\lambda, \forall\lambda \Rightarrow e \in M_\lambda$ [since M_λ strict-honest, $\forall\lambda$] $\Rightarrow e \in \cap_\lambda M_\lambda$. □

The intersection of all strict-honest N-subgroups of E is the smallest strict-honest N-subgroup of E. We denote it by S. If $S \subset E$, then E has proper strict-honest N-subgroups, otherwise E is the only strict-honest N-subgroup of E itself.

Lemma 3.7. *If E and $E^{'}$ are N-groups, f is a N-homomorphism from E to $E^{'}$, then for each strict-honest N-subgroup $B^{'}$ of $E^{'}$, f^{-1}($B^{'}$) is a strict-honest N-subgroup of E.*

Proof. Let for some $n \in N$, $x \in E$, $nx(\neq 0) \in f^{-1}(B^{'})$. Then $f(nx) = nf(x)(\neq 0) \in B^{'}$, $f(x) \in B^{'}$. Since $B^{'}$ is strict-honest in $E^{'}$, it follows that $x \in f^{-1}\ (B^{'})$. Hence $f^{-1}\ B^{'}$ is strict-honest in E. □

Corollary 3.5. *If S is the smallest strict-honest N-subgroup of an N-group E, then for each N-endomorphism f of E, $f^{-1}(S) \supset S \supset f(S)$.*

Proof. Since $f^{-1}(S)$ is a strict-honest N-subgroup of E, $f^{-1}(S) \supset S$. Hence $P \supset f(P)$. □

4. Relations of Strict-Honest and Super-honest N-subgroups

In this section we study the relation between strict-honest N-subgroups and super-honest N-subgroups with help of the structure essentially closed N-subgroups.

Lemma 4.1. *If M is super-honest in E then M is strict-honest in E.*

Proof. To show M is strict-honest in E. So for some $n \in N$ and $e \in E$. $0 \neq ne \in M$ to show $e \in M$. If possible $e \notin M$. Then $n = 0$ [since M is super-honest in E], a contradiction as $0 \neq ne$. □

Lemma 4.2. *If M is an N-subgroup of E such that ${}^{C}T(E) \subseteq M$, then M is an essential N-subgroup of ${}^{C}Cl(M)$.*

Proof. Let A be N-subgroup of ${}^{C}Cl(M)$. We assume $A \cap M = 0$. Let $m \in A$, then $m \in {}^{C}Cl(M)$ implies $nm \in M$, for all $n \in N$. Also $nm \in A \quad \forall n \in N$, implies $nm \in M \cap A = 0$. This gives $nm = 0$ for all $n \in N$. Thus $m \in {}^{C}T(E) \subseteq M$. So $m \in A \cap M \Rightarrow m = 0$. This gives $A = 0$. Thus M is an essential N-subgroup of ${}^{C}Cl(M)$. □

Lemma 4.3. *Let M be an N-subgroup of E. Then M is essentially closed N-subgroup of E satisfying ${}^{C}T(E) \subseteq M$ implies M is c-closed.*

Proof. If M is a essentially closed N-subgroup of E such that ${}^{C}T(E) \subseteq M$, then by above we get M is c-closed. □

Theorem 4.1. *If M is an ideal of an N-group E then M is super-honest in E if and only if M is essentially closed in E ,${}^{C}T(E) \subseteq M$ and ${}^{C}T(E/M) \supset T_N(E/M)$.*

Proof. M is a super-honest N-subgroup of E implies M is essentially closed [4]. Again $a \in T_N(E) \Rightarrow (0 : a) \neq 0 \Rightarrow x(\neq 0) \in (0 : a)$ so $xa = 0$. If $a \in M$ then it is done. If $a \notin M$ i.e. $a \in E \setminus M$ then $x = 0$ [since M is super-honest in E]. Hence contradiction to $x \neq 0$. So $a \in M$. Thus $T_N(E) \subseteq M$. And so ${}^{C}T(E) \subseteq M$, because ${}^{C}T(E) \subseteq T_N(E)$. Now $T_N(E/M) = \bar{0}$. Since $T_N(E) = \{a \in E \quad | \quad (0 : a) \neq 0\}$.So $T_N(E/M) = \{\bar{a} \in E/M \quad | \quad (\bar{0} : \bar{a}) \neq 0\}$. Let $\bar{a} \in T_N(E/M)$, $a \notin M \Rightarrow (\bar{0} : \bar{a}) \neq 0 \Rightarrow \exists \quad x(\neq 0)$ such that $x \in (\bar{0} : \bar{a}) \Rightarrow x\bar{a} = \bar{0} \Rightarrow xa + M = M \Rightarrow xa \in M$ where $a \notin M \Rightarrow x = 0$, since M is super-honest in E. Therefore $\forall \bar{a} \in E/M$, $(\bar{0} : \bar{a}) = 0$ and so $T_N(E/M) = \bar{0} = M$. So ${}^{C}T(E/M) \supset T_N(E/M)$ holds as $\bar{x} \in T_N(E/M) = M \Rightarrow x \in M \Rightarrow nx \in M$ for all $n \in N \Rightarrow x + M = M = \bar{0}$ for all $n \in N \Rightarrow \bar{x} \in {}^{C}T(E/M)$.
Next consider $a \in E \setminus M$ with $na \in M$ for some $n \in N$. If $n \neq 0$, then $n\bar{a} = na + M \in M = \bar{0} \Rightarrow n \in (\bar{0} : \bar{a}) \Rightarrow \bar{a} \in T_N(E/M)$. So $\bar{a} \in {}^{C}T(E/M)$. Thus $(n\bar{a}) = \bar{0}$, for all $n \in N$. i.e. $na \in M$ for all $n \in N$. So $a \in {}^{C}Cl(M) = M$ [by Lemma 4.3], a contradiction . □

Corollary 4.1. *If M is an ideal of an N-group E then M is super-honest in E if and only if M c-closed and ${}^{C}T(E/M) \supset T_N(E/M)$.*

Corollary 4.2. *If M is an c-closed ideal of E and ${}^{C}T(E/M) \supset T_N(E/M)$ then M is strict-honest in E.*

Theorem 4.2. *Let $M \subseteq E$ be an strict-honest ideal such that ${}^{C}T(E) \subseteq M$ and ${}^{C}T(E/M) \supset T_N(E/M)$, then M is super-honest in E.*

Proof. As $M \subseteq E$ is strict-honest ideal such that ${}^{C}T(E) \subseteq M$, so it is c-closed [by Corollary 3.1] and again ${}^{C}T(E/M) \supset T_N(E/M)$, so M is super-honest in E [by Corollary 4.1]. □

ACKNOWLEDGMENT

Our sincere acknowledgement to Prof. Helen K. Saikia, Gauhati University.

REFERENCES

[1] A. ABIANS, D. REINHART: *Honest subgroups of abelian groups*, Rend. Circ Math. Palermo, **12** (1963), 35—356.

[2] M.-H. CHANG: *Note on Superhonest modules,* Chin. j. of math., **14** (1986), 109–119.

[3] H. K. SAIKIA, N. HAZARIKA: *Super–honest N-subgroups*, Adv.in Alg., **5** (2012), 31–41.

[4] G. PILZ: *Near-rings* , North Holland publishing Company, Amsterdam, 1977.

[5] A. W. GOLDIE: *Torsion free modules and rings*, Journal of Algebra, **1** (1964), 268–287.

[6] P. JARA: *Honest submodules*, Czech. Math. Jour., Oxford, **57** (2007), 225–241.

[7] S. V. JOUBERT, M. J. SCHOEMAN: *Super honestry for Modules and Abelian groups,* Chinese J. of Math., **13** (1985), 1–13.

[8] T. H. FAY, S. V. JOUBERT: *Isolated submodules and skew fields*, Applied Categorical Structures, **8** (2000), 317–326.

[9] N. HAZARIKA, H. K. SAIKIA: *Singular and Semi-simple character in E-injective N-groups with weakly descending chain conditions,* Africa Mathematica, **29** (2018), 1065–1072.

DEPARTMENT OF MATHEMATICS
THE ASSAM ROYAL GLOBAL UNIVERSITY
GUWAHATI-781035, INDIA
E-mail address: navalakhmi@gmail.com

DEPARTMENT OF MATHEMATICS
THE ASSAM ROYAL GLOBAL UNIVERSITY
GUWAHATI-781035, INDIA
E-mail address: narayan.nayak05@gmail.com