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# EFFECT OF REFRACTORY TIME PERIOD ON INFORMATION PROCESSING OF A LEAKY INTEGRATE-AND-FIRE NEURON IN DISTRIBUTED DELAY FRAMEWORK

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ABSTRACT. A neuron encodes the neuronal information in term of either in time of interval of two consecutive spikes or in number of spikes generated in a unit time interval. This mechanism is known as temporal information encoding scheme. Statistical study of temporal information processing scheme results the first passage time (FPT) problem. After generation of a spike, the neuron does start processing of information for few milliseconds. This small time duration is known as refractory time period in which a neuron remains idle. In this article, we investigate the effect of Gaussian distributed refractory time period on inter-spike-interval distribution of LIF model in distributed delay framework (DDF) with hypo-exponential distributed delay kernel functions and obtain the combination of parameter values for which the effect of refractory time period becomes negligible.

## 1. INTRODUCTION

Information processing in a neuron occurs in term spikes [1]. This encoded information in a spike transmits from one neuron to other neuron in form of spikes sequence [9,10]. The neuron receives different kinds of ion and molecule, known as neurotransmitter, which increases the membrane potential. When the

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membrane potential value reaches to certain threshold value, an epoch of potential generates which epoch is termed as a spike. After a spike generation, the membrane potential suddenly reset to it's the lowest value. After reset the neuron takes few times (in msec.) to restart information processing. This time duration, between a spike generations to restart information processing is known as refractory time period [12, 15]. Refractory time period depends on neuronal internal environment as well as external environment. Teeter at. al. [18] used different values refractory time period in generalized leaky integrate-and-fire neuron model.

Temporal encoding scheme fpor neuronal information processing uses the time interval between two consecutive spikes, for encoding information. Time duration between two consecutive spikes is known as inter-spike-interval whereas its distribution is termed inter-spike-interval distribution (ISI distribution). Mathematical study of temporal encoding scheme results into a very popular first passage time problem (FPT problem). Its mathematical formulation can be given as [7, 13, 19]

$$T = \inf\{t : t > 0, V(t) > V_{threshold}, V(0) = V_0 < V_{threshold}\}. \tag{1.1}$$

Karmeshu et. al. [19] have suggested distributed delay framework (DDF) to include the previous membrane potential values on its present evolution and investigated the neuronal encoding with temporal encoding scheme. Choudhary et. al. [4–7] have investigated the LIF model in DDF for exponential distributed delay kernel, gamma distributed delay kernel and hypo-exponential distributed delay kernel with rate coding scheme. Choudhary et. al. [6, 7] have also computed the stationary state membrane potential distribution for LIF model in DDF with above mentioned delay kernel functions and noticed that the stationary state membrane potential of LIF neuron is unaffected due to such kind of distributed delay kernel functions. In this article, we study the effect of refractory time period on information processing of LIF neuron in DDF with hypo-exponential distributed delay kernel functions and compare the findings with the simple LIF neuron model driven by stochastic input stimulus. Incorporation of refractory time period $t_{ref}$ on temporal encoding scheme modifies the mathematical formulation for FPT problem which can be given as given as [2, 3, 14]

$$T = \inf\{t + t_{ref} : t > 0, V(t) > V_{threshold}, V(0) = V_0 < V_{threshold}\}. \tag{1.2}$$

## 2. LEAKY INTEGRATE-AND-FIRE NEURON MODEL

The leaky integrate-and-fire neuron model is one of the most popular threshold based neuron model. It is widely used to mimic various neuronal activities. It is very helpful in implementing the brain like structure at hardware level as well as software level, both. In order to describe the membrane potential change rate,generalized form for LIF model can be given as [1–3, 8]

$$\frac{dV}{dt} = -\beta V + \mu + \xi(t). \tag{2.1}$$

Here $\beta$ is known as membrane potential decay constant. $\mu+\xi(t)$ is the stochastic input stimulus with $\mu$ as mean of input value and $\xi(t)$ as a delta correlated Gaussian white noise i.e. if intensity of noise be $\sigma$, then it yields $<\xi(t) = 0$ and $<\xi(t_i)\xi(t_j) = \frac{\sigma^2}{2}>$.

## 3. LIF NEURON MODEL IN DDF

Karmeshu et. al. [19] has suggested a kernel function method which is also known as distributed delay framework (DDF). The kernel function used in the DDF is known as memory kernel. The inclusion of a memory kernel function in LIF model given in Eq. 2.1, results into an LIF model in DDF given as:

$$\frac{dV(t)}{dt} = -\beta \int_0^t K(t-\tau)V(\tau)d\tau + \mu + \xi(t). \tag{3.1}$$

Incorporation of hypo-exponential distribution function in Eq. 3.1 as a delay kernel function results into a new model in given below

$$\frac{dV(t)}{dt} = -\frac{\beta\lambda_E\lambda_I}{\lambda_E - \lambda_I} \int_0^t (e^{-\lambda_E(t-\tau)} - e^{-\lambda_I(t-\tau)})V(\tau)d\tau + \mu + \xi(t). \tag{3.2}$$

In extended space the model becomes

$$\frac{dV}{dt} = -\frac{\beta\lambda_E\lambda_I}{\lambda_E - \lambda_I}, \tag{3.3}$$

$$\frac{dU_1}{dt} = -\lambda_E U_1 + V, \tag{3.4}$$

$$\frac{dU_2}{dt} = -\lambda_I U_2 + V. \tag{3.5}$$

Choudhary et. al. [7] have investigated the temporal information processing of the neuron model shown by Eq. 3.3 and suggested that the model is capable to generate uni-modal, bi-modal, multi-modal ISI distribution patterns. Few of these ISI distribution patterns exhibit long tail behavior and power-law behavior can observed in the long tail of such kind of ISI distribution patterns.

## 4. SIMULATION STUDY

Simulation based study has been performed for LIF neuron model given in Eq. 2.1 and LIF neuron in DDF with hypo-exponential distributed delay kernel functions as given in Eq. 3.3. These equations form a system of coupled stochastic differential equations (SDEs). Monte Carlo numerical simulation technique has been used to perform the study. There a number of numerical simulation schemes are suggested for solving SDEs in literature which results similar qualitative results [11]. We apply Euler-Maruyama numerical simulation scheme for considered neuron models in DDF.

We compute the inter-spike-intervals for these considered neuron models. A small time interval is added with inter-spike-interval as a refractory time period (i.e. a small time interval in which the neuron does not participate into neuronal information processing). Both the models are investigated in the presence of Gaussian distributed refractory time period [20, 21]. Inter-spike-interval distributions with different set of other parameter values are shown in Fig. 3 to Fig. 2 in next section.

## 5. RESULT ANALYSIS

We investigate the ISI distribution of LIF neuron in DDF with hypo-exponentially distributed delay (HEDD) kernel function and compare the findings with the simple LIF neuron with stochastic input as given in Eqs. 2.1 and 3.3, respectively.

Fig. 3 to Fig. 2 illustrates the ISI distribution pattern for the LIF neuron model with Gaussian distributed refractory time period. Here mean of the Gaussian distribution is considered as 4 ms and 1 ms as the standard deviation. Fig. 3 shows the ISI distribution pattern with small excitatory and inhibitory arrival rates ISI distribution patterns for LIF neuron in DDF is more consistent than simple LIF neuorn.This occurs due to the Gaussian distributed refractory time period which reduces the effect of noisy environment.

Fig 4 shows the ISI distribution pattern for increased excitatory arrival rates. Increased excitatory arrival rate along with Gaussian distributed refractory time

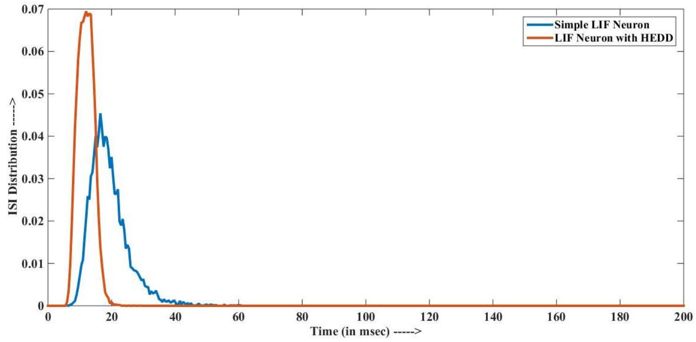


FIGURE 1. ISI Distribution with $\beta = 0.2$, $\mu = 0.2$, $\sigma = 0.1$, $\lambda_E = 0.2$ and $\lambda_I = 0.5$

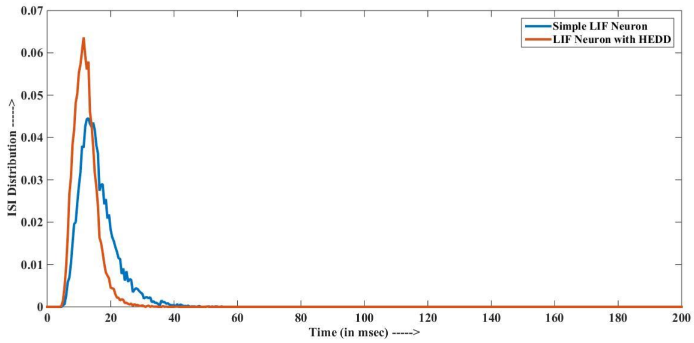


FIGURE 2. ISI Distribution with $\beta = 0.2$, $\mu = 0.2$, $\sigma = 0.2$, $\lambda_E = 0.8$ and $\lambda_I = 0.2$

period is bringing the homogeneity in the spiking activity. The rage of ISI distribution pattern for both the neuron models are almost same.

Fig. 1 shows the ISI distribution pattern for increased inhibitory arrival rates as compared with excitatory arrival rates and membrane decay constant with increased noise. The noise is bringing the long tail behavior where as Gaussian distributed refractory time period is bring the similar quantitative behavior in the ISI distribution pattern. Fig. 2 shows the perfect matching of qualitative and quantitative behavior in ISI distribution pattern for the considered neuron models. This ISI distribution pattern is obtained for negligible excitatory membrane potential. Here, only inhibitory membrane potential along with refractory time period are representing the similar distribution pattern as simple LIF neuron model. This provides a combination of parameter values and refractory time

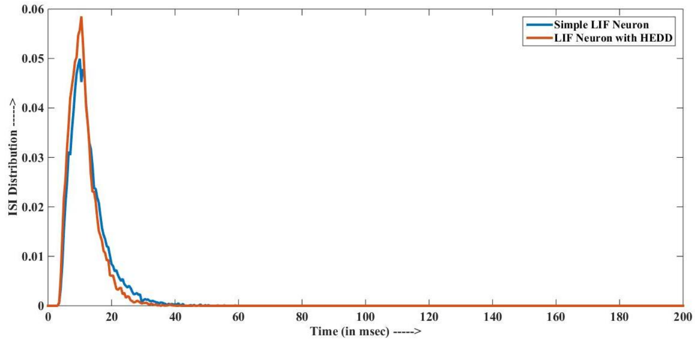


FIGURE 3. ISI Distribution with $\beta = 0.3$, $\mu = 0.2$, $\sigma = 0.5$, $\lambda_E = 0.5$ and $\lambda_I = 1$

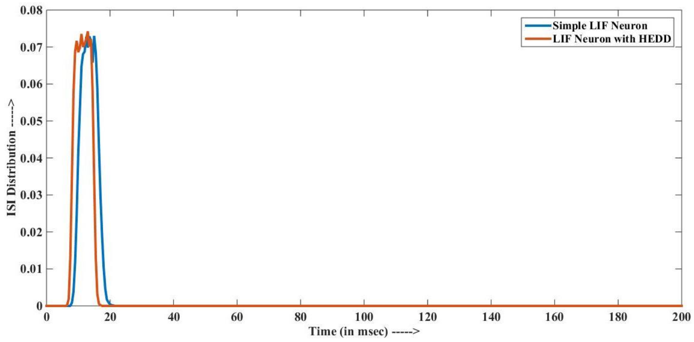


FIGURE 4. ISI Distribution with $\beta = 0.1$, $\mu = 0.1$, $\sigma = 0.05$, $\lambda_E = 0.01$ and $\lambda_I = 1$

period where the LIF neuron has no effect of distributed delay on its spiking behavior.

## 6. Conclusion

LIF neuron model can be considered as the backbone in artificial intelligence. It is widely used for implementing the neuronal networks at software level as well as hardware level. LIF neuron model has been investigated in various forms to understand the real neuronal activity. Distributed delay kernel function brings the memory element in the neuronal information processing. LIF neuron model with distributed delay is more closure to the realistic neuron as compared with other threshold based neuron model. We have investigated the LIF neuron model in distributed delay framework with refractory time period. The refractory time has been considered as the Gaussian distributed, which is found more satisfactory as ISI distribution patterns are more closure to the experimental studies. Moreover, we notice the set of parameter values with Gaussian distributed refractory time period where effect of previous values of membrane potential becomes negligible. This study suggests a way to include the refractory time period and also its choice. This study can be extended for software level and hardware level implementation of LIF neuron model.

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