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SYNTHESIS METHODS OF OPTIMAL DISCRETE CORRECTIVE FUNCTIONS

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ABSTRACT. In this work, the task of constructing the optimal corrective function for the control material is solved; classes of functions of discrete functions depending on n variables that satisfy the restrictions of I-conservation of given sets and II-monotonicity conditions are studied in detail. In section 1, we give the necessary definitions and information from the theory of functions of discrete functions depending on n variables and the statement of the problem. In section 2 describes the construction of an optimal corrector for various combinations of restrictions I, II or the absence of restrictions. In section 3 we solve the problem of constructing an optimal corrector for various combinations of constraints I-II. Constraint II defines the condition of monotonicity. Therefore, the main task of this section is to construct a monotonic function delivering an extremum to a linear functional φ and monotonic on the entire lattice E_n^k; an algorithm for constructing such a function is also indicated here. In section 4 we prove theorems justifying the application of the constructed algorithm. The algorithm is practically effective. It is built in a form convenient for computer implementation.

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1. STATEMENT OF THE PROBLEM OF CORRECTING DISCRETE FUNCTIONS DEPENDING ON N VARIABLES

Consider the set $P_k^n-$of all functions depending on n variables and taking values from the set $S = [0, 1, \ldots, k-1]$. Consider also the totality of subsets $L_1, L_2, \ldots, L_t$ of a set . They say that a family $[f]$ of functions $k-$ of significant logic preserves the totality L, if it follows $\alpha_i \in L_j$, $i = 1, n$. from the condition that $f(\alpha_1, \alpha_2, \ldots, \alpha_n) \in L_j$, $j = \overline{1, t}$. In what follows, we will consider $[f]$ as a set of discrete functions depending on n variables that preserve $L^* = [[0], \ldots, [k-1], [0,1], \ldots, [0, k-1]]$.

Let a partial order be given on the set:

$$0 < 1, 0 < 2, \ldots, 0 < k-1 \tag{1.1}$$

In the set S^n- of sets $\tilde{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\alpha_l \in [0, 1, \ldots, k-1]$, order (1.1) induces a partial order:

$$\tilde{\beta} = (\beta_1, \beta_2, \ldots, \beta_n) \leq \tilde{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n), \tag{1.2}$$

if $\beta_i < \gamma_i$ by (1.1), $i = \overline{1, n}$.

Definition 1.1. *A function $f(\tilde{x})$ from is monotonic in order* (1.1) *if, for any tuples $\tilde{\alpha}$ and $\tilde{\beta}$ from S^n such that $\tilde{\alpha} \leq \tilde{\beta}$, it is true $f(\tilde{\alpha}) \leq f\left(\tilde{\beta}\right)$.*

Definition 1.2. *A function $f(\tilde{x})$ from P_k^n is called symmetric if it does not change its value under any permutations of variables.*

Let be $f(\tilde{x}) \in P_k^n$, $\tilde{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $f(\tilde{\alpha}) \in \beta$, $\beta \in [0, 1, \ldots, k-1]$. Let also the coordinates $\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_p}$ in the set $\tilde{\alpha}$ are not equal β. Consider the set $[\tilde{\gamma}]$ of sets obtained from the set $\tilde{\alpha}$ by substituting β the value in the place of any of $\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_p}$.

Definition 1.3. *A function $f(\tilde{x}) \in P_k^n$ satisfies constraint IV if $f(\tilde{\gamma}) = \beta$, $\tilde{\gamma} \in [\tilde{\gamma}]$ and $\beta \in [0, 1, \ldots, k-1]$.*

Definition 1.4. *The family of functions that preserve the set L^* is called σ_1; monotonic functions - by a class σ_2; preserving L^* and monotonous - by class σ_3; symmetrical - by class σ_4; satisfying the restriction IV - by the class σ_5 preserving L^* and symmetric - by the class σ_6; preserving L^*, monotonic and symmetric - by class σ_7; preserving L^*, satisfying constraint IV, monotone and symmetric - by class σ_8.*

Obviously $\sigma_3 = \sigma_1 \cap \sigma_2$, $\sigma_6 = \sigma_1 \cap \sigma_4$, $\sigma_7 = \sigma_2 \cap \sigma_6$, $\sigma_8 = \sigma_5 \cap \sigma_7$

Definition 1.5. *Two sets $\tilde{\alpha}$, $\tilde{\beta}$ of S^n are called comparable if $\tilde{\alpha} \geq \tilde{\beta}$ or $\tilde{\beta} \geq \tilde{\alpha}$.*

Definition 1.6. *A set $\tilde{\alpha}$ of $M \subseteq S^n$ is called maximal (minimal) in M if there is no set $\tilde{\beta}$ of M such that $\tilde{\beta} \geq \tilde{\alpha}$ $\left(\tilde{\beta} \leq \tilde{\alpha}\right)$.*

Definition 1.7. *In the set $M \subseteq S^n$, a set $\tilde{\alpha}$ immediately follows $\tilde{\beta}$, if $\tilde{\alpha}$ and $\tilde{\beta}$ are comparable and there does not exist such that $\tilde{\alpha} \leq \tilde{\gamma} \leq \tilde{\beta}$.*

Consider the structure S^n generated by order $0 < 1, 0 < 2, \ldots, 0 < k-1$. In S^n there is a single minimal element $\tilde{0} = (0, \ldots, 0)$ and $(k-1)^n$ incomparable maximum elements: sets, all coordinates of which belong to the set $[1, 2, \ldots, k-1]$.

Divide into levels $U_0, U_1, \ldots, U_n$. The level U_j is composed of sets in which j the coordinates take values from $[1, 2, \ldots, k-1]$, and the rest $(n-j)$ of the coordinates are zero. It is obvious that $U_0 = [(0, \ldots, 0)]$, U_n it consists of all the maximum elements S^n, and the power U_j is equal $C_n^j (k-1)^j$.

According to (1.2), in S^n a chain we will call a set $\left\{\tilde{\alpha}_{i_1}, \tilde{\alpha}_{i_2}, \ldots, \tilde{\alpha}_{i_p}\right\}$ such that $\left\{\tilde{\alpha}_{i_1} < \tilde{\alpha}_{i_2} < \ldots < \tilde{\alpha}_{i_p} \text{ and } \tilde{\alpha}_{i_j} \in U_{i_j},\ j = \overline{0, p},\ 1 \leq p \leq n+1,\ i_j = i_{j-1} + 1\right\}$.

Definition 1.8. *Sets $\tilde{\alpha}$, $\tilde{\beta}$ are called quasi-comparable if there is $\tilde{\gamma}$ a set such that $\tilde{\gamma} \geq \tilde{\alpha}$ and $\tilde{\gamma} \geq \tilde{\beta}$.*

When defining classes $\sigma_1, \sigma_2, \ldots, \sigma_8$, we assume that functions from these classes are defined on the whole set S^n. Such functions are called defined everywhere. Let an arbitrary function $f(\tilde{x})$ from P_k^n:

$$f(\tilde{x}) = \begin{cases} 0, & \text{if } x \in A_0, \\ 1, & \text{if } x \in A_1, \\ 2, & \text{if } x \in A_2, \\ \vdots \ \ldots & \ldots \ \ldots \ \ldots \\ k-1, & \text{if } \tilde{x} \in A_{k-1}; \end{cases}$$

here $A_0, A_1, \ldots, A_{k-1} \subseteq S^n$; $(A_1 \cup A_2 \cup \ldots \cup A_{k-1}) \cap A_0 = \emptyset$ and $A_i \cap A_j = \emptyset$, i, $j = \overline{1, k-1}$, $i \neq j$ (generally speaking $S^n \backslash (A_0 \cup \ldots \cup A_{k-1}) \neq \emptyset$).

Definition 1.9. *Not everywhere a certain function $f(\tilde{x})$ belongs to a class σ_i, $i = \overline{1, 8}$. if there is an everywhere defined function $g(\tilde{x}) \in \sigma_i$ for which $A_0 \subseteq \{\tilde{\alpha} : g(\tilde{\alpha}) = 0\}$, $A_1 \subseteq \{\tilde{\alpha} : g(\tilde{\alpha}) = 1\}, \ldots, A_{k-1} \subseteq \{\tilde{\alpha} : g(\tilde{\alpha}) = k-1\}$.*

It should be noted that $f(\tilde{x})$ for $\sigma_1, \sigma_3, \sigma_6, \sigma_7, \sigma_8$ out takes place $\tilde{0} \in A_0, \ldots$ $\ldots, (k-1) \in A_{k-1}$. We denote $E^n(0,1), \ldots, E^n(0, k-1)$ by the set of all length sets whose coordinates take values from the sets $[0,1], \ldots, [0, k-1]$, respectively.

Let $S_1^n = S^n \backslash \left(E^n(0,1) \bigcup \ldots \bigcup E^n(0, k-1)\right)$.

Definition 1.10. *An algorithm A that calculates the predicate P value from an object S is called incorrect if the result of the calculation can be one of the following values: 0 - refusal to calculate; 1 - property is fulfilled $1; \ldots; k-1$ property is fulfilled $k-1$.*

Usually considered recognition algorithms that calculate P for an object S are incorrect [1-2].

Formulation of the problem. Let a set of tasks $[Z]$, algorithms $[A]$ for solving problems $[Z]$, many $[R(Z)]$ solutions to problems $[R_A(Z)]$ and many solutions Z using algorithms $[A]$ from. It is not necessary that $R_A(Z) = R(Z)$.

The last statement is equivalent to the statement that the algorithms $[A]$ are heuristic or incorrect.

Consider an operator F with a scope $[R_{A_1}(Z)] \times \ldots \times [R_{A_m}(Z)]$ and a scope. $[R(Z)]$.

In other words, F it translates the solution of the problem Z obtained by the algorithms $A_1, A_2, \ldots A_m$ into an element of the set $\tilde{R}(Z)$, which is also called the solution to Z.

The quality of correction is determined by the distance between the sets $\left[\tilde{R}(Z)\right]$ and $[R(Z)]$.

The distance can be set in various ways, which leads to various mathematical problems. Obviously, the main problem is the construction of an optimal corrector F, that is, a corrector that minimizes the above distance [3-8]. To solve this problem, it is necessary to assign some information $J(Z)$ about the tasks from $[Z]$ the presented to the solution. In addition, you must specify exactly what heuristic information A will be used. We denote such information by $J(A)$.

Variants of mathematical settings are possible. The sets $[J(Z)]$, $[J(A)]$, $[F]$ – the set of admissible correctors are given, and the functional of the quality of adjustment φ is determined.

(i) Indicate the algorithms $A_1, A_2, \ldots A_m$ and the corrector F on which the lower bound of the quality functional is implemented.

(ii) For given heuristics $A_1, A_2, \dots A_m$, find the minimizing corrector F.

This article discusses the second problem, as well as some generalizations that occupy intermediate positions between the first and second tasks.

2. Constructing optimal corrections under constraints not related to the concept of monotony

Consider a special type of adjustment quality functionals - linear quality functionals. Let the predicate $P(S) = \alpha$ and incorrect algorithms $A_1, A_2, \dots, A_n$ be calculated $P(S)$ equal to $\alpha_1, \alpha_2, \dots, \alpha_n$ and $F(\alpha_1, \alpha_2, \dots, \alpha_n) = \beta$ respectively. Obviously, $\alpha \in [1, \dots, k-1]$ and α_i, $\beta \in [0, 1, \dots, k-1]$, $i = \overline{1,\ n}$.

The linear quality functional $\varphi(x, y)$ is determined by the penalty matrix, where $x \in [1, 2, \dots, k-1]$, $y \in [0, 1, \dots, k-1]$. The penalty is a function of the true value $P(S)$ and of the value of the predicate calculated by the corrector. With the correct correction at the facility, S the penalty is zero, with incorrect correction the penalty is determined according to Table 1.

$\alpha \backslash \beta$	0	1	$\cdots$	$k-1$
1	φ_{10}	φ_{11}	$\cdots$	$\varphi_{1(k-1)}$
2	φ_{20}	φ_{21}	$\cdots$	$\varphi_{2(k-1)}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$k-1$	$\varphi_{(k-1)0}$	$\varphi_{(k-1)1}$	$\cdots$	$\varphi_{(k-1)(k-1)}$

TABLE 1. Incorrect correction the penalty

All values in the table are not negative. In addition, usually $\varphi_{i0} = \varphi_{ji}$, i, $j = \overline{1, k-1}$. The following penalty tables are most commonly used:

(i) $\varphi_{ij} = 1$, $\varphi_{i0} = 0,5$, i, $j = \overline{1, k-1}$;

(ii) $\varphi_{ij} = 1$, $j = \overline{0, k-1}$, $i = \overline{1, k-1}$, $i \neq j$.

Let the control material form objects $S_1, S_2, \dots, S_q$ for which the property P and $P(S_i) = \alpha_i$, $\quad i = \overline{1, q}$ is calculated in advance.

The results of the algorithms $A_1, A_2, \dots, A_n$ with which you can calculate the value of P for control objects are shown in Table 2. Here $\alpha_{ij} \in [0, 1, \dots, k-1]$, $\alpha_p \in [1, 2, \dots, k-1]$, $i = \overline{1,\ t}$, $j = \overline{1,\ n}$, $p = \overline{1,\ q}$

Let $A = \{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_t\}$ where $\tilde{\alpha}_i = (\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_n})$, $i = \overline{1,t}$. On the set S^n, we consider the class of discrete functions depending on n variables. For an arbitrary class σ function $f(\tilde{x})$ defined on the set A, we introduce the functional, where

$$n_f = \sum_{i=1}^{t} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, f\left(\tilde{\alpha}_i\right)\right)$$

and $p_0 = 0$.

Definition 2.1. *The functional n_f. is called the linear quality functional.*

When solving the problem of synthesis of the optimal corrector, a corrector (correcting function) is selected that satisfies the given constraints and is optimal in terms of the linear quality functional.

S	A_1	A_2	$\cdots$	A_n	$P(S)$
S_1	α_{11}	α_{12}	$\cdots$	α_{1n}	α_1
S_2	α_{21}	α_{22}	$\cdots$	α_{2n}	α_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_{p_i}	α_{p_i1}	α_{p_i2}	$\cdots$	α_{p_in}	α_p
S_{p_i+1}	$\alpha_{(p_i+1)1}$	$\alpha_{(p_i+1)2}$	$\cdots$	$\alpha_{(p_i+1)n}$	α_{p_i+1}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_{p_2}	α_{p_21}	α_{p_22}	$\cdots$	α_{p_2n}	α_{p_2}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_{p_i+1}	$\alpha_{(p_i+1)1}$	$\alpha_{(p_i+1)2}$	$\cdots$	$\alpha_{(p_i+1)n}$	α_{p_i+1}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_q	α_{q1}	α_{q2}	$\cdots$	α_{qn}	α_q

TABLE 2. Selecting corrector function

Let $n_\sigma = \min_{g:g\in\sigma} n_g$. The following problem W_σ : is of interest: to construct a function $g(\tilde{x}) \in P_k^n$ that is not everywhere defined from a class σ from a given table 2 of incorrect algorithms in such a way that $n_g = n_\sigma$.

To solve the problems of optimal adjustment, knowledge of combinatorial characteristics is essential $\sigma_i, \ \ i = 1, 2, 3$. Some of the most important characteristics include the number of functions that depend on variables $x_1, x_2, \ldots, x_n$

and belong to the corresponding classes, and the number of elementary steps necessary for a complete decoding of functions.

In this chapter, decryption problems are solved $W_{\sigma_1}, W_{\sigma_1}, \ldots, W_{\sigma_8}$, and estimates of the number of class functions $\sigma_1, \sigma_2, \sigma_3$ are also constructed.

I. The algorithm in the absence of restrictions. For all $\tilde{\alpha}_i \in A$, we calculate the functional

$$r = \min \left\{ r_0 = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \right.$$
$$\left. r_1 = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 1\right), \ldots, r_{k-1} = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, k-1\right) \right\}$$

(see Table 2). We define $g\left(\tilde{x}\right)$ the value $\tilde{\alpha}_i$ on the set: if $r_0 \neq r_1 \neq \ldots \neq r_{k-1}$, then, putting $v = \min\left(r_0, r_1, \ldots, r_{k-1}\right)$, we can admit $g\left(\tilde{\alpha}_i\right) = p$; if $v = r_p, \quad p \in [0, 1, \ldots, k-1]$, $(g\left(\tilde{a}_i\right)$, is assumed to be equal to one of the indices such that a minimum of value is realized $v)$.

II. The algorithm F_{σ_1} over the class σ_1. Without loss of generality, we assume that on sets $\tilde{0}, \tilde{1}, \ldots, \left(k\tilde{-}1\right)$ the function $g\left(\tilde{x}\right)$ takes values $0, 1, \ldots, k-1$, respectively.

Let $A' = A \backslash \left(\left\{\tilde{0}\right\} \cup \left\{\tilde{1}\right\} \cup \ldots \cup \left\{k\tilde{-}1\right\}\right)$. For all sets $\tilde{\alpha}_{ij}$ such that

$$\tilde{\alpha}_{i1} \in A' \cap E^n\left(0, 1\right),$$
$$\tilde{\alpha}_{i1} \in A' \cap E^n\left(0, 1\right),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\tilde{\alpha}_{ik-1} \in A' \cap E^n\left(0, k-1\right),$$
$$\tilde{\alpha}_{i1} \in A' \cap E_1^n,$$

we calculate the functionals

$$r^1 = \min\left\{ r_0 = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \quad r_1 = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 1\right), \right\},$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots ,$$

$$r^{k-1} = \min\left\{ r_0, \;\; r_{k-1} = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \;\; r_1 = \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, k-1\right), \right\},$$

$$r^k = \min\left\{r_0, r_1, \dots, r_{k-1}\right\}$$

respectively. Define the values $g\left(\tilde{x}\right)$ on the sets $\tilde{\alpha}_{ij}$:

$$g\left(\tilde{\alpha}_{i1}\right) = \begin{cases} 0, & \text{if } r^1 = r_0, \\ 1, & \text{if } r^1 = r_1, \end{cases} \quad r_0 \neq r_1,$$

$$g\left(\tilde{\alpha}_{i2}\right) = \begin{cases} 0, & \text{if } r^2 = r_0, \\ 1, & \text{if } r^2 = r_2, \end{cases} \quad r_0 \neq r_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots ,$$

$$g\left(\tilde{\alpha}_{ik-1}\right) = \begin{cases} 0, & \text{if } r^{k-1} = r_0, \\ k-1, & \text{if } r^{k-1} = r_{k-1}, \end{cases} \quad r_0 \neq r_{k-1},$$

$$g\left(\tilde{\alpha}_{ik}\right) = \begin{cases} 0, & \text{if } r^k = r_0, \\ 1, & \text{if } r^k = r_1, \\ \vdots & \\ k-1, & \text{if } r^{k-1} = r_{k-1}, \end{cases} \quad r_0 \neq r_1 \neq \dots \neq r_{k-1}.$$

If the corresponding conditions $r_0 \neq r_1$, $r_0 \neq r_{k-1}$, $r_0 \neq r_1 \neq \dots \neq r_{k-1}$ are not satisfied, then it is $g\left(\tilde{\alpha}_{ij}\right)$ determined similarly to the case without restrictions.

III. The algorithm F_{σ_4} over the class σ_4. For each set $A_{q_1}, A_{q_2}, \dots, A_{q_{k-1}}$ such that $A_{q_1}, A_{q_2}, \dots, A_{q_{k-1}} \neq \varnothing$, and consisting of all sets $\tilde{\alpha}_i \in A$ for which q_1 the coordinates are 1, q_2 the coordinates are 2, etc. q_{k-1}, the coordinates are equal $(k-1)$, we build the functional r:

$$r = \min\left\{ r_0 = \sum_{i:\tilde{\alpha}_i \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi(\alpha_j, 0), \right.$$

$$r_1 = \sum_{i:\tilde{\alpha}_i \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi(\alpha_j, 1), \ldots,$$

$$\left. r_{k-1} = \sum_{i:\tilde{\alpha}_i \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi(\alpha_j, k-1) \right\}$$

(see Table 2). Here $0 \leq q_i \leq n$ and $\sum_{i=1}^{k-1} q_i \leq n$.

We define $g(\tilde{x})$ the value on the set $A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}$: if $r_0 \neq r_1 \neq \ldots \neq r_{k-1}$, then

$$g(\tilde{x}) = \begin{cases} 0, & \text{if } r = r_0, \\ 1, & \text{if } r = r_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ k-1, & \text{if } r = r_{k-1}, \end{cases},$$

for all $\tilde{\alpha} \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}$, if the condition $r_0 \neq r_1 \neq \ldots \neq r_{k-1}$ is not fulfilled, then setting $v = \min(r_0, r_1, \ldots, r_{k-1})$ it can be assumed $g(\tilde{\alpha}) = t$ for all $\tilde{\alpha} \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}$ if $v = r_t$, $t \in [0, 1, \ldots, k-1]$ ($g(\tilde{a})$ for all $\tilde{\alpha} \in A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}$ it is assumed to be equal to one of the indices t such that r_t a minimum of value is realized v).

IV. The algorithm F_{σ_6} over the class σ_6. Without loss of generality, we assume that on sets $\tilde{0}, \tilde{1}, \ldots, (\widetilde{k-1})$ the functions $g(\tilde{x})$ that are optimal in terms of the quality functional take values $0, 1, \ldots, k-1$, respectively.

Let $A' = A \backslash \left(\{\tilde{0}\} \bigcup \{\tilde{1}\} \bigcup \ldots \bigcup \{\widetilde{k-1}\}\right)$. Consider the totality of all sets $A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}}$ such that $A_{q_1}, A_{q_2}, \ldots, A_{q_{k-1}} \neq \varnothing$, and consisting of all sets $\tilde{\alpha} \in A$ for which q_1 the coordinates are 1; q_2 coordinates are equal $2, \ldots;$ q_{k-1} coordinates are equal $k-1$.

Here $0 \leq q_1, q_2, \ldots, q_{k-1} \leq n$ and $q_1 + q_2 + \ldots + q_{k-1} \leq n$. For all sets $A_{q_1,0,\ldots,0}$, $A_{0,q_2,0,\ldots,0}, \ldots, A_{0,\ldots,0,q_{k-1}}$, $A_{q_1,\ldots,q_{k-1}}$, $q_1, q_2, \ldots, q_{k-1} > 0$, we calculate the functionals

$$r^1 = \min\left\{ r_0 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,0,\dots,0}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \right.$$

$$\left. r_1 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,0,\dots,0}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 1\right) \right\}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$r^{k-1} = \min\left\{ r_0 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,0,\dots,0,q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \right.$$

$$\left. r_1 = \sum_{i:\tilde{\alpha}_i \in A_{0,\dots,0,q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, k-1\right) \right\},$$

$$r^k = \min\left\{ r_0 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,0,\dots,0,q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 0\right), \right.$$

$$r_1 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,\dots,q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, 1\right), \dots$$

$$\left. \dots r_1 = \sum_{i:\tilde{\alpha}_i \in A_{q_1,\dots,q_{k-1}}} \sum_{j=p_{i-1}+1}^{p_i} \varphi\left(\alpha_j, k-1\right) \right\}.$$

We define the values $g\left(\tilde{x}\right)$ on the sets

$$A_{q_1,0,\dots,0}, A_{0,q_2,0,\dots,0}, \dots, A_{0,\dots,0,q_{k-1}}, A_{q_{k-1}}, A_{q_1,\dots,q_{k-1}}, q_1, \dots, q_{k-1} > 0,$$

$$g\left(\tilde{\alpha}\right) = \begin{cases} 0, \text{ if } \ r^1 = r_0, \\ 1, \text{ if } \ r^1 = r_1, \quad \text{for all } \ \tilde{\alpha} \in A_{q_1,0,\dots,0,} r_0 \neq r_1; \end{cases}$$

$$g\left(\tilde{\alpha}\right) = \begin{cases} 0, \text{ if } \ r^2 = r_0, \\ 2, \text{ if } \ r^2 = r_2, \quad \text{for all } \ \tilde{\alpha} \in A_{q_2,0,\dots,0,} r_0 \neq r_2; \end{cases}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$g(\tilde{\alpha}) = \begin{cases} 0, & \text{if } r^{k-1} = r_0, \\ k-1, & \text{if } r^{k-1} = r_{k-1}, \quad \text{for all } \tilde{\alpha} \in A_{0,\ldots,0,q_{k-1}} r_0 \neq r_{k-1}; \end{cases}$$

$$g(\tilde{\alpha}) = \begin{cases} 0, & \text{if } r^{k} = r_0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots , & \\ k-1, & \text{if } r^{k} = r_{k-1}, \quad \text{for all } \tilde{\alpha} \in A_{q_1,\ldots,q_{k-1}} \quad r_0 \neq r_1 \neq \ldots \neq r_{k-1}; \end{cases}$$

If the corresponding conditions $r_0 \neq r_1$, $r_0 \neq r_2, \ldots, r_0 \neq r_{k-1}$ are not satisfied, then $g(\tilde{\alpha})$ on sets $A_{q_1,0,\ldots,0}, A_{0,q_2,0,\ldots,0}, \ldots, A_{0,\ldots,0,q_{k-1}}, A_{q_{k-1}}, A_{q_1,\ldots,q_{k-1}}$ they are defined similarly to the case described in section 3.

3. Building optimal monotone proofreaders

Methods are given for constructing not everywhere defined corrective functions that are optimal with respect to the linear quality functional for classes σ_2, σ_3.

Consider the set A. Let $\{B_{\tilde{\alpha}}\}$ the collection of all sets A of, comparable with $\tilde{\alpha} \in U_n$, and $\{B_{\tilde{\alpha}}\}-$ the family of all sets $B_{\tilde{\alpha}}$. $\{B_{\tilde{\alpha}}\}$ in we single out $\{A_{\tilde{\alpha}}\} \subseteq \{B_{\tilde{\alpha}}\}$ the set of all sets $A_{\tilde{\alpha}}$ for which there does not exist $B_{\tilde{\alpha}}$ from $\{B_{\tilde{\alpha}}\}$ such that $\{A_{\tilde{\alpha}}\} \subseteq \{B_{\tilde{\alpha}}\}$. $\{A_{\tilde{\alpha}}\}$ we introduce a system of main neighborhoods.

Definition 3.1. *The main neighborhood of the first order $S_1(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$ of a set $A'_{\tilde{\alpha}}$ is the set $\{A_{\tilde{\alpha}}\}$ of all sets $A'_{\tilde{\alpha}}$, such that $A_{\tilde{\alpha}} \cap A'_{\tilde{\alpha}} \neq \emptyset$.*

Let a neighborhood of the $(p-1)$th order of the set $S_p(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$ be defined.

Definition 3.2. *The main neighborhood of the $p-$th order $S_p(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$ of a set $A_{\tilde{\alpha}}$ is the set $\{A_{\tilde{\alpha}}\}$ of all sets $A'_{\tilde{\alpha}}$ of for $\{A_{\tilde{\alpha}}\}$ which one of the following conditions holds:*

1. $A_{\tilde{\alpha}} \cap A'_{\tilde{\alpha}} \neq \emptyset,\ A'_{\tilde{\alpha}} \in S_{p-1}(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$;

2. $A'_{\tilde{\alpha}} \subseteq \bigcup A_{\tilde{\beta}} E$, $A'_{\tilde{\alpha}} \subseteq A_{\beta}$ where $A_{\tilde{\beta}}$ satisfies the first condition.

Definition 3.3. *The system of principal neighborhoods $S_1, S_2, \ldots, S_p$ is finite if for some p the condition $S_p(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\}) = S_{p+1}(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$ for all pairs is satisfied $(A_{\tilde{\alpha}}, \{A_{\tilde{\alpha}}\})$.*

It is easy to see that in $\{A_{\tilde{\alpha}}\}$ for all $A_{\tilde{\alpha}}$ the system of neighborhoods is finite.

When constructing algorithms for the correcting functions from $\delta_2, \delta_3, \delta_7, \delta_8$, we consider pairwise disjoint principal neighborhoods $S_{p_1}, S_{p_2}, \ldots, S_{p_t}$ such that $\bigcup\limits_{i=1}^{t} S_{p_i} = \{A_{\tilde{\alpha}}\}$ and $S_{p_{i-1}} \subset S_{p_i} = S_{p_{i+1}} = \ldots$, $i = \overline{1,t}$.

For arbitrary $A' \subseteq A$ we define the amount $r_{oA}, r_{1A}, \ldots, r_{k-1A} : r_{\alpha A'} =$
$= \sum\limits_{\tilde{\alpha}_i : \tilde{\alpha}_i \in A'} \sum\limits_{j=p_{i-1}+1}^{p_i} \varphi(\alpha_j, \alpha)$, where $\alpha \in [0, 1, \ldots, k-1]$.

Let $L = \{L_1, L_2, \ldots, L_t\}$, $L_i \subseteq S^n$, $i = \overline{1,t}$.. The set L corresponds to the set $\tilde{\alpha}$ if $\tilde{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_t)$ and $\alpha_i \in [0, 1, \ldots, k-1]$ $i = \overline{1,t}$.

1. The algorithm F_{δ_2} over the class δ_2. For each neighborhood $S_{p_i} = \left\{A_i\tilde{\beta}_1, A_i\tilde{\beta}_2, \ldots, A_i\tilde{\beta}_{q_i}\right\}$, $i = \overline{1,t}$ where $\tilde{\beta}_i \in U_n$, and all sets of $A_i\tilde{\beta}_j$ are comparable to $\tilde{\beta}_j$, $j = \overline{1, q_i}$.

1. We build $\tilde{U}_{i_1}, \tilde{U}_{i_1}, \ldots, \tilde{U}_{i_m}$ a set such that, where $\tilde{U}_{i_j} = \tilde{S}_{p_i} \cap \tilde{U}_{i_j}$, where $\tilde{S}_{p_i} = \bigcup\limits_{j=1}^{q_i} A_i\tilde{\beta}_j$ and $i_1 > i_2 > \ldots > i_m$.

Let $\tilde{U}_{ij} = \{\alpha_{j_1}, \alpha_{j_2}, \ldots, \alpha_{j_m}\}$, $j = \overline{1,m}$, $M = \left\{\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_{q_i}\right\}$ and $\tilde{M}$ a lot of all the sets $(\gamma_1, \gamma_2, \ldots, \gamma_{q_i})$ for which $\gamma_p \in [1, 2, \ldots, k-1]$, $p = 1, 2, \ldots, q_i$.

2. By induction on $p, p = \overline{1,m}$, we construct a family $\{N_{mp}\}$ of sets N_{m_p} of length sets m_p.

First step. For each set $\tilde{y} = (\gamma_1, \gamma_2, \ldots, \gamma_{q_i}) \in \tilde{M}$, we construct many $N_{\tilde{\gamma}}$ sets $\delta_{m_1} = (\delta_{11}, \delta_{12}, \ldots, \delta_{1m_1})) : \delta_{1i} = 0, i \in [1, 2, \ldots, m_1]$ if $\tilde{\alpha}_{1_i} \in U_{i_1}$ it is comparable with sets $\tilde{\beta}_p, \tilde{\beta}_t \in M$ such that the coordinates γ_p and $\tilde{y}_t$ of $\tilde{y}$, corresponding to $\tilde{\beta}_p$ and $\tilde{\beta}_t$, are not equal to each other ($\gamma_p \neq \gamma_t$): $\delta_{1i} \in [0, j], i \in [1, 2, \ldots, m], j = \overline{1, k-1}$, in the case when all sets of M, are comparable $\tilde{\alpha}_{1i} \in \tilde{U}_{i_1}$ with correspond $j-$ to the other coordinate on $\tilde{\gamma}$.

Let it be built $\{N_{m_p}\}$.

$(p+1)$-th step. For all sequences $\left\{\tilde{\gamma}\tilde{\delta}_{m_1}, \ldots, \tilde{\delta}_{m_p}\right\}$ such that, $\tilde{\gamma} \in \tilde{M}, \tilde{\delta}_{m_1} \in N_{\tilde{\gamma}}, \tilde{\delta}_{m_2} \in N_{\tilde{\delta}_{m_1}}, \ldots, \tilde{\delta}_{m_p} \in N_{\tilde{\delta}_{m_{p-1}}}$, we construct $N_{\tilde{\delta}_{m_p}}$ a set of sets $\tilde{\delta}_{m_{p+1}} = \left(\delta_{(p+1)1}, \ldots, \delta_{(p+1)m_{p+1}}\right)$, $\tilde{\delta}_{(p+1)i} = 0$, $i \in \{1, 2, \ldots, m_{p+1}\}$, $i \in \{1, 2, \ldots, m_{p+1}\}$, if:

(a) there is a set $\tilde{\alpha}_{ij} \in \tilde{U}_{i_l}, l \in [1, 2, \ldots, p], j \in \{1, 2, \ldots, m_l\}$ such that $\tilde{\alpha}_{l_j}$ is comparable to $\tilde{\alpha}_{(p+1)i} \in U_{i_{p+1}}$ and $j-$ the coordinate of the set $\tilde{\delta}_{m_l}$ corresponding $\tilde{\alpha}_{l_j}$ to is zero;

(b) $\tilde{\alpha}_{(p+1)i}$ it is comparable with sets $\tilde{\beta}_p, \tilde{\beta}_t \in M$ such that the coordinates γ_p and γ_t of $\tilde{\gamma}$ corresponding to $\tilde{\beta}_p$ and $\tilde{\beta}_t$ are not equal to each other ($\gamma_p \neq \gamma_t$); $\delta_{(p+1)} \in [0, t]$, $i \in \{1, 2, \ldots, m_{p+1}\}, t = \overline{1, k-1}$, in the case when all sets of $M \cup \bigcup_{j=1}^{k} \tilde{U}_{i_j}$, comparable to $\tilde{\alpha}_{(p+1)i}$ correspond to $t-$ other coordinates of $(\gamma_1, \gamma_2, \ldots, \gamma_{q_i}, \delta_{11}, \ldots, \delta_{1m_1}, \delta_{21}, \ldots, \delta_{p1}, \ldots, \delta_{pm_p})$.

3. We construct sequences $E = \{\tilde{\gamma}, \tilde{\delta}_{m_1}, \tilde{\delta}_{m_2}, \ldots, \tilde{\delta}_{m_m}\}$ of length $m+1$ such that $\tilde{\gamma} \in \tilde{M}, \tilde{\delta}_{m_1} \in N_{\tilde{\gamma}}, \tilde{\delta}_{m_2} \in N_{\tilde{\delta}_{m_1}}, \ldots, \delta_{m_m} \in N_{\tilde{\delta}_{m_{m-1}}}$. 4. For each sequence E, we construct a class σ_2 function $f_E(\tilde{x})$ defined $\tilde{S}_{p_i}$ on as follows: $f_E(\tilde{\alpha}_{pi}) = \delta_{pt}$, where $\tilde{\alpha}_{pt} \in \tilde{U}_{i_p}, \delta_{pt} - t-$ the coordinate of the set $\tilde{\delta}_{m_p}$ in E, $p = \overline{1, m}$, $t = \overline{1, m_p}$.

5. For all $f_E(\tilde{x})$, we calculate the functional $r_{f_E} = \sum_{i=1}^{m} \sum_{j=1}^{m_i} r_{\tilde{\delta}_{ij}} \{\tilde{\alpha}_{ij}\}$.

6. Of all $f_E(\tilde{x})$, we fix all $g_E(\tilde{x})$ that correspond to the minimum r_{g_E} among all the functionals.

7. We construct all functions $g(\tilde{x})$ on the set A in such a way $g(\tilde{x})$ that it is equal to one $g_E(\tilde{x})$ on S_{p_t}, $i = \overline{1, t}$.

The algorithm F_{δ_3} over the class δ_3 is constructed similarly to the algorithm F_{δ_2}.

4. Substantiation of synthesis algorithms for optimal correctors

1. The functions defined on the set in A the absence of restrictions, and the functions from the class σ_1 on each set of A are determined independently of the values of these functions on the remaining sets of A.

Therefore, when constructing these functions on the basis of Table 2, the algorithms that construct the functions calculate the quality functional on each set of the set A, regardless of the values of the functional on other sets. Moreover, on each set, the function is determined so that the functional is minimal.

The sum of the minimum values of the functionals on each set is a functional corresponding to the constructed function that is optimal with respect to the linear quality functional. Therefore, the algorithms described in paragraphs 1 and 2 are optimal.

2. The functions defined on the set of A classes σ_4 and σ_6 on each set of $A_{p_1, p_2, \ldots, p_{k-1}}$ sets of A containing units p_1, p_2, twos, $\ldots, p_{k-1}$ values $k-1$ are determined independently of the values of these functions on the remaining

sets $A_{p'_1,p'_2,\ldots,p'_{k-1}}$, moreover, all these sets are pairwise disjoint and $f(\tilde{x}) = \left\{\tilde{\alpha} : f(\tilde{x}) \in A_{p_1,p_2,\ldots,p_{k-1}}\right\}$ where $\alpha \in [0,1,\ldots,k-1]$.

Therefore, when constructing functions from these classes by algorithms F_{σ_4} and F_{σ_6}, the values of the functional on each set $A_{p_1,p_2,\ldots,p_{k-1}}$ are calculated independently of the values of the functional on other sets. Moreover, in each set, the function is determined so that the functional is minimal.

The sum of these functionals will correspond to a function that is optimal with respect to the linear quality functional, and this shows that the algorithms described in paragraphs 3.

3. Let the set A be divided into neighborhoods $S_{k_1}, S_{k_2}, \ldots, S_{k_t}$. Class functions σ_2 defined on a set A are defined on each set $\tilde{S}_{k_i} = \bigcup\limits_{A_{\tilde{\beta}} \in S_{k_i}} A_{\tilde{\beta}}$, regardless of the values of these functions on the remaining sets $S_{k_1}, S_{k_2}, \ldots, S_{k_{i-1}}, \ldots$ $\ldots, S_{k_{i+1}}, \ldots, S_{k_t}$, $i = \overline{1,t}$.

Therefore, when constructing these functions based on table (2), the algorithm F_{σ_2} calculates the value of the functional on each set S_{k_i}, $i = 1,2,\ldots,t$, regardless of the functional on other sets S_{k_j}.

Let $[f]_{S_{k_i}}$ – the set of all class σ_2 functions f defined on $\tilde{S}_{k_i}$. Let us show that the algorithm F_{σ_2} constructs a set $[f]_{S_{k_i}}$.

Let $S_{k_i} = \left\{A_{\tilde{\beta}_1}, A_{\tilde{\beta}_2}, \ldots, A_{\tilde{\beta}_p}\right\}$, where $\tilde{\beta}_j \in U_n$, and all sets $A_{\tilde{\beta}_j} \subseteq A$ of are comparable to $\tilde{\beta}_j$, $j = 1,2,\ldots,p$. Obviously, class σ_2 functions f defined on the set $A_{\tilde{\beta}}$, $i = 1,2,\ldots,p$ take values from $[0,1]$ or $[0,2], \ldots$, or $[0,k-1]$ in the case when, $f\left(\tilde{\beta}_i\right) = 1$ or $f\left(\tilde{\beta}_i\right) = 2, \ldots, f\left(\tilde{\beta}_i\right) = k-1$ respectively.

Therefore, the algorithm F_{σ_2} builds functions on S_{k_i}, starting with sets $\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_p$, moreover $f\left(\tilde{\beta}_i\right) \in [1,2,\ldots,k-1]$, $i = \overline{1,p}$. Then, the values of the functions on the sets $A_{\tilde{\beta}_1}, A_{\tilde{\beta}_2}, \ldots, A_{\tilde{\beta}_p}$ and on all possible intersections are determined. For anyone $\tilde{\alpha} \in U_n$ for whom the set $A_{\tilde{\alpha}}$ of all sets A of comparable to $\tilde{\alpha}$ is not empty, there exists $\tilde{\beta} \in \{\beta_1, \beta_2, \ldots, \beta_p\}$ such that $A_{\tilde{\alpha}} \subseteq A_{\tilde{\beta}}$. It follows that to construct a function on it is $A_{\tilde{\beta}}$ enough to know the value of this function on $\tilde{\beta}$.

Let the algorithm F_{σ_2} compute function $f(\tilde{x})$ values on sets $\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_p$. Consider $\tilde{U}_{tj} = \{\tilde{\alpha}_{j1}, \tilde{\alpha}_{j2}, \ldots, \tilde{\alpha}_{jm}\}$, $\quad j = \overline{1,m'}$. For each j, $j = 1,2,\ldots,p$, the induction k, $k = 1,2,\ldots,m'$ algorithm F_{σ_2} constructs a function f on $U_{i_k} \cap A_{\tilde{\beta}_j}$ in such a way that, $f(\tilde{\alpha}) \in \left\{0, f\left(\tilde{\beta}_j\right)\right\}$ where $\tilde{\alpha} \in \tilde{U}_{i_k} \cap A_{\tilde{\beta}_j}$. Obviously in the process, F_{σ_2}, $f(\tilde{\alpha}) = 0$ if $\tilde{\alpha} \in A_{\tilde{\beta}_i} \cap A_{\tilde{\beta}_j}$, $f\left(\tilde{\beta}_j\right) \neq f\left(\tilde{\beta}_j\right)$, $i \neq j$, $i, j \in [1, 2, \ldots, p]$.

Let $f(\tilde{x}) \in \{f\}_{S_{k_i}}$ it be such that $f(\tilde{\alpha}_{ij}) = \tilde{\beta}_{ij}$, $i = \overline{1, m'}$, $j = \overline{1, m_i}$ and $[E]-$ the set of all sequences E of length sets $m' + 1$ obtained by applying the algorithm F_{σ_2} to Table 2. We show that $[E]$ there exists a sequence in $E = \{\tilde{\gamma}, \tilde{\alpha}_{m_1}, \tilde{\alpha}_{m_2}, \ldots, \tilde{\alpha}_{m_m}\}$ where $\tilde{\gamma} \in \tilde{M}$, $\tilde{\alpha}_{m_j} = \left(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{im_j}\right)$, $j = \overline{1, m'}$, $\tilde{\alpha} \in N_{\tilde{\gamma}}$, $\tilde{\alpha}_{m_2} \in N_{\tilde{\alpha}_{m_1}}, \ldots, \tilde{\alpha}_{m_m} \in N_{\tilde{\alpha}_{m_{m'-1}}}$

Such that $\tilde{\alpha}_{m_i} = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{im_i})$, $i = \overline{1, m'}$. Suppose the contrary. We put $\tilde{\alpha}_{m_i} \neq (\beta_{i1}, \beta_{i2}, \ldots, \beta_{im_1})$, more precisely, $\alpha_{ij} \neq \beta_{ij}$ and $\beta_{ij} = 0$.

Then, if $\beta_{ij} \neq 0$ and $\alpha_{ij} = 0$, there are sets of M, comparable with $\tilde{\alpha}_{ij} \in \tilde{U}_{i_j}$ and corresponding to various non-zero coordinates of $\tilde{\gamma}$, or there is a set $\tilde{\alpha} \in U_{i_l}$, $l \in [1, 2, \ldots, m]$ that is comparable to $\tilde{\alpha}_{ij}$ and corresponds to the zero coordinate of $\tilde{\alpha}_{m_l}$.

In the case when $\beta_{ij} = \delta_1$, $\alpha_{ij} = \delta_2$ ($\beta_{ij} = \delta_2$, $\alpha_{ij} = \delta_1$) all sets comparable to $\tilde{\alpha}_{ij}$ and following $\tilde{\alpha}_{ij}$, correspond to the coordinates $\delta_2\,(\delta_1)$ from sets in E. The latter contradict the monotonicity of the function f. The obtained contradiction shows that the algorithm F_{σ_2} constructs $[f]_{S_k}-$ the set of all functions f from σ_2 given in $\tilde{S}_{k_i}$. And since F_{σ_2} he chooses from $[f]_{S_k}$ one for $g(\tilde{x})$ which $r_g = \min\limits_{f \in \{f\}_{S_k}} r_f$ he is fair.

Theorem 4.1. *The algorithm is optimal according to Table 2. The proof of the optimality of the algorithms is carried out similarly.*

5. Conclusion

In the present work, we studied the set of correctors from functional closure. A corrector was selected by solving an extreme problem on a control set of objects: a $k-$valued logic function is selected that optimally corrects the predicate value on a finite subset of objects for which these predicates are known and the results of their calculations by algorithms are known

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