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EDGE IRREDUNDANT FUNCTION IN INTUITIONISTIC FRACTIONAL GRAPH

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ABSTRACT. In this paper, the concept of edge irredundant function and maximal edge irredundant function in an intuitionistic fractional graph has been introduced and its parameters such as intuitionistic fractional edge irredundance number and upper intuitionistic fractional edge irredundance number of intuitionistic fractional graphs are found. Also we introduced the edge irreducible function and maximal edge irreducible function of intuitionistic fractional graphs with suitable illustrations.

1. INTRODUCTION

Let a graph $G = (V, E)$ be a connected graph. The open neighborhood $N(v)$ and the closed neighborhood $N[v]$ of v are defined by $N(v) = \{u \in V : uv \in E\}$ and $N[v] = N(v) \cup v$ [2] respectively. Let $g : E \to [0, 1]$ be a function which weights are assigns to each edge of a graph in the interval [0,1]. We say g is an edge irredudant function if for every edge $e \in E(G)$ with $g(e) > 0$ there exists an edge $x \in N[e]$ such that $g(N[x]) = 1$. An edge irredundant function g is maximal if for all function $f : E \to [0, 1]$ with $f > g$, f is not an edge irredundant function. The fractional edge irredundance number and the upper fractional edge irredundance number is $ir'_f(G) = \min\{|g| : g$ is an maximal edge irredundant function

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of G} and $IR_f^{'}(G) = \max\{|g| : g$ is an maximal edge irredundant function of G} in [1]. An intuitionistic fuzzy graph[IFG] or intuitionistic fractional graph[IFG] $G = (V, E)$ is a pair (V, E) in which V is a finite set and E is an intuitionistic fuzzy set of 2-element subsets of V. Alternatively, one might allow V to be an intuitionistic fuzzy set as well. That is an intuitionistic fuzzy graph(IFG) [3] is of the form $G = (V, E, \mu, \nu)$, where $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\nu_1 : V \to [0, 1]$ denote the degree of membership and non - membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, for every $v_i \in V$ $(i = 1, 2, \dots, n)$ and $E \subset V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\nu_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)], \nu_2(v_i, v_j) \le \max[\nu_1(v_i), \nu_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E(i, j = 1, 2, \dots, n)$.Motivated by this concept, we introduce the concept of edge irredundant function and edge irreducible function of an intuitionistic fractional graph. $\mu-$edge irredundant function, $\nu-$edge irredundant function, edge irredundant function, maximal edge irredundant function, edge irreducible function and maximal edge irreducible function of an intuitionistic fractional graph are defined in Section 2 and also to derived a linear programming problem, for finding out intuitionistic fractional edge irredundance number and upper intuitionistic fractional edge irredundance number of G. In Section 3, the relationship between maximal edge irredundant function and maximal edge irreducible function of an intuitionistic fractional graph have been studied. Section 4 concludes the paper.

2. Preliminaries

Definition 2.1. *A function $f_{\mu_2} : E \to [0, 1]$ is called a μ-edge irredundant function of an IFG $G = (V, E)$ if for every $e \in E$ such that $\sum_{u \in N_{\mu_2}[w]} f_{\mu_2}(u) = 1$ for any $w \in N_{\mu_2}[e]$.*

Definition 2.2. *A function $f_{\nu_2} : E \to [0, 1]$ is called a ν-edge irredundant function of an IFG $G = (V, E)$ if for every $e \in E$ where $\nu_2(e) \neq 1$ such that $\sum_{u \in N_{\nu_2}[w]} f_{\nu_2}(u) < 1$ for any $w \in N_{\nu_2}[e]$.*

Definition 2.3. *A function $f_{\mu_2,\nu_2} : E \to [0, 1]$ is called an edge irredundant function an IFG G if it is μ-edge irredundant and ν-edge irredundant function of G with $0 \le f_{\mu_2}(e) + f_{\nu_2}(e) \le 1$ for each $e \in E$ or A function $f = f_{\mu_2,\nu_2} : E \to [0, 1]$ is called* *an edge irredundant function of an IFG $G = (V, E)$ in which V is a intuitionistic fuzzy set and E is a 2-element subsets of V if for every $e \in E$ where $\mu_2(e) \geq 0, \nu_2(e) \neq 1$ such that $\sum_{u \in N_{\mu_2}[w]} f_{\mu_2}(u) = 1$, $\sum_{u \in N_{\nu_2}[w]} f_{\nu_2}(u) < 1$ for any $w \in N[e]$ with $0 \leq f_{\mu_2}(e) + f_{\nu_2}(e) \leq 1$ for each $e \in E$.*

Definition 2.4. *An edge irredundant function $f = f_{\mu_2,\nu_2}$ of an IFG G is called a maximal edge irredundant function of G if for all function $g : E \to [0, 1]$ with $g < f, g$ is not an edge irredundant function.*

Definition 2.5. *The intuitionistic fractional edge irredundance number of an IFG G, denoted by $ir^{'}_{if}(G)$ is defined as, $ir^{'}_{if}(G) = \min\{|f| : f$ is an maximal edge irredundant function of G} where $|f| = \sum_{e \in E} f(e) = (\sum_{e \in E} f_{\mu_2}(e), \sum_{e \in E} f_{\nu_2}(e))$ or $ir^{'}_{if}(G) = (ir^{'}_{if_{\mu_2}}(G), ir^{'}_{if_{\nu_2}}(G))$ where $ir^{'}_{if_{\mu_2}}$ is a f_{μ_2}-intuitionistic fractional edge irredundance number and $ir^{'}_{if_{\nu_2}}$ is a f_{ν_2}-intuitionistic fractional edge irredundance number of G.*

Definition 2.6. *The upper intuitionistic fractional edge irredundance number of an IFG G, denoted by $IR^{'}_{if}(G)$ is defined as, $IR^{'}_{if}(G) = \max\{|f| : f$ is an maximal edge irredundant function of G} where $|f| = \sum_{e \in E} f(e) = (\sum_{e \in E} f_{\mu_2}(e), \sum_{e \in E} f_{\nu_2}(e))$ or $IR^{'}_{if}(G) = (IR^{'}_{if_{\mu_2}}(G), IR^{'}_{if_{\nu_2}}(G))$ where $IR^{'}_{if_{\mu_2}}$ is a f_{μ_2}-upper intuitionistic fractional edge irredundance number and $IR^{'}_{if_{\nu_2}}$ is a f_{ν_2}-upper intuitionistic fractional edge irredundance number of G.*

Observation 2.1. *The problem of finding the μ-edge irredundant intuitionistic fractional edge irredundance number $(ir^{'}_{if_{\mu_2}})$ is equivalent to finding the optimal solution of the following linear programming problem.*

$$\text{Minimize } Z = \sum_{e_i \in E(G)} f_{\mu_2}(e_i)$$

$$\text{Subject to } \sum_{x \in N[e_i]} f_{\mu_2}(x) \geq 1 \quad \text{and} \quad 0 \leq f_{\mu_2}(e_i) \leq 1 \forall e_i \in E(G).$$

The problem of finding the μ-edge irredundant upper intuitionistic fractional edge irredundance number $(IR^{'}_{if_{\mu_2}})$ is equivalent to finding the optimal solution of the following linear programming problem.

$$\text{Maximize } Z = \sum_{e_i \in E(G)} f_{\mu_2}(e_i)$$

$$\text{Subject to } \sum_{x \in N[e_i]} f_{\mu_2}(x) \leq 1 \quad \text{and} \quad 0 \leq f_{\mu_2}(e_i) \leq 1 \forall e_i \in E(G).$$

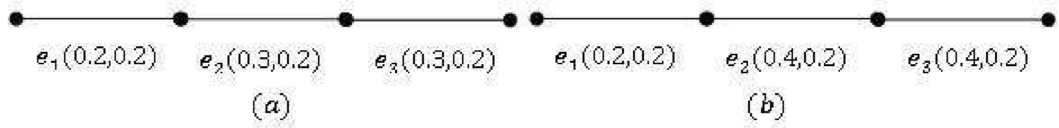

FIGURE 1. Examples of edge irreducible functions

The problem of finding the ν-edge irredundant intuitionistic fractional edge irredundance number $(ir'_{if_{\nu_2}})$ is equivalent to finding the optimal solution of the following linear programming problem.

$$\text{Minimize } Z = \sum_{e_i \in E(G)} f_{\nu_2}(e_i)$$
$$\text{Subject to } \sum_{x \in N[e_i]} f_{\nu_2}(x) \geq 1 \quad \text{and} \quad 0 \leq f_{\nu_2}(e_i) \leq 1 \forall e_i \in E(G).$$

The problem of finding the ν-edge irredundant upper intuitionistic fractional edge irredundance number $(IR'_{if_{\nu_2}})$ is equivalent to finding the optimal solution of the following linear programming problem.

$$\text{Maximize } Z = \sum_{e_i \in E(G)} f_{\nu_2}(e_i)$$
$$\text{Subject to } \sum_{x \in N[e_i]} f_{\nu_2}(x) \leq 1 \quad \text{and} \quad 0 \leq f_{\nu_2}(e_i) \leq 1 \forall e_i \in E(G)$$

Definition 2.7. *Let $G = (V, E)$ be a IFG. A function $f : E \to [0, 1]$ is edge irreducible, if for every $e \in E$ where $\nu_2(e) \neq 1$ such that*

$$\sum_{u \in N_{\mu_2}[e]} f_{\mu_2}(u) \leq 1, \qquad \sum_{u \in N_{\nu_2}[e]} f_{\nu_2}(u) < 1$$

for any $u \in N[e]$.

Definition 2.8. *An edge irreducible function f on an IFG G, there exist an maximal edge irreducible function f' with $f \leq f'$ such that $f(e) \leq f'(e)$ for every $e \in E$.*

In Figure 1, the IFG G with two different edge irreducible function f and f' as shown in (a) and (b). Note that f is edge irreducible function and f' is maximal edge irreducible function of an IFG G whereas $f \leq f'$.

Remark 2.1. *All edge irredundant functions an IFG G are edge irreducible function of G.*

3. Maximal Edge irredundant functions

Lemma 3.1. *If $f : E \to [0,1]$ is a maximal edge irreducible function of an IFG G if and only if f is a maximal edge irredundant function of G.*

Proof. The proof is trivial from the Definition 2.4 and 2.8. □

Lemma 3.2. *If $f : E \to [0,1]$ is not an edge irredundant an IFG G but edge irreducible function. Then there exists a edge e such that $f(e) < 1$ and f_e is maximal edge irreducible function.*

Proof. We assume that $f : E \to [0,1]$ is not an edge irredundant function of an IFG G. Then for every $e \in E$ where $\nu_2(e) \neq 1$ such that

$$\sum_{u \in N_{\mu_2}[e]} f_{\mu_2}(u) \neq 1, \sum_{u \in N_{\nu_2}[e]} f_{\nu_2}(u) < 1$$

for any $u \in N[e]$. Since f is irreducible,

$$\sum_{u \in N_{\mu_2}[e]} f_{\mu_2}(u) < 1, \sum_{u \in N_{\nu_2}[e]} f_{\nu_2}(u) < 1$$

for any $u \in N[e]$. Consider the values of d_1 and d_2 can be determined by the minimum of some edge whose f-neighborhood sum is strictly less than one and it's subtract from one as defined as follows:

$$d_1 = \min\{1 - \sum_{u \in N_{\mu_2}[e]} f_{\mu_2}(u) \mid u \in E, \sum_{u \in N_{\mu_2}[e]} f_{\mu_2}(u) < 1\} \tag{1}$$

$$d_2 = \min\{1 - \sum_{u \in N_{\nu_2}[e]} f_{\nu_2}(u) \mid u \in E, \sum_{u \in N_{\nu_2}[e]} f_{\nu_2}(u) < 1\} \tag{2}$$

Now we define

$$f_{e_i}(\mu_2(u)) = \begin{cases} f(\mu_2(u)) + d_1 & \text{if } u = e_i \\ f(\mu_2(u)) & \text{otherwise} \end{cases} \tag{3}$$

and

$$f_{e_j}(\nu_2(u)) = \begin{cases} |d_2 - f(\nu_2(u))| & \text{if } u = e_j \\ f(\nu_2(u)) & \text{otherwise} \end{cases}, \tag{4}$$

where e_i and e_j are taken from d_1 and d_2.

Since

$$f_{e_i}(\mu_2(e_i)) = f(\mu_2(e_i)) + d_1 \leq \sum_{u \in N_{\mu_2}[e_i]} \mu_2(u) + d_1 \leq 1$$

and

$$0 \leq f_{e_j}(\nu_2(e_j)) = |d_2 - f(\nu_2(e_j))| \leq |d_2 - \sum_{u \in N_{\nu_2}[e_j]} \nu_2(u))| < 1$$

where $f_e = f_{e_i,e_j} : E \to [0,1]$.

Note that since any edge in $N_{\mu_2}[e]$ has a f_{e_i}-neighborhood sum of 1,

$$f_{e_i}(\sum_{u \in N_{\mu_2}[e]} \mu_2(u)) \leq 1, f_{e_j}(\sum_{u \in N_{\nu_2}[e]} \nu_2(u)) < 1$$

for every edge u for which $f(\sum_{u \in N_{\mu_2}[e]} \mu_2(u)) \leq 1, f(\sum_{u \in N_{\nu_2}[e]} \nu_2(u)) < 1$.

Thus f_e is maximal irreducible of G. □

Remark 3.1. *We will define the new function f_e constructed from f, shown in $(3),(4)$. Note however, that f_e is well-defined only if the set on the right-hand side of $(1),(2)$ is nonempty. That is, there must be some edge whose f-neighborhood sum is strictly less than one.*

Lemma 3.3. *Let f be an edge irredundant function of an IFG G that is not maximal edge irredundant function. Then there exists a edge e such that $f(e) < 1$ and f_e is edge irreducible.*

Proof. Since f is not maximal edge irredundant of an IFG G, then there exist an edge irredundant function g and a edge e such that $g(e) > f(e)$. Since g is edge irredundant, there must exist a edge $u \in N[e]$ with $g(N_{\mu_2}[u]) = 1, g(N_{\nu_2}[u]) < 1$ where $N_{\mu_2}[u] = \sum_{w \in N_{\mu_2}[u]} g_{\mu_2}(w), N_{\nu_2}[u] = \sum_{w \in N_{\nu_2}[u]} g_{\nu_2}(w)$. Since $g(e) > f(e)$, it follows that $f(N_{\mu_2}[u]) < 1$ and $f(N_{\nu_2}[u]) < 1$. Thus $f_e = f_{e_i,e_j} : E \to [0,1]$ is well-defined.

Note also that since $f(e) + d_1 \leq f(N_{\mu_2}[u]) + d_1 \leq 1$ and $0 \leq |d_2 - f(e)| \leq |f(N_{\nu_2}[u]) - d_2|, f$ maps E into $[0,1]$. Assume that $f_e(e) > 0$ for some edge e. Note that $f_e(e) > 0$ implies $g(e) > 0$ since $g \geq f$. Thus there exists $z \in N[e]$ with $g(N_{\mu_2}[z]) = 1, g(N_{\nu_2}[z]) < 1$ where

$$N_{\mu_2}[z] = \sum_{u \in N_{\mu_2}[z]} g_{\mu_2}(u), N_{\nu_2}[z] = \sum_{u \in N_{\nu_2}[z]} g_{\mu_2}(u).$$

But $g(N_{\mu_2}[z]) = 1, g(N_{\nu_2}[z]) < 1$ implies $f_{e_i}(N_{\mu_2}[z]) \leq 1, f_{e_j}(N_{\nu_2}[z]) < 1$. since if $w \in N[z]$ and $g(N_{\mu_2}[z]) = 1, g(N_{\nu_2}[z]) < 1$ then $f(N_{\mu_2}[z]) < 1, f(N_{\nu_2}[z]) < 1$ so $f_{e_i}(N_{\mu_2}[z]) = f(N_{\mu_2}[z]) + d_1 \leq 1, f_{e_j}(N_{\nu_2}[z]) = f(N_{\nu_2}[z]) - d_2 < 1$ and if $w \notin N[z]$

and $g(N_{\mu_2}[z]) = 1, g(N_{\nu_2}[z]) < 1$ then $f_{e_i}(N_{\mu_2}[z]) = f(N_{\mu_2}[z]) \leq g(N_{\mu_2}[z]) = 1, f(N_{\nu_2}[z]) = f_{e_j}(N_{\nu_2}[z]) \leq g(N_{\nu_2}[z]) < 1$. Hence f_e is edge irreducible. □

Lemma 3.4. *An edge irredundant function f on an IFG G is maximal if and only if for no edge e, is f_e edge irreducible.*

Proof. Let f be edge irredundant, and assume no f_e is edge irreducible of an IFG G. By Lemma 3.3, f must be maximal. Conversely, assume f is maximal edge irredundant. Then by Lemma 3.1, f is maximal edge irreducible, and so no f_e can be edge irreducible. □

Theorem 3.1. *In any IFG G, the non membership value of fractional edge irredundance number $ir'_{if_{\nu_2}}(G)$ is always zero.*

Proof. Follows from the definition 2.2 and observation 2.1. □

Theorem 3.2. *In any IFG G, the membership value of fractional edge irredundance number $ir'_{if_{\mu_2}}(G)$ and the non membership value of upper fractional edge irredundance number $IR'_{if_{\nu_2}}(G)$ is always equal. That is $ir'_{if_{\mu_2}}(G) = IR'_{if_{\nu_2}}(G)$.*

Proof. Follows from the observation 2.1. □

Illustration:

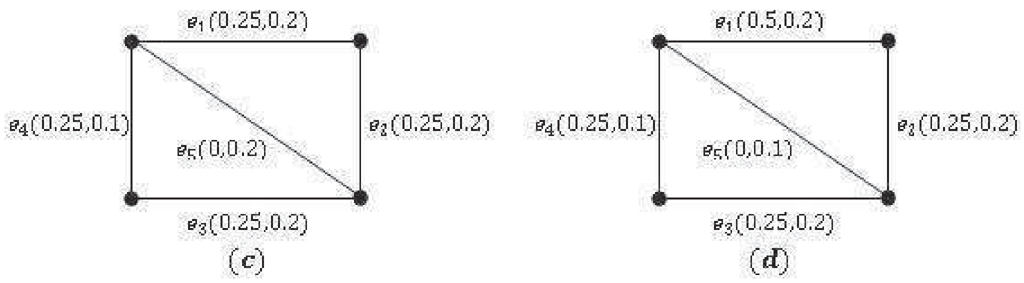

FIGURE 2

Now we formulate L.P.P. for the graph in Figure 2(c), to finding the intuitionistic fractional edge irredundance number $ir'_{if}(G)$ and upper intuitionistic fractional edge irredundance number $IR'_{if}(G)$ of G, which is equivalent to finding the optimal solution of the L.P.P. for the Figure 2(c). Consider the graph in Figure 2(c), we formulate the following L.P.P.:

$$\text{Minimize } Z = \sum_{e_i \in E(G)} f_{\mu_2}(e_i)$$

$$\text{Subject to } f_{\mu_2}(e_1) + f_{\mu_2}(e_2) + f_{\mu_2}(e_3) + f_{\mu_2}(e_4) + f_{\mu_2}(e_5) \geq 1$$

$$\text{and } 0 \leq f_{\mu_2}(e_i) \leq 1 \text{ forall } e_i \in E(G)$$

That is

$$\text{Minimize } Z = f_{\mu_2}(e_1) + f_{\mu_2}(e_2) + f_{\mu_2}(e_3) + f_{\mu_2}(e_4) + f_{\mu_2}(e_5)$$

$$\text{Subject to } \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} f_{\mu_2}(e_1) \\ f_{\mu_2}(e_2) \\ f_{\mu_2}(e_3) \\ f_{\mu_2}(e_4) \\ f_{\mu_2}(e_5) \end{pmatrix} \geq \begin{pmatrix} 1 \end{pmatrix}$$

$$\text{and } 0 \leq f_{\mu_2}(e_i) \leq 1 \text{ for all } e \in E(G).$$

Then $ir'_{if_{\mu_2}}(G) = 1$. Similarly we have to find $ir'_{if_{\nu_2}}(G) = 0, IR'_{if_{\mu_2}}(G) = 1$ and $IR'_{if_{\nu_2}}(G) = 1$. Hence $ir'_{if}(G) = (1, 0)$ and $IR'_{if}(G) = (1, 1)$.

Edge	**f in Figure 2(c), $d_1 = 0.25(e_1)$ and $d_2 = 0.1(e_5)$**						**$f_e = f_{e_1,e_5}$ in Figure 2(d)**			
(e_i)	$f_{\mu_2}(e_i)$	$f_{\nu_2}(e_i)$	$f(N_{\mu_2}[e_i])$	$f(N_{\nu_2}[e_i])$	d_1	d_2	$f_{e_{1_{\mu_2}}}(e_i)$	$f_{e_{1_{\nu_2}}}(e_i)$	$f_{e_1}(N_{\mu_2}[e_i])$	$f_{e_5}(N_{\nu_2}[e_i])$
e_1	0.25	0.2	0.75	0.7	**0.25**	0.3	0.5	0.2	1	0.6
e_2	0.25	0.2	0.75	0.8	0.25	0.2	0.25	0.2	1	0.7
e_3	0.25	0.2	0.75	0.7	0.25	0.3	0.25	0.2	0.75	0.6
e_4	0.25	0.1	0.75	0.7	0.25	0.3	0.25	0.1	1	0.6
e_5	0	0.2	1	0.9	–	**0.1**	0	0.1	1.25	0.8

For an IFG G with an edge irredundant function, maximal edge irredundant function such as f its shown in Figure 2(c) and no f_e can be edge irreducible function its shown in Figure 2(d) of the above Figure(Refer:Table).

4. Conclusion

The study of intuitionistic fractional edge irredundance parameters is a relatively new and rapidly developing field. In this paper, we introduce the concept of intuitionistic fractional edge irredundance number, upper intuitionistic fractional edge irredundance number and maximal edge irreducible function for an intuitionistic fractional graphs have been discussed. Further real life problems can be premeditated and solutions can be predicted by using these intuitionistic fractional graphs. In future we will study some more properties and applications of edge irredundant function in intuitionistic fractional graphs.

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