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# BIPOLAR VALUED FUZZY $d$-ALGEBRA

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ABSTRACT. In this paper, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we discuss the relation between bipolar fuzzy subalgebra and their level cuts. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

## 1. INTRODUCTION

The concept of fuzzy subsets of a set was introduced by Zadeh, L.A. [9] in 1965. After that, there are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, etc. In fuzzy sets the membership degree of elements range over the interval [0,1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set and membership degree 0 indicates that an element does not belong to fuzzy set. The membership degrees on the interval (0,1) indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. The concept of bipolar-valued fuzzy sets, first introduced by Zhang, W.R. [10] in 1994, is an

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extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property.

Neggers, J. and Kim, H.S. [8] introduced and studied the concept of $d$-algebra, which is another generalization of BCK-algebras and investigated relations between $d$-algebras and BCK-algebras. After that, Jun, Y.B., Neggers, J. and Kim, H.S. [7] introduced the concepts of fuzzy $d$-subalgebra, fuzzy $d$-ideal and fuzzy $d^*$-ideal, and investigated relations among them. Further, they discussed $d$-ideals in $d$-algebras.

In this paper, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we discuss the relation between bipolar fuzzy $d$-algebra and their level cuts. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

## 2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

**Definition 2.1.** *( [8]) A nonempty set $X$ with a constant $0$ and a binary operation $*$ is called a $d$-algebra, if for all $x, y \in X$ it satisfies the following axioms:*

*(dA1)* $x * x = 0$

*(dA2)* $0 * x = 0$

*(dA3)* $x * y = 0$ *and* $y * x = 0 \Rightarrow x = y$.

**Definition 2.2.** *( [8]) Let $Y$ be a non-empty subset of a $d$-algebra $X$, then $Y$ is called subalgebra of $X$ if $x * y \in Y$, for all $x, y \in Y$.*

**Definition 2.3.** *( [8]) Let $X$ and $Y$ be two $d$-algebras. A mapping $f : X \to Y$ is called a homomorphism if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$.*

**Definition 2.4.** *( [9]) Let $X$ be a non-empty set. A fuzzy subset $\mu$ of the set $X$ is a mapping $\mu : X \to [0, 1]$.*

**Definition 2.5.** *( [10]) Let $X$ be the universe of discourse. A bipolar-valued fuzzy set $\mu$ in $X$ is an object having the form $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$, where $\mu^- : X \to [-1, 0]$ and $\mu^+ : X \to [0, 1]$ are mappings.*

*For the sake of simplicity, we shall use the symbol $\mu = (X; \mu^-, \mu^+)$ for the bipolar-valued fuzzy set $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$ and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.*

**Definition 2.6.** *( [10]) Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set and $s \times t \in [-1, 0] \times [0, 1]$, the sets $\mu_s^N = \{x \in X/\mu^-(x) \leq s\}$ and $\mu_t^P = \{x \in X/\mu^+(x) \geq t\}$ are called negative $s$-cut and positive $t$-cut respectively. For $s \times t \in [-1, 0] \times [0, 1]$, the set $\mu_{(s,t)} = \mu_s^N \cap \mu_t^P$ is called $(s, t)$-set of $\mu = (X; \mu^-, \mu^+)$.*

**Definition 2.7.** *Let $\mu = (X; \mu^-, \mu^+)$ and $\sigma = (X; \sigma^-, \sigma^+)$ be two bipolar fuzzy sets of a universe of discourse $X$.*
*The intersection of $\mu$ and $\sigma$ is defined as*
$(\mu^- \cap \sigma^-)(x) = \min\{\mu^-(x), \sigma^-(x)\}$ *and* $(\mu^+ \cap \sigma^+)(x) = \min\{\mu^+(x), \sigma^+(x)\}$.
*The union of $\mu$ and $\sigma$ is defined as*
$(\mu^- \cup \sigma^-)(x) = \max\{\mu^-(x), \sigma^-(x)\}$ *and* $(\mu^+ \cup \sigma^+)(x) = \max\{\mu^+(x), \sigma^+(x)\}$.
*A bipolar set $\mu$ is contained in another bipolar set $\sigma$, $\mu \subseteq \sigma$ if and only if $\mu^-(x) \geq \sigma^-(x)$ and $(\mu^+(x) \leq \sigma^+(x)$, for all $x \in X$.*

**Definition 2.8.** *Let $f : X \to Y$ be a homomorphism from a set $X$ onto a set $Y$ and let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set of $X$ and $\sigma = (Y; \sigma^-, \sigma^+)$ be two bipolar fuzzy set of $Y$, then the homomorphic image $f(\mu)$ of $\mu$ is $f(\mu) = ((f(\mu))^-, (f(\mu))^+)$ defined as for all $y \in Y$.*

$$(f(\mu))^-(x) = \begin{cases} \max\{\mu^-(x)/x \in f^{-1}(y) & \textit{if } f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

*and*

$$(f(\mu))^+(x) = \begin{cases} \max\{\mu^-(x)/x \in f^{-1}(y) & \textit{if } f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}.$$

*The pre-image $f^{-1}(\sigma)$ of $\sigma$ under $f$ is a bipolar set defined as $(f^{-1}(\sigma))^-(x) = \sigma^-(f(x))$ and $(f^{-1}(\sigma))^+(x)) = \sigma^+(f(x))$, for all $x \in X$.*

## 3. Bipolar Fuzzy $d$-algebra

In this section, we introduce and study the concept of bipolar fuzzy subalgebra of $d$-algebra and we characterize bipolar fuzzy subalgebra to the crisp $d$-algebra. Further, we prove that the homomorphic image and inverse image of a bipolar fuzzy subalgebra is a bipolar fuzzy subalgebra.

Throughout this section $X$ stands for a $d$-algebra unless otherwise mentioned. Now, we introduce the following.

**Definition 3.1.** *A Bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ in $X$ is called a bipolar fuzzy subalgebra if it satisfies the following properties: for any $x, y \in X$,* $(i)$. $\mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\}$
$(ii)$. $\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\}$.

**Example 1.** *Consider a $d$-algebra $X = \{0, 1, 2, 3\}$ with the following Cayley table*

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 3 | 3 | 0 |

*Define a bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$, where $\mu^- : X \to [-1, 0]$ and $\mu^+ : X \to [0, 1]$ as*

$$\mu^-(x) = \begin{cases} -0.7 & \text{when } x = 0 \\ -0.2 & \text{when } x \neq 0 \end{cases}$$

*and*

$$\mu^+(x) = \begin{cases} 0.8 & \text{when } x = 0 \\ 0.1 & \text{when } x \neq 0 \end{cases}.$$

*Then $\mu$ is a bipolar fuzzy subalgebra.*

**Proposition 3.1.** *If $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra of $X$, then $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$, for all $x \in X$.*

*Proof.* Let $x \in X$. Now, $\mu^-(0) = \mu^-(x * x) \leq \max\{\mu^-(x), \mu^-(x)\} = \mu^-(x)$ and $\mu^+(0) = \mu^+(x * x) \geq \min\{\mu^+(x), \mu^+(x)\} = \mu^+(x)$. Thus $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$. □

**Theorem 3.1.** *Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set. Then the two level cuts $\mu^N_{s_1}, \mu^P_{t_1}$ and $\mu^N_{s_2}, \mu^P_{t_2}$ are equal i.e., $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$ if and only if there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$.*

*Proof.* Suppose $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$. Suppose if possible there exist $x \in X$ such that

$$s_1 \geq \mu^-(x) \geq s_2 \quad \text{and} \quad t_1 \leq \mu^+(x) \leq t_2.$$

Now, $\mu^-(x) \leq s_1 \Rightarrow x \in \mu^N_{s_1} = \mu^N_{s_2} \Rightarrow \mu^-(x) \leq s_2$ and $\mu^+(x) \leq t_2 \Rightarrow x \in \mu^P_{t_2} = \mu^P_{t_1} \Rightarrow \mu^+(x) \leq t_1$, which is a contradiction. Thus there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$.

Conversely, suppose that there is no $x \in X$ such that $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$. Suppose $\mu^N_{s_1} \neq \mu^N_{s_2}$ and $\mu^P_{t_1} \neq \mu^P_{t_2}$. That implies there exist $x \in \mu^N_{s_1} \& x \notin \mu^N_{s_2}$ and there exist $y \in \mu^P_{t_1} \& y \notin \mu^P_{t_2}$. This implies $\mu^-(x) \leq s_1 \& \mu^-(x) \geq s_2$ and $\mu^+(x) \geq t_1 \& \mu^+(x) \leq t_2$, i.e., $s_1 \geq \mu^-(x) \geq s_2$ and $t_1 \leq \mu^+(x) \leq t_2$. Which is a contradiction. Thus $\mu^N_{s_1} = \mu^N_{s_2}$ and $\mu^P_{t_1} = \mu^P_{t_2}$. □

**Theorem 3.2.** *A bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ of $X$ is a bipolar fuzzy subalgebra of $X$ if and only if the level cuts are subalgebras i.e., for all $s \times t \in [-1, 0] \times [0, 1], \emptyset \neq \mu^N_s$ and $\emptyset \neq \mu^P_t$ are subalgebras of $X$.*

*Proof.* Suppose $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra. Let $s \times t \in [-1, 0] \times [0, 1]$ such that $\mu^N_s \neq \emptyset$ and $\mu^P_t \neq \emptyset$. Let $x, y \in \mu^P_t$ and $g, h \in \mu^N_s$. Therefore $\mu^+(x) \geq t, \mu^+(y) \geq t, \mu^-(g) \leq s$ and $\mu^-(h) \leq s$. Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra, we have

$(i)$ $\mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\} \geq t$ and
$(ii)$ $\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\} \geq s \Rightarrow x * y \in \mu^P_t$ and $g * h \in \mu^N_s$.

Thus $\mu^N_s$ and $\mu^P_t$ are subalgebras of $X$.

Conversely suppose that the level cuts $\mu^N_s$ and $\mu^P_t$ are subalgebras of $X$. Let $x, y \in X$. Then $\mu^+(x), \mu^+(y) \in [0, 1]$ and $\mu^-(x), \mu^-(y) \in [-1, 0]$. Choose $t = \min\{\mu^+(x), \mu^+(y)\}$ and $s = \max\{\mu^-(x), \mu^-(y)\}$. That implies $\mu^+(x) \geq t, \mu^+(y) \geq t, \mu^-(x) \leq s$ and $\mu^-(y) \leq s$, i.e., $x, y \in \mu^P_t$ and $x, y \in \mu^N_s$, and further $x * y \in \mu^P_t$ and $x * y \in \mu^N_s$, $\mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\}$ and $\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\}$.

Thus $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra. □

**Theorem 3.3.** *Let $Y$ be a subalgebra of $X$, then for any $s \times t \in [-1, 0] \times [0, 1]$ there exist a bipolar fuzzy subalgebra $\mu$ of $X$ such that $\mu^N_s = Y$ and $\mu^P_t = Y$.*

*Proof.* : Let $Y$ be a sub algebra of $X$. Define a bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ as

$$\mu^-(x) = \begin{cases} -1 & \text{if } x \in Y \\ s & \text{if } x \notin Y \end{cases}$$

and

$$\mu^+(x) = \begin{cases} t & \text{if } x \in Y \\ 0 & \text{if } x \notin Y \end{cases}.$$

Clearly $\mu_s^N = Y$ and $\mu_t^P = Y$ Let $x, y \in X$. If $x, y \in Y$, then $x * y \in Y$. So, $\mu^-(x) = \mu^-(y) = \mu^-(x * y) = -1$ and $\mu^+(x) = \mu^+(y) = \mu^+(x * y) = t$. Therefore, $(i).\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\}$ and $(ii)$ $\mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\}$.

If $x, y \notin Y$, then $\mu^-(x) = \mu^-(y) = s$ and $\mu^+(x) = \mu^+(y) = 0$.
Therefore, $(i)$ $\mu^-(x*y) \leq \max\{\mu^-(x), \mu^-(y)\}$ and $(ii)$ $\mu^+(x*y) \geq \min\{\mu^+(x), \mu^+(y)\}$.
If at most one of $x, y \in Y$, then at least one of $\mu^-(x) \& \mu^-(y)$ is equal to $s$ and $\mu^+(x) \& \mu^+(y)$ is equal to $0$. Therefore,$(i).\mu^-(x * y) \leq \max\{\mu^-(x), \mu^-(y)\}$ and $(ii)$ $\mu^+(x * y) \geq \min\{\mu^+(x), \mu^+(y)\}$.

Thus $\mu$ bipolar fuzzy subalgebra of $X$ such that $\mu_s^N = Y$ and $\mu_t^P = Y$. □

**Theorem 3.4.** *If $\mu = (X; \mu^-, \mu^+)$ and $\sigma = (X; \sigma^-, \sigma^+)$ are two bipolar fuzzy subalgebras, then $\mu \cap \sigma$ is a bipolar fuzzy subalgebra.*

*Proof.* Let $x, y \in X$. Now,

$$\begin{aligned}(\mu^- \cap \sigma^-)(x * y) &= \min\{\mu^-(x * y), \sigma^-(x * y)\} \\ &\leq \min\{\max\{\mu^-(x), \mu^-(y)\}, \max\{\sigma^-(x), \sigma^-(y)\} \\ &\leq \min\{\max\{\mu^-(x), \sigma^-(x), \max\{\mu^-(y), \sigma^-(y)\} \\ &= \min\{(\mu^- \cap \sigma^-)(x), (\mu^- \cap \sigma^-)(y)\}.\end{aligned}$$

Also,

$$\begin{aligned}(\mu^+ \cap \sigma^+)(x * y) &= \min\{\mu^+(x * y), \sigma^+(x * y)\} \\ &\geq \min\{\min\{\mu^+(x), \mu^+(y)\}, \min\{\sigma^+(x), \sigma^+(y)\} \\ &\geq \min\{\max\{\mu^+(x), \sigma^+(x), \max\{\mu^+(y), \sigma^+(y)\} \\ &= \min\{(\mu^+ \cap \sigma^+)(x), (\mu^+ \cap \sigma^+)(y)\}.\end{aligned}$$

Thus $\mu \cap \sigma$ is bipolar fuzzy subalgebra. □

**Corollary 3.1.** *The intersection of arbitrary family of bipolar fuzzy subalgebras is a bipolar fuzzy subalgebra.*

In general union of two fuzzy $\Gamma$-semirings may not be a fuzzy $\Gamma$-semiring.

**Example 2.** *Consider a d-algebra* $X = \{0, 1, 2\}$ *with the following Cayley table*

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 1 | 1 | 2 |

*Define a bipolar fuzzy set* $\mu = (X; \mu^-, \mu^+)$*, where* $\mu^- : X \to [-1, 0]$ *and* $\mu^+ : X \to [0, 1]$ *as*

$$\mu^-(x) = \begin{cases} -0.7 & \text{if } x = 0 \\ -0.5 & \text{if } x = 1 \\ -0.4 & \text{if } x = 2 \end{cases} \quad \text{and} \quad \mu^+(x) = \begin{cases} 0.9 & \text{if } x = 0 \\ 0.8 & \text{if } x = 1 \\ 0.6 & \text{if } x = 2 \end{cases}.$$

*Define a bipolar fuzzy set* $\sigma = (X; \sigma^-, \sigma^+)$*, where* $\sigma^- : X \to [-1, 0]$ *and* $\sigma^+ : X \to [0, 1]$ *as*

$$\sigma^-(x) = \begin{cases} -0.7 & \text{if } x = 0 \\ -0.5 & \text{if } x = 1 \\ -0.4 & \text{if } x = 2 \end{cases} \quad \text{and} \quad \sigma^+(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.7 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \end{cases}.$$

*Clearly,* $\mu$ *and* $\sigma$ *are bipolar fuzzy subalgebras. Here* $(\mu^+ \cup \sigma^+)(1 * 0) = 0.6$ *is not greater than or equal to* $0.8 = \min\{\mu^+ \cup \sigma^+)(1), (\mu^+ \cup \sigma^+)(0)\}$*. Therefore* $\mu^+ \cup \sigma^+$ *is not a bipolar fuzzy subalgebra. Thus union of bipolar fuzzy subalgebras is not a bipolar fuzzy subalgebra.*

In particular, we have the following theorem.

**Theorem 3.5.** *Let* $\mu = (X; \mu^-, \mu^+)$ *and* $\sigma = (X; \sigma^-, \sigma^+)$ *be two bipolar fuzzy subalgebras, then* $\mu \cup \sigma$ *is a bipolar fuzzy sub algebra only if* $\mu \subseteq \sigma$ *or* $\sigma \subseteq \mu$*.*

*Proof.* Suppose $\mu \subseteq \sigma$. Let $x, y \in X$. Now,

$$\begin{aligned}(\mu^- \cap \sigma^-)(x * y) &= \max\{\mu^-(x * y), \sigma^-(x * y)\}\\ &= \sigma-(x * y)\\ &\leq \max\{\sigma^-(x), \sigma^-(y)\}\\ &\leq \max\{\max\{\mu^-(x), \sigma^-(x)\}, \max\{\mu^+(y), \sigma^-(y)\}\\ &= \max\{(\mu^- \cup \sigma^-)(x), (\mu^- \cup \sigma^-)(y)\}.\end{aligned}$$

Also,

$$\begin{aligned}(\mu^+ \cap \sigma^+)(x * y) &= \max\{\mu^+(x * y), \sigma^+(x * y)\}\\ &= \sigma^+(x * y)\\ &\geq \min\{\sigma^+(x), \sigma^+(y)\}\\ &\geq \min\{\max\{\mu^+(x), \sigma^+(x)\}, \max\{\mu^+(y), \sigma^+(y)\}\}\\ &= \max\{(\mu^+ \cap \sigma^+)(x), (\mu^+ \cap \sigma^+)(y)\}.\end{aligned}$$

Similarly, we can prove if $\sigma \subseteq \mu$. Thus $\mu \cup \sigma$ is bipolar fuzzy subalgebra. □

**Theorem 3.6.** *Let $f$ be a homomorphism from a $d$-algebra $X$ onto a $d$-algebra $Y$. Let $\sigma$ be a bipolar fuzzy subalgebra of $Y$, then the pre-image $f^{-1}(\sigma)$ of $\sigma$ is a bipolar fuzzy subalgebra of $X$.*

*Proof.* Let $x, y \in X$. Now,

$$\begin{aligned}(f^{-1}(\sigma))^-(x * y) &= \sigma^-(f(x * y))\\ &\leq \sigma^-(f(x) * f(y))\\ &\leq \max\{\sigma^-(f(x)), \sigma^-(f(y))\}\\ &= \max\{(f^{-1}(\sigma))^-(x), (f^{-1}(\sigma))^-(y)\}.\end{aligned}$$

Also,

$$\begin{aligned}(f^{-1}(\sigma))^+(x * y) &= \sigma^+(f(x * y))\\ &\geq \sigma^+(f(x) * f(y))\\ &\geq \min\{\sigma^+(f(x)), \sigma^+(f(y))\}\\ &= \min\{(f^{-1}(\sigma))^+(x), (f^{-1}(\sigma))^+(y)\}.\end{aligned}$$

Thus $f^{-1}(\sigma)$ is a bipolar fuzzy sub algebra of $X$. □

**Theorem 3.7.** *Let $f$ be a homomorphism from a $d$-algebra $X$ onto a $d$-algebra $Y$. Let $\mu$ be a bipolar fuzzy subalgebra of $X$, then the homomorphic image $f(\mu)$ of $\mu$ is a bipolar fuzzy subalgebra of $Y$.*

*Proof.* Let $x, y \in Y$. Suppose neither $f^{-1}(x)$ nor $f^{-1}(y)$ is non-empty. since $f$ is homomorphism and so there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$ it follows that $a * b \in f^{-1}(x * y)$. Now,

$$\begin{aligned}
(f(\mu))^-(x * y) &= \max\{\mu^-(z)/z \in f^{-1}(x * y)\} \\
&\leq \max\{\mu^-(a * b)/a \in f^{-1}(x), b \in f^{-1}(y)\} \\
&\leq \max\{\max\{\mu^-(a), \mu^-(b)\}/a \in f^{-1}(x), b \in f^{-1}(y)\} \\
&= \max\{\max\{\mu^-(a)/a \in f^{-1}(x)\}, \max\{\mu^-(b)/b \in f^{-1}(y)\}\} \\
&= \max\{(f(\mu))^-(x), (f(\mu))^-(y)\}.
\end{aligned}$$

Also,

$$\begin{aligned}
(f(\mu))^+(x * y) &= \max\{\mu^+(z)/z \in f^{-1}(x * y)\} \\
&\geq \max\{\mu^+(a * b)/a \in f^{-1}(x), b \in f^{-1}(y)\} \\
&\geq \max\{\min\{\mu^+(a), \mu^+(b)\}/a \in f^{-1}(x), b \in f^{-1}(y)\} \\
&= \min\{\max\{\mu^+(a)/a \in f^{-1}(x)\}, \max\{\mu^+(b)/b \in f^{-1}(y)\}\} \\
&= \min\{(f(\mu))^+(x), (f(\mu))^+(y)\}.
\end{aligned}$$

Thus $f(\mu)$ is a bipolar fuzzy subalgebra of $Y$. □

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