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MATCHING DOMINATION IN FUZZY LABELING TREE

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ABSTRACT. A matching is a set of non-adjacent edges. If every vertex of fuzzy graph is M-saturated then the matching is said to be complete or perfect. In this paper, we introduce the new concept of Global matching domination in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts in Fuzzy labeling tree.

1. INTRODUCTION

One of the remarkable mathematical inventions of the 20^{th} century is that of fuzzy sets by Lotfi A. Zadeh in 1965 [12]. His aim was to develop a mathematical theory to deal with uncertainty and imprecision. The advantage of replacing the classical sets by Zadeh's fuzzy sets is that it gives greater accuracy and precision in theory and more efficiency and system compatibility in applications.

The distinction between sets and fuzzy sets is that the sets divided the Universal set into two subsets namely members and Non-members while fuzzy sets assigns a membership value to each element of the universal set ranging from zero to one.

Rosenfeld [3] introduced the notion of fuzzy graphs in the year 1975. The rigorous study of dominating sets in Graph Theory began around 1960. The

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domination problems were studied from 1950's onwards, but the rate of research on domination significantly increased in the mid 1970's. Domination can be useful tool in many chemical structures and also there is many applications of domination theory in wireless communication networks, business networks and making decisions. The study of domination set in graphs was begun by Ore and Berge. A.Somasundaram and S. Somasundaram [4] discussed domination in fuzzy graphs. A.Nagoorgani and T. Chendrasekaran [2] introduced the concept of Domination in fuzzy Graphs. S.Yahya Mohamed, S.Suganthi [5–8] introduced some parameters of fuzzy labelling tree using matching and Perfect matching. Min-Jen and Jenq-Jong Lin [1] defined domination numbers of Trees. Some recent works in generalization of fuzzy graph theory can be found in [9–11]. In this paper, we introduce the new concept of Global matching domination in fuzzy labeling tree and its spanning tree. We discussed some properties of Fuzzy labeling tree using Neighbours, Matching bridge and supportive edges. Here we consider complete fuzzy labeling tree with even number of vertices.

2. Preliminaries

Definition 2.1. *Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$. A **fuzzy graph** $G = (\alpha, \beta)$ is a pair of functions $\alpha : V \to [0,1]$ and $\beta : V \times V \to [0,1]$ where for all u,v ∈V, we have* $\beta(u,v) \leq \min\{\alpha(u), \alpha(v)\}$.

Definition 2.2. *Let $G : (\alpha, \beta)$ be a fuzzy graph and F is a subset of G. If nodes of F is contained (or) equal to the nodes of G then F is said to be a **fuzzy subgraph**.*

Definition 2.3. *A fuzzy sub graph F of the fuzzy labeling graph G is said to be **fuzzy spanning sub graph(FSS)** of G if nodes of fuzzy sub graph is equal to the nodes of fuzzy graph.*

Definition 2.4. *A fuzzy graph G is said to be **fuzzy simple labeling graph (FSG)** if G does not contain a line with same ends and multiple lines.*

Definition 2.5. *A fuzzy simple graph G is said to be **fuzzy complete labeling graph (FCLG)** if every pair of nodes of the graph are joined by line. A FCLG with n nodes are denoted by k_n.*

Definition 2.6. *A fuzzy labelling graph G is said to be **fuzzy connected labelling graph (FCG)** if there exists a path between all pair of nodes of G.*

Definition 2.7. *A cyclic graph G is said to be **fuzzy cyclic graph** if it has fuzzy labeling.*

3. Main Results

3.1. Global Matching Domination.

Definition 3.1. *A subset M of $\beta(v_i, v_{i+1})$, $1 \leq i \leq n$ is called a **matching** in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be saturated under M.*

Definition 3.2. *If every vertex of fuzzy graph is $M-$saturated then the matching is said to be **complete or perfect**. It is denoted by C_M.*

Definition 3.3. *Let M be a matching in fuzzy labeling graph. An **M-alternating path** in G is a path whose edges alternatively in $\beta - M$ and M.*

Definition 3.4. *A graph $G = (\alpha, \beta)$ is said to be fuzzy labeling tree (FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree inwhich every pair of nodes contains a path inwhich the lines are alternatively in β and $\beta - M$.*

Definition 3.5. *A set of edges μ of β is said to be an **edge dominating set** if every edge in $\beta - \mu$ is adjacent to atleast one edge in μ. The minimum number of elements in the edge dominating set is called the **edge domination number**. It is denoted by $\mu_{DN}(G)$.*

Definition 3.6. *Let $G = (\alpha, \beta)$ be a fuzzy labelling tree and M be a matching in FLT. The set M is said to be **matching dominating set** if every edge in $\beta - M$ is adjacent to atleast one edge in M. The number of elements in the minimal matching dominating set is called **matching domination number**. It is denoted by $\mu_{MDN}(G)$.*

Example 1. *Consider the fuzzy graph given in Figure 1. Here, matching dominating set $\{e_2, e_5\}$ and $\mu_{MDS}(G) = 2$.*

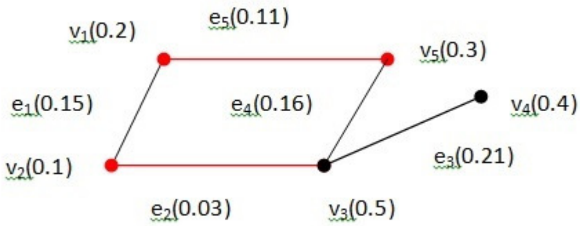

FIGURE 1. FLT

Definition 3.7. *Let $G = (\alpha, \beta)$ be a fuzzy labelling tree and M be a perfect matching in FLT. The set M is said to be* ***global matching dominating set*** *if every edge in $\beta - M$ is adjacent to atleast one edge in M. The number of elements in the minimal global matching dominating set is called* ***Global matching domination number***. *It is denoted by $\mu_{GMDN}(G)$.*

Example 2. *Consider a fuzzy graph given in Figure 2. Here global matching dom-*

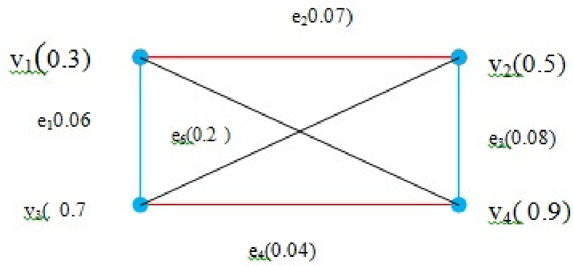

FIGURE 2. FLT

inating set is $\{e_1, e_3\}$ and $G_{GMDN}(G) = 2$.

3.2. **Neighbours of an Edge.**

Definition 3.8. *The* ***open neighbour*** *of an edge e in a fuzzy tree is the set of all edges adjacent to e in FLT. It is denoted as* ***ON(e)***. *The Closed Neighbour of an edge e is the union of e and its open neighbour and it is marked as $clN(e)$.*

Theorem 3.1. *Every Open neighbour $ON(e)$ in complete fuzzy tree with $n > 2$ form a spanning subgraph of FLT which may or may not contain cycle.*

Proof. Let us consider a fuzzy labeling tree G and take e be any arbitrary edge in G. Now we find the Open neighbour for e. Here open neighbour is the set of all edges adjacent to e in G. Here Open neighbour form a subgraph which contains all vertices of fuzzy tree because every vertex is adjacent to all other vertices in complete fuzzy tree. This subgraph of fuzzy tree may or may not contain a cycle because an edge e not include in the neighbour. Hence every Open neighbour $ON(e)$ in complete fuzzy tree with $n > 2$ may or may not contain a cycle but in both cases we have a spanning subgraph of FLT. □

Example 3. *Consider fuzzy graphs given in Figure 3 and Figure 4.*

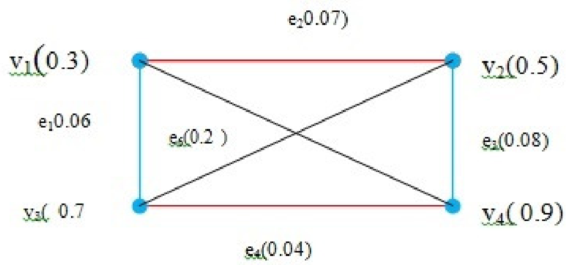

FIGURE 3. FLT

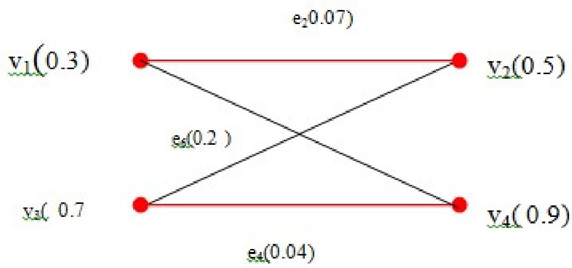

FIGURE 4. $ON(e_1)$

Theorem 3.2. *Every closed neighbour $clN(e)$ in complete fuzzy tree with $n > 2$ form a spanning subgraph with cycle of FLT .*

Proof. Let us consider a fuzzy labeling tree G and take e be any arbitrary edge in G. Now we find the closed neighbour for e. Here closed neighbour is the union of edge e and its open neighbour. Here closed neighbour form a cycle which contains all vertices of fuzzy tree because every vertex is adjacent to all other vertices in complete fuzzy tree. This cycle is a subgraph of fuzzy tree with all vertices of G. Hence every closed neighbour $clN(e)$ in complete fuzzy tree with $n > 2$ form a cyclic spanning subgraph of FLT. □

Definition 3.9. *An edge in a fuzzy labelling tree is said to be* ***matching bridge*** *if it belongs to any one of the perfect matching.*

Definition 3.10. *Two distinct edges are called* ***Duplicated Edges*** *in a fuzzy labelling tree if they have same neighbours.*

Definition 3.11. *An edge e is said to be* ***Supportive Edge*** *if it is the common neighbour for two or more edges in fuzzy labelling tree.*

Example 4. *Consider fuzzy graphs given in Figure 5. Here the edges* $\{e_1(0.15), e_4(0.16)\}$

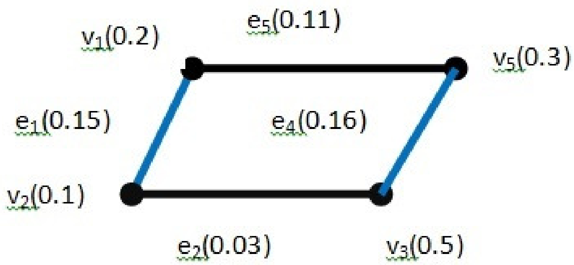

FIGURE 5. FLT

are Duplicated Edges and $\{e_2(0.03), e_5(0.11)\}$ *are Supportive Edges.*

Definition 3.12. *An edge* e *is called a* ***Base Edge*** *for spanning tree if it is adjacent to supportive edge in the fuzzy labeling tree.*

Theorem 3.3. *Every spanning tree of fuzzy labelling tree contains at least one supportive edges.*

Proof. Consider a complete fuzzy labelling tree G. By the definition of fuzzy labeling tree, a fuzzy graph $G = (\alpha, \beta)$ is said to be fuzzy labeling tree (FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains a path in which the lines are alternatively in β and $\beta - M$. So that G contains a spanning tree T and also every pair of vertices contains an alternating path in T. Since G is complete, each vertex is adjacent to every other vertices. Hence T contains at least one supportive edge in it. □

Theorem 3.4. *Every spanning tree of fuzzy labelling tree with n vertices has* $\frac{n}{2}$ *Matching bridges and* $\frac{n-2}{2}$ *Supportive edges.*

Proof. Consider a complete fuzzy labelling tree G.
By the definition, G contains a spanning tree T. But we know that every tree with n vertices has $n-1$ edges. So T contains $n-1$ edges. Here an edge in a Spanning tree is matching bridge if it belongs to any one of the perfect matching. Now we have sum of the matching bridges**(MB)** and supportive edges**(SE)** equal the total number of edges in a spanning tree T. But we have every complete fuzzy graph with n vertices has $n/2$ edges in its perfect matching. (ie.)

$$\begin{aligned} MB + SE &= (n-1) \\ &\Rightarrow \frac{n}{2} + SE = (n-1) \\ &\Rightarrow SE = (n - 1 - \frac{n}{2} \\ &\Rightarrow \text{ Supportive Edges } = \frac{n-2}{2}. \end{aligned}$$

Hence every spanning tree of fuzzy labelling tree with n vertices has $\frac{n}{2}$ matching bridges and $\frac{n-2}{2}$ Supportive edges. □

4. Conclusion

In this paper, we introduced the new concept of matching Domination and Global matching Domination in fuzzy labeling tree. Also we defined duplicate and supportive edges and we discussed some properties using these concepts in spanning sub graphs of fuzzy labeling tree. In Future, we will find chromatic number of fuzzy labelling tree using matching and perfect matching.

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