UNAVAILABILITY OF THE MACHINES IN TWO STAGE FLOW SHOP SCHEDULING MODEL TO MINIMIZE RENTAL COST WITH JOB BLOCK CRITERIA AND TRANSPORTATION TIME

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ABSTRACT. This paper deals to minimize the rental cost under specified rental policy for n jobs, 2-machines flow shop scheduling problem including breakdown interval and job block criteria. Further, the transportation time of moving the jobs from one machine to another machine is considered. The objective of the paper is to find an algorithm to minimize the rental cost under specified rental policy with breakdown interval, job block criteria and transportation time. The method is made clear with the help of a numerical example.

1. INTRODUCTION

Scheduling problems involves placing items (jobs) in a certain sequence (order) for service. For example in general, in a job-shop there are n-jobs and m different machines and each job must be processed on m machines in the same order with no passing between machines. The problem before us is how to order the jobs on all the machines so that the total elapsed time i.e. the time between the start of first job on the first machine and the completion of last job on the last machine is minimum. The scheduling problem practically depends upon the important factors namely, transportation time(which includes loading time, moving time and unloading time), Job block criteria (which is due to priority of one job over the another) and machines break down (due to failure of a component of machines for certain interval of time or the machines are supposed

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to stop their working for a certain interval due to some external imposed policy such as stop of flow of electric current to the machines may be a government policy due to shortage of electricity production. Before 1954, the concept of breakdown of machines had not considered by any author. In 1954 Johnson had considered the effect of breakdown of machines on the completion times of jobs in an optimal sequence. Maggu and Dass (1977) introduced the concept of equivalent job blocking in the theory of scheduling . Singh T.P. studied scheduling problem in wider sense involving the concept of job block criteria, transportation time and break down interval. T.P. Singh, D. Gupta studied $n \times 2$ general flow shop problem to minimize the rental cost under predefined rental policy.

D. Gupta, P. Sehgal discussed unavailability in nx2 flow shop scheduling to minimize rental cost with job block criteria.

In the present paper we extend the study made by D. Gupta and P. Sehgal (2018) by introducing the concept of transportation time.

2. Practical Situation

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machines. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up gradation to new technology. Further, when the machines on which jobs are to be processed are planted at different places, the transportation time which include the loading time, moving time and unloading time etc. has significant role in production concern. Sometimes the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria becomes important. The break down of the machines (due to delay in material, tool unavailability, failure of electric

current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) has significant role in the production concern.

3. NOTATIONS

S: Sequence of jobs $1, 2, 3, \ldots, n$.
A_i: Processing time of ith job on machine A.
B_i: Processing time of ith job on machine B.
t_i: transportation time of ith job from machine A to machine B
β: Equivalent job for job-block.
L: Length of the break-down interval.
$A_i^{'}$: Expected processing time of ith job after break-down effect on machine A.
$B_i^{'}$: Expected processing time of ith job after break-down effect on machine B.
S_i: Sequence obtained from Johnson's procedure to minimize rental cost.
C_j: Rental cost per unit time of machine j.
U_i: Utilization time of B (2 nd machine) for each sequence Si
$T_1(S_i)$: Completion time of last job of sequence Si on machine A.
$T_2(S_i)$: Completion time of last job of sequence Si on machine B.
$R(S_i)$: Total rental cost for sequence Si on all machines.
$C_T(S_i)$: Completion time of 1st job of each sequence Si on machine A.

4. ASSUMPTIONS

1. We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.
2. Jobs are independent of each other.
3. Machine break down interval is deterministic, .i.e. the break-down intervals are well known in advance. This simplifies the problem by ignoring the stochastic cases where the break-down interval is random.
4. Pre-emption is not allowed .i.e. once a job started on a machine, the process on the machine can't be stopped unless the job is completed.

5. PROBLEM FORMULATION

Let n jobs are to be processed on two machines in the order A B. Let A_i and B_i be the processing time of i-th jobs on machine A and B respectively. Let t_i

$(i = 1, 2, \ldots, n)$ be the transportation time of moving the i-th job from machine A to machine B. The mathematical model of the given problem in matrix can be stated as: Let the machines cease working in the time interval (a, b) for the

TABLE 1. Mathematical Model

Jobs	Machine A		Machine B
i	A_i	t_i	Bi
1	A_1	t_1	B_1
2	A_2	t_2	B_2
3	A_3	t_3	B_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	A_n	t_n	B_n

length of time L = (b-a) due to break down of machines. Let β (k,m) be the equivalent job block. Now, our objective is to obtain the optimal sequence of jobs in order to minimize the rental cost under specified rental policy.

6. ALGORITHM

Based on the equivalent job block theorem by Maggu & Das(1977) and by considering the effect of break-down interval (a ,b) on different jobs, the algorithm which minimize the total rental cost of machines under specified rental policy with the minimum makespan can be depicted as below:

Step 1: Define two fictitious machines G and H with their processing times Gi and Hi respectively follows:
$$G_i = A_i + t_i$$
$$H_i = B_i + t_i$$

Step 2: Take an equivalent job β=(k,m) and define processing time as follows:
$$G_\beta = G_k + G_m - min(G_m, H_k)$$
$$H\beta = H_k + H_m - min(G_m, H_k)$$

Step 3: Define a new reduced problem with processing time G_i and H_i where job block (k,m) is replaced by single equivalent job β with processing time G_β and H_β as obtained in step2.

Step 4: Apply Johnson(1954) technique and obtain an optimal sequence of given jobs.

Step 5: Prepare a flow time table for the sequence obtained in step 4 and read

the effect of break-down interval (a ,b) on different jobs.

Step 6: Form a reduced problem with processing times A_i and B_i If the break-down interval (a, b) has effect on job i then

$A_i' = A_i' + L, B_i' = B_i' + L$ where L = b-a, the length of break-down interval. If the break-down interval (a, b) has no effect on job i then $A_i' = A, B_i' = B_i$.

Step 7: Now repeat the procedure from step 1 to step 4 to get the sequence S_i.

Step 8: Observe the processing time of 1st job of S1 on the first machine A. Let it be α.

Step 9: Obtain all the jobs having processing time on A greater than α. Put these job one by one in the 1 st position of the sequence S_1 in the same order. Let these sequences be $S_2, S_3, S_4, \ldots, S_5$.

Step 10: Prepare in-out flow table only for those sequence Si (i=1, 2,âĂęr)which have job block β(k, m) and evaluate total completion time of last job of each sequence, i.e., $T_1(S_i)$ & $T_2(S_i)$ on machine A and B respectively.

Step 11: Evaluate completion time CT (S_i) of 1st job of each of above selected sequence S_i on machine A.

Step 12: Calculate utilization time Ui of 2nd machine for each of above selected sequence S_i as:
$U_i = T_2(S_i) - C_T(S_i) for i = 1, 2, 3, r.$

Step 13: Find Min U_i, i=1, 2, âĂęr. let it be corresponding to i = m, then S_m is the optimal sequence for minimum rental cost.

Min rental cost $= T_1(S_m) \times C_1 + U_m \times C_2,$
where C_1 and C_2 are the rental cost per unit time of 1st and 2nd machines. respectively.

7. NUMERICAL ILLUSTRATION

Consider 5 jobs and 2 machines problem to minimize the rental cost. The processing times with transportation time from one machine to another machines are given as in Table 2.

Rental costs per unit time for machines M_1 and M_2 are 12 and 13 units respectively and jobs block(2,5) is considered as equivalent job β. Also given that the breakdown interval is (50,60).

Step 1: $G_i = A_i + t_i$ $H_i = B_i + t_i$ (Table 3)

TABLE 2

Jobs	A_i	t_i	B_i
1	11	6	9
2	10	5	8
3	12	4	10
4	13	3	12
5	15	4	9

TABLE 3

Jobs	G_i	H_i
1	17	15
2	15	13
3	16	14
4	16	15
5	19	13

Step 2&3: The processing times of equivalent job block $\beta = (2,5)$ by using Maggu and Das criteria are given by:

$G_\beta = 15 + 19\text{-}13 = 21$ and $H_\beta = 13 + 13\text{-}13 = 13$

TABLE 4

Jobs	G_i	H_i
1	17	15
β	21	13
3	16	14
4	16	15

Step 4: Using Johnson's two machines algorithm, the optimal sequence is

S = 1, 4, 3, β i.e. S = 1, 4, 3, 2, 5.

Step 5: The in-out flow table for the sequence S = 1- 4- 3- 2- 5 is as in Table 5.

Step 6: On considering the effect of break down interval (50, 60) the new reduced problem is given as in Table 6.

TABLE 5

Jobs	A In-out	t_i	B In-out
1	0-11	6	17-26
4	11-24	3	27-39
3	24-36	4	40-50
2	36-46	5	51-59
5	46-61	4	65-74

TABLE 6

Jobs	A	t_i	B
1	11	6	9
2	10	5	18
3	12	4	20
4	13	3	12
5	25	4	9

Step 7: Now on repeating the procedure to get the optimal sequence for modified scheduling problem (see Table 7).

TABLE 7

Jobs	G_i	H_i
1	17	15
2	15	23
3	16	24
4	16	15
5	19	13

The processing times of equivalent job block $\beta = (2,5)$ by using Maggu and Das criteria are given by:

$G_\beta = 15 + 29 - 23 = 21$ and $H_\beta = 23 + 13 - 23 = 13$ (Table 8)

Using Johnson's two machines algorithm, the optimal sequence is

$S_1 = $ 4-3-1-2- 5

Step 8: The processing time of 1st job on S1 = 6, i.e., $\alpha = 6$.

Step 9: The other optimal sequences for minimizing rental cost are

TABLE 8

Jobs	G_i	H_i
1	17	15
β	15	23
3	16	24
4	16	15

S_2 = 3-4-1-2 -5; S_3 = 1-4-3-2-5;
S_4 = 2-4-3-1-5; S_5 = 5-4-3-1-2

Step 8: The in-out flow tables for sequences S1, S2 ,S3, S4 and S5 are as follows:
For S_1 =4-3-1-2-5.

TABLE 9

Jobs	A In-out	t_i	B In-out
4	0-13	3	16-28
3	13-25	4	30-50
1	25-36	6	50-59
2	36-46	5	59-77
5	46- 71	4	77- 86

Total elapsed time on machine A = $T_1(S_1)$ =71.
Total elapsed time on machine B = $T_2(S_1)$ = 86.
Utilization time of 2nd machine (B)= U_1 = 86-16 = 70units; S_2 = 3-4-1-2-5.

TABLE 10

Jobs	A In-out	t_i	B In-out
3	0-12	4	16-36
4	12-25	3	36-48
1	25-36	6	48-57
2	36-46	5	57-75
5	46- 71	4	75- 84

Total elapsed time on machine A = $T_1(S_2)$ = 71.
Total elapsed time on machine B = $T_2(S_2)$ = 84.
Utilization time of 2nd machine (B)= U_2 = 84-16 = 68units; S_3 = 1-4-3-2-5.
Total elapsed time on machine A = $T_1(S_3)$ = 71.

TABLE 11

Jobs	A In-out	t_i	B In-out
1	0-11	6	17-26
4	11-24	3	27-39
3	24-36	4	40-60
2	36-46	5	60-78
5	46- 71	4	78- 87

Total elapsed time on machine B = $T_2(S_3)$ = 87.
Utilization time of 2nd machine (B)= U_3 = 87-17 = 70units; S_4 = 2-4-3-1-5.

TABLE 12

Jobs	A In-out	t_i	B In-out
2	0-10	5	15-33
4	10-23	3	33-45
3	23-35	4	45-65
1	35-46	6	65-74
5	46- 71	4	74- 84

Total elapsed time on machine A = $T_1(S_4)$ = 71.
Total elapsed time on machine B = $T_2(S_4)$ = 84.
Utilization time of 2nd machine (B)= U_4 = 84-15 = 69units; S_5 = 5-4-3-1-2.

TABLE 13

Jobs	A In-out	t_i	B In-out
5	0-25	4	29-38
4	25-38	3	41- 53
3	38-50	4	54- 74
1	50-61	6	74- 83
2	61- 71	5	83-101

Total elapsed time on machine A = $T_1(S_5)$ = 71.
Total elapsed time on machine B = $T_2(S_5)$ = 101.
Utilization time of 2nd machine (B)= U_5 = 101-29 = 72units.

The total utilization of machine A is fixed 71 units and minimum utilization of B is 68 units for the sequence S_2. Therefore the optimal sequence is S_2 = 3-4-1-2-5.

Therefore minimum rental cost is = 71 x 12 + 68 x 13 = 1736 units.

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