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ENCOURAGED OR DISCOURSED ARRIVALS OF AN $M/M/1/N$ QUEUEING SYSTEM WITH MODIFIED RENEGING

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ABSTRACT. In this paper, we develop a finite capacity single server Markovian queueing system with encouraged or discouraged arrivals and a modified reneging policy of the customers. The Customer arrivals follow Poisson process with mean arrival rate λ and service times follow exponential distribution with parameter μ. We develop the steady state equations, these equations are solved using iterative process. Performance measures of the system are derived under steady state. Also, numerical computations are presented.

1. INTRODUCTION

The primary aim of this paper is to explore the steady state behavior of the queueing model with encouraged or discouraged arrivals and impatient customers. To attract a greater number of customers and hence to improve the business, organizations will encourage the arrivals of the customers by giving a variety of offers. This type of encouraged arrival pattern is termed as reverse balking. During this time customers are willing to stay for a longer period of time in the queue. So even if the queue length increases the chance of leaving the queue without getting the service will be low and this phenomenon is called reverse reneging. But when the queue length reaches a threshold value say, k, customer arrivals will be discouraged by long queues. As a result, the arriving

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customer either decides not to join the queue (balk) on anticipating a long wait or depart after joining the queue(renege) on getting impatient due to waited for a long time in the queue.

Som and Seth [10] presented the steady state analysis of a finite capacity $M/M/1/N$ queue considering the encouraged arrivals and reverse reneging They obtained some system performance measures under steady state. One may refer [5] and [6] for related papers. .Ammar et al. (4) studied the transient solution for the busy period of an $M/M/1$ queueing model where balking and reneging can occur only when the queue size exceeds a threshold value k. Vasanta Kumar et al.[7] studied an $M/M/1$ queueing system with customer reneging during vacation and breakdown times under multiple vacation policy. Swathi and Vasanta Kumar [8] considered a Markovian queueing system with reneging under single and multiple vacation policies. Swathi et al. [9] considered an $M/M/1$ queuing system with server vacations, breakdowns under the scenario of customer balking and reneging. Related work on reneging under different vacation policies may be found in [1], [2] and [3].

In this paper we present the steady state analysis of an $M/M/1/N$ queueing model with encouraged arrivals and reverse reneging as long as the queue length is less than or equal to a threshold value k, later with customer balking and reneging as a result of customer impatience on anticipation of long wait. Rest of the paper is organized as follows: The mathematical model is presented in Section 2. Steady state equations and performance measures are obtained in Section 3 and the effect of variation in the system parameters on the performance measures are studied through numerical examples in Section 4. Conclusions are presented in Section 5, followed by important references.

2. MATHEMATICAL MODEL

We consider a single server queueing system with finite capacity N. Customer arrivals follow Poisson process with mean arrival rate λ. Customers are served on first-cum first-served discipline with the service times following an exponential distribution with parameter μ. As long as the queue length is less than or equal to the threshold value k, the arrivals are as per the encouraged arrival rate $\lambda(1+\eta)$ and the customer will wait for a longer length of time T_1 in the queue. If it has not begun by that time, he will leave the queue without getting service.

This time T_1 follows an exponential distribution with parameter α. Average rate of reneging of the customer is given by

$$R_1(n) = (N-(n-1))\alpha, 1 \leq n \leq k.$$

When the queue length exceeds the threshold k, the new arrival joins the queue with probability p and balk with probability $(1-p)$. After joining the queue, each customer will wait a certain length of time T_2 for service to begin. In this case the tolerance time T_2 is assumed to follow an exponential distribution with parameter ξ . Average rate of reneging of the customer is given by

$$R_2(n) = (n-k)\xi, k+1 \leq n \leq N.$$

3. Steady state equations and Performance measures of the model

Let P_n be the probability that there are n customers in the system. The steady state equations governing the system are given below:

(3.1) $$\lambda(1+\eta)P_0 = (\mu + N\alpha)P_1$$

(3.2) $$[\mu+\lambda(1+\eta)+(N-(n-1))]P_n = \lambda(1+\eta)P_{n-1}+(\mu+(N-n)\alpha)P_{n+1}, 1 \leq n \leq k-1.$$

(3.3) $$(\lambda p+\mu)P_k = \lambda(1+\eta)P_{k-1} + (\mu+\xi)P_{k+1},$$

(3.4) $$[\lambda p+\mu+(n-k)\xi]P_n = \lambda p P_{n-1} + (\mu+((n-(k-1))\xi)P_{n+1}, k+1 \leq n < N$$

(3.5) $$[\mu+(N-k)\xi]P_N = \lambda p P_{N-1}.$$

On solving the equations (3.1) to (3.5) iteratively, we get

(1) The probability of n units in the system is given by

$$P_n = \frac{\lambda^n(1+\eta)^n}{\Pi_{i=1}^{n}(\mu+(N-(i-1))\alpha)}P_0,$$

when $1 \leq n \leq k$, and

$$P_n = \frac{\left[\sum_{l=0}^{n-(k-1)}\left\{(\lambda p)^l \prod_{i=1}^{n-l-(k+1)}(\mu+i\xi)\right\}\right](k-1)\alpha + (\lambda p)^{n-k}}{R\prod_{i=1}^{n-k}(\mu+i\xi)}\lambda^k(\lambda p)^k P_0,$$

when $k+1 \leq n < N$. Here, $R = \prod_{i=1}^{k}(\mu+(N-(i-1))\alpha)$.

(2) The probability that the system is full that is N units in the system is given by

$$P_N = \frac{\lambda p}{(\mu + (N-k)\xi)} P_{N-1}.$$

Using the normalizing condition

$$P_0 + \sum_{n=1}^{k} P_n + \sum_{n=k+1}^{N} P_n = 1 \ ,$$

we get

$$P_0 = \left[1 + L1 + (L2 + L3)\lambda^k (1+\eta)^k\right]^{-1} ,$$

where

$$L_1 = \sum_{n=1}^{k} \frac{\lambda^n (1+\eta))^n}{\prod_{i=1}^{n} (\mu + (N-(i-1))\alpha)}$$

$$L_2 = \frac{\sum_{n=k+1}^{N-1} \left[\sum_{l=0}^{n-(k+1)} \left((\lambda p)^{l \prod_{i=1}^{n-l-(k+1)} (\mu + i\xi)}\right)\right] (k-1)\alpha + (\lambda p)^{n-k}}{R \prod_{i=1}^{n-k} (\mu + i\xi)}$$

and

$$L_3 = \frac{\lambda p}{(\mu + (N-k)\xi)}$$

$$\cdot \frac{\left\{\sum_{l=0}^{N-(k+2)} (\lambda p)^{l \prod_{i=1}^{N-l-(k+2)} (\mu + i\xi)}\right\} (k-1)^{\alpha} + (\lambda p)^{N-(k+1)}}{R \ \prod_{i=1}^{N-k} (\mu + i\xi)}.$$

(3) Expected number of units in the system during the reverse balking is given by

$$L(S_1) = \sum_{n=1}^{k} n P_n.$$

(4) Expected number of units in the system during balking and reneging is given by

$$L(S_2) = \sum_{n=k+1}^{N} n P_n.$$

(5) Expected number of units in the system is given by

$$L(S) = L(S_1) \ + L(S_2).$$

(6) Expected waiting time in the system in the respective stages is given by

$$W(S_1) = \frac{L(S_1)}{(\lambda\,(1+\eta))}$$

and

$$W(S_2) = \frac{L(S_2)}{(\lambda p)}.$$

(7) The total waiting time in the system is given by

$$W(S) = W(S_1) + W(S_2).$$

4. Numerical examples

In this section, the expected queue length and waiting time in the queue are computed and tabulated for specific values of the system parameters.

The following values of the parameters are considered in all the tables, while changing the corresponding parameter of the table:

$$\lambda = 0.6, \mu = 0.9, \eta = 0.2, \alpha = 0.6, \xi = 0.3, p = 0.8, k = 5, N = 10.$$

TABLE 1. Expected system length and expected waiting time in the system for variation in k, λ and μ

k	$L(S)$	$W(S)$	λ	$L(S)$	$W(S)$	μ	L_s	$W(S)$
3	0.3305	0.6060	0.3	0.0882	0.2901	0.7	2.5322	5.1893
4	0.7185	1.4138	0.4	0.2228	0.6172	0.8	1.8636	3.7980
5	1.4065	2.8472	0.5	0.5783	1.3648	0.9	1.4065	2.8472
6	2.2571	4.6067	0.6	1.4065	2.8472	1.0	1.0861	2.1810
7	2.7171	5.5774	0.7	3.1371	5.5167	1.1	0.8565	1.7043
8	2.0791	4.2483	0.8	6.4561	10.0001	1.2	0.6886	1.3550

It is observed from the Table 1 that, with increase in the values of k , both expected system length and waiting time in the system increases up to k=7 and then decreases, this is because of balking and reneging after queue length exceeds k, whereas they increase with increase in the values of $lambda$, and decrease with increase in the values of μ. One may notice from the table 2 that, with increase in the values of η and α , both expected system length and waiting time increases, whereas they decrease with increase in ξ . It can be verified from

TABLE 2. Expected system length and expected waiting time in the system for variation in η , α and ξ

η	L(S)	W(S)	α	L(S)	W(S)	ξ	L(S)	W(S)
0.10	0.9495	1.9166	0.3	0.8218	1.5470	0.2	3.2308	6.6477
0.15	1.1592	2.3431	0.4	0.9950	1.9480	0.3	1.4065	2.8472
0.20	1.4065	2.8472	0.5	1.1941	2.3882	0.4	0.7343	1.4467
0.25	1.0958	3.4390	0.6	1.4065	2.8472	0.5	0.4446	0.8432
0.30	2.0323	4.1289	0.7	1.6266	3.3176	0.6	0.3050	0.5524
0.35	2.4215	4.9283	0.8	1.8516	3.7951	0.7	0.2319	0.4000

TABLE 3. Expected system length and expected waiting time in the system for variation in p and N

p	L(S)	W(S)	N	L(S)	W(S)
0.3	1.0345	5.2487	8	2.6674	4.4522
0.4	1.0954	4.2319	9	1.1938	4.4779
0.5	1.1624	3.2677	10	1.4065	2.8472
0.6	1.2362	2.0178	11	0.7676	1.5239
0.7	1.3173	2.8472	12	0.3874	0.7380

the Table 3 that both expected system length and waiting time increases with increase in p and decreases with increase in N.

5. CONCLUSION

An $M/M/1/N$ queueing system with encouraged or discouraged arrivals with modified reneging policy is studied. Using the steady state equations explicit expressions for the expected system length and expected waiting time in the system are obtained and the variation in their numerical values for changes in the values of the system parameters is studied.

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