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A NOTE ON PRINCIPAL ROUGH IDEALS OF A ROUGH MONOID

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ABSTRACT. In this paper, we portray the principal ideals of a commutative regular rough monoid of idempotents (T, Δ) and (T, ∇) and we give the sufficient condition for the principal ideals (T, Δ) and (T, ∇) also we describe some of its properties with respect to the defined binary operations Δ and ∇. We delineate these ideas through examples.

1. INTRODUCTION

Zadeh [22] presented the idea of fuzzy sets in 1965. Foundation of Rough sets was introduced by Pawlak [16] in 1982 to perform deficient data in the data framework and it is characterized as an ordered pair of lower and upper approximation. The idea of Rough fuzzy and fuzzy rough sets was examined in 1990 by Dubois and Parade [6]. Kuroki and Wang [11] described the upper and lower approximations and its properties of normal subgroup. Bonikowaski [3] in 1994 and Iwinski [9] in 1997 have examined algebraically some properties of rough sets. Biswas and Nanda [1] dealt the idea of groups and subgroups in rough sense. The idea of rough ideal in semigroup was presented by Kuroki [12] in 1997. Howie [8] described the basic ideas of semigroups in 2003. Rough sets can be applied in many fields like data analysis, pattern recognition and to generate decision rules etc [2, 5, 7, 14, 20]. Kondo [10] described the notion of the structure on generalized rough sets in 2006. Fiala [15] examined a few models

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in groups of groupoid identities in 2008. R.Chinram [19] in 2009, presented the idea of gamma semigroups. Wang and Chen [4] discussed few properties of rough groups in 2010. Liu [21] described algebraically about special lattice in rough sets in 2011. Praba et al. [17,18] described the idea of lattice on T and a commutative rough monoid under the operation praba Δ in 2013. Manimaran et al. [13] examined the thought of a regular rough ∇ monoid of idempotents in 2014.

In this paper we explore the principal ideals of a commutative rough monoid of idempotents (T, Δ) and (T, ∇) for the given information system $I = (U, A)$ where U is a finite(universal)set and A is a nonempty attribute set in fuzziness.

2. Basic Defintions

In this section we present few basics of rough sets and monoids

2.1. **Rough sets.** Let $I = (U, A)$ be an approximation space, U is a non empty finite (universe) set of objects and R is an indiscernible relation(arbitrary). For any $x \in U$ the equivalence class is defined as $[x]_R = \{y \in U \mid (x, y) \in R\}$ and for any arbitrary subset Y of U, $RS(Y) = (\underline{R}(Y), \overline{R}(Y))$ be the rough set of Y where $\underline{R}(Y) = \{y \in U \mid [y]_R \subseteq Y\}$ and $\overline{R}(Y) = \{y \in U \mid [y]_R \cap Y \neq \phi\}$ are the lower and upper approximations individually.

2.2. **Algebraic structures.**

Definition 2.1. *[8] For a semigroup $(S, *)$, a (nonempty) subset I of S is said to be a right or left ideal, if it satisfies $I * S \subseteq I$ or $S * I \subseteq I$ individually.*

Definition 2.2. *[8] If a nonempty subset I of S is both right and left ideal then I is said to be an ideal of S.*

Theorem 2.1. *[18] [13]*

- (T, Δ) *is a regular rough monoid of idempotents.*
- (T, ∇) *is a commutative regular rough ∇ monoid of idempotents.*

In the following section, we discuss the principal ideals of the rough monoids (T, Δ) and (T, ∇).

3. Principal Ideals of a Commutative Rough Monoid of Idempotents

In this section, we consider an information system $I = (U, A)$. Now for any $Y \subseteq U$ $RS(Y) = (\underline{P}(Y), \overline{P}(Y))$ and let $T = \{RS(Y)|Y \subseteq U\}$ be the set of all rough sets. and let $E = \{Y_1, Y_2, ...Y_n\}$ be the indiscernible classes induced by $Ind(P)$.

Let us consider, E_Y be the set of equivalence classes contained in Y and P_Y be the set of pivot elements of Y.

Theorem 3.1. *For any $Y \subseteq U$, the principal ideal generated by $RS(Y)$ in T (with respect to Δ) is $RS(Y)\Delta T = T_1$ where $T_1 = \{RS(X) \mid X \in (Y \cup P(E \setminus E_Y) \cup P(P_{\overline{Y}}))\}$.*

Proof. Clearly the elements of $RS(Y)\Delta T$ contained in the right hand side set. Conversely, for $RS(X) \in RHS$ such that $X = Y \cup Z_1 \cup Z_2$ and $RS(X) = RS(Y \cup Z_1 \cup Z_2)$ where $Z_1 \in P(E \setminus E_Y)$ and $Z_2 \in P(P_{\overline{Y}})$, therefore $RS(X) = RS(Y)\Delta RS(Z_1 \cup Z_2) \in LHS$. ☐

Theorem 3.2. *For any $Y \subseteq U$, the principal ideal generated by $RS(Y)$ in T (with respect to ∇) is given by $RS(Y)\nabla T = T_2$ where $T_2 = \{RS(X) \mid X \in (P(E_Y) \cup P(Z_Y))\}$ where $Z_Y = \{y \in U \mid [y]_p \cap Y \neq \phi\}$.*

Proof. For $RS(X) \in RHS$ and $X = Z_1 \cup Z_2$ where $Z_1 \in P(E_Y)$ and $Z_2 \in P(Z_Y)$ then $RS(X) = RS(Z_1 \cup Z_2)$ and $RS(Y\nabla(Z_1 \cup Z_2)) = RS(Y)\nabla RS(Z_1 \cup Z_2) = RS(Z_1 \cup Z_2) = RS(X) \in LHS$. Conversely, $RS(Y\nabla Z) = RS(\phi)$ if $Y \cap Z = \phi$, where $RS(\phi) \in RHS$. If $Y \cap Z$ contains one or more indiscernible classes then $RS(Y\nabla Z) \in RS(P(E_Y))$. If $Y \cap Z$ contains one or more elements of Z_Y then $RS(Y\nabla Z) \in RS(P(Z_Y))$. If $Y \cap Z$ contains one or more indiscernible classes or one or more elements of Z_Y then $RS(Y\nabla Z) \in RS(P(E_Y) \cup P(Z_Y))$. Therefore $RS(Y\nabla Z) \in RHS$, which proves the theorem. ☐

Remark 3.1. *Note that these principal ideals are submonoid of idempotents with respect to Δ and ∇.*

Lemma 3.1. *If x_i,x_j belongs to the same indiscernible class then $RS(x_i)\Delta T = RS(x_j)\Delta T$.*

Proof. On the contrary, let $RS(x_i)\Delta T \neq RS(x_j)\Delta T$ then $RS(x_i) \neq RS(x_j)$ where $RS(Y_i) = RS(x_i)\Delta T = \{RS(Z_i) \mid Z_i \in X_i \cup P(E \setminus E_{X_i}) \cup P(P_{\overline{X_i}})\}$ and $RS(Y_j) =$

$RS(x_j)\Delta T = \{RS(Z_j) \mid Z_j \in X_j \cup P(E \setminus E_{X_j}) \cup P(P_{\overline{X_j}})\}$ implies that $RS(Y_i) \cap RS(Y_j)$ contains one or more $RS(Z_i)$ in common but not all. This implies that x_i and x_j does not belongs to the same indiscernible class which is a contradiction to the assumption. Therefore, $RS(x_i)\Delta T = RS(x_j)\Delta T$. □

Lemma 3.2. *If x_i,x_j belongs to the different indiscernible class then $RS(x_i)\nabla T \neq RS(x_j)\nabla T$.*

Proof. Let X_i and X_j be the two distinct indiscernible classes containing x_i and x_j respectively then $RS(Y_i) = RS(x_i)\nabla T = \{RS(Z) \mid Z \in P(E_{X_i}) \cup P(Z_i)\}$ and $RS(Y_j) = RS(x_j)\nabla T = \{RS(Z) \mid Z \in P(E_{X_j}) \cup P(Z_j)\}$. Since both X_i and X_j are distinct therefore $Y_i \cap Y_j$ contains one or some elements of $P(Z_i)$ and $P(Z_j)$ but not the some elements of $P(E_{X_i})$ and $P(E_{X_j})$ implies that $RS(Y_i)$ do not contains all elements of $RS(Y_j)$. Therefore, $RS(x_i)\nabla T \neq RS(x_j)\nabla T$. □

3.1. Properties.

(1) If z_1, z_2 do not belongs to the same indiscernible classes, then $RS(z_1)\Delta T \neq RS(z_2)\Delta T$.
(2) $RS(\phi) \notin RS(Y)\Delta T$ for $Y \neq \phi$.
(3) If $Y \subseteq Z$ then $RS(Y)\Delta T \subseteq RS(Z)\Delta T$.
(4) If Y_i and Y_j are two indiscernible classes in U such that $|Y_i|\ and\ |Y_j| > 1$ and if $y_i \in Y_i$ and $y_j \in Y_j$ then $|RS(y_i)\Delta T| = |RS(y_j)\Delta T|$.
(5) If z_1, z_2 belongs to the same indiscernible class then $RS(z_1)\nabla T = RS(z_2)\nabla T$.
(6) $RS(U) \notin RS(Y)\nabla T$ for $Y \neq U$ and $RS(\phi) \in RS(Y)\nabla T$ for $Y \neq \phi$.
(7) If $Y \subseteq Z$ then $RS(Y)\nabla T \subseteq RS(Z)\nabla T$.
(8) If Y_i and Y_j are two indiscernible classes in U such that $|Y_i|\ and\ |Y_j| > 1$ and if $y_i \in Y_i$ and $y_j \in Y_j$ then $|RS(y_i)\nabla T| = |RS(y_j)\nabla T|$.

4. Examples

Example 1. *Let us consider the information system $I = (U, A)$ where $U = \{y_1, y_2, y_3, y_4\}$ and $A = \{a_1, a_2, a_3, a_4\}$ where each $a_i(i = 1$ to $4)$ is a fuzzy set of attributes whose membership values are shown in Table 1.*

Let $X = \{y_1, y_2, y_3, y_4\} \subseteq U$ *then the equivalence classes induced by* $IND(P)$ *are given below:*

$$\begin{aligned} Y_1 = [y_1]_p &= \{y_1, y_3\} \\ Y_2 = [y_2]_p &= \{y_2, y_4\}. \end{aligned}$$

TABLE 1. Data Information table

A/U	a_1	a_2	a_3	a_4
y_1	0.2	0.3	1	0
y_2	0.8	0.4	0.1	0.9
y_3	0.2	0.3	1	0
y_4	0.8	0.4	0.1	0.9

$T = \{RS(\phi), RS(U), RS(Y_1), RS(Y_2), RS(\{y_1\}), RS(\{y_2\}), RS(Y_1 \cup \{y_2\}), RS(\{y_1\} \cup Y_2), RS(\{y_1\} \cup \{y_2\})\}$

Example 2. *From example(1), the principal ideals generated by* $RS(X)$ *with respect to* Δ

- $< RS(Y_1) >$ *is* $\{RS(U), RS(Y_1), RS(Y_1 \cup \{y_2\})\}$.
- $< RS(Y_2) >$ *is* $\{RS(U), RS(Y_2), RS(\{y_1\} \cup Y_2)\}$.
- $< RS(\{y_1\}) >$ *is* $\{RS(\{y_1\}), RS(Y_1), RS(\{y_1\} \cup Y_2), RS(\{y_1\} \cup \{y_2\}), R(Y_1 \cup \{y_2\}), RS(U)\}$.
- $< RS(\phi) >$ *is* $\{RS(\phi), RS(U), RS(Y_1), RS(Y_2), RS(\{y_1\}), RS(\{y_2\}), RS(Y_1 \cup \{y_2\}), RS(\{y_1\} \cup Y_2), RS(\{y_1\} \cup \{y_2\})\}$.
- $< RS(\{y_2\}) >$ *is* $\{RS(\{y_2\}), RS(Y_2), RS(\{y_1\} \cup \{y_2\}), RS(\{y_1\} \cup Y_2), RS(Y_1 \cup \{y_2\}), RS(U)\}$.
- $< RS(U) >$ *is* $\{RS(U)\}$.

Example 3. *From example(1), the principal ideals generated by* $RS(X)$ *with respect to* ∇

- $< RS(Y_1) >$ *is* $\{RS(\phi), RS(\{y_1\}), RS(Y_1)\}$.
- $< RS(\{x_1\}) >$ *is* $\{RS(\phi), RS(\{y_1\})\}$.
- $< RS(U) >$ *is* $\{RS(\phi), RS(U), RS(Y_1), RS(Y_2), RS(\{y_1\}), RS(\{y_2\}), RS(Y_1 \cup \{y_2\}), RS(\{y_1\} \cup Y_2), RS(\{y_1\} \cup \{y_2\})\}$.

5. CONCLUDING REMARKS

In this paper, we examined the idea of principal ideals of a commutative rough monoid of idemopotents (T, Δ) and (T, ∇) with respect to the given information system I and proved the existence theorem of those principal ideals and the same concepts are illustrated with examples.

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