ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7027–7036
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.52 Spec. Issue on MEM-2020

THE PACIFYING AND SHRINKING EDGES OF SOME GRAPHS

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ABSTRACT. Given a vertex v of a graph G, an edge e of G^c is called a pacifying edge of v if the addition of the edge e to G decreases the eccentricity of v the most. We identify the pacifying edges of the vertices of cycles and a more general class of graphs called Symmetric Even Graphs. An e in G^c is called a shrinking edge of G if the addition of e to G decreases the radius of G the most. Shrinking edges of the above mentioned classes are identified.

1. INTRODUCTION

Centrality is one of the most important concepts in graph theory as it measures the relative importance of a vertex in the graph. Eccentricity measures how far is a vertex is from the furthest in the graph. In some cases it is desirable to reduce the eccentricity of a vertex by introducing additional edges to the graph. This is in fact an optimization problem as the reduction in the eccentricity caused by different set of edges are different. Hence the objective becomes to determine the minimum number of edges that may be added so that the eccentricity is reduced the most. One special case of this problem is when addition of only a single edge is permissible. That is we determine a pair of non-adjacent vertices such that adding an edge between them reduces the eccentricity to a minimum. This has applications in various types of networks such as computer networks where the problem is to connect an optimal pair of nodes such that

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2010 *Mathematics Subject Classification.* 05C12, 05C35.

Key words and phrases. Pacifying Edges, Shrinking Edges, Symmetric Even Graphs.

the maximum distance of a node to any part of the network is reduced, social networks where a link between two persons reduces the maximum distance of a person in the network.

We consider only finite simple connected graphs. For the graph G, $V(G)$ denotes the vertex set and $E(G)$ denotes the edge set. For a vertex w of G the *eccentricity*, $e_G(w)$ is $\max_{u \in V} d(u, w)$. When the underlying graph is obvious we shall use V, E and $e(w)$ for $V(G)$, $E(G)$ and $e_G(w)$ respectively. The *radius* of G is the minimum eccentricity among the vertices of G and the *diameter* of G is the maximum eccentricity among the vertices of G. A vertex v is an *eccentric vertex* of u if $e(u) = d(u, v)$. A graph G is called *even* if for each vertex u of G there is a unique eccentric vertex, $\bar{u}$, such that $d(u, \bar{u}) = diam(G)$. Even graphs are referred to as diametrical graphs in [5] and as self-centered unique eccentric point graphs in [7].

Graphs having extremal properties with respect to distance parameters like radius and diameter have been studied extensively. Ore in [6] defined a graph to be *diameter maximal* if the addition of any edge to the graph decreases the diameter of the graph and gave a characterisation of such graphs. Caccetta and Smyth [1] gave a general form of diameter maximal graphs with edge connectivity k, diameter d, number of vertices n and having the maximum number of edges.

A graph G is *diameter minimal* if the deletion of any edge increases the diameter of G. This class of graphs were studied by many authors [1–3, 8].

A graph G is called *radius minimal* if radius of $G - e$ is greater than radius of G for every edge of G. Gliviak [2] proved that a graph is radius minimal if and only if it is a tree.

Any graph G such that radius of $G + e \leq$ radius of G for every $e \in G^c$ is called a *radially maximal* graph. Vizing in [9] found an upper bound on the number of edges in radially maximal graphs and a lower bound was found by Nishanov. Nishanov studied some properties of radially maximal graphs with radius $r \geq 3$ and diameter $2r - 2$.

Knor characterized unicyclic, non-selfcentric, radially-maximal graphs on the minimum number of vertices. He further proved that the number of such graphs is $\frac{1}{48}r^3 + \mathcal{O}(r^2)$. It was conjectured that if G is a non-selfcentric radially-maximal graph with radius $r \geq 3$ on the minimum number of vertices then G is planar, has exactly $3r-1$ vertices, the maximum degree of G is 3 and the minimum degree of G is 1. Knor with the help of exhaustive computer search proved this result for $r=4$ and 5.

In this paper we focus on the extremal properties of the individual vertices of the graph.

2. PACIFYING EDGES

For a vertex $w \in G$ an edge uv is called a *pacifying edge* of w if $e_{G+uv}(w) \leq e_{G+xy}(w)$ for all $xy \in E(G)$. It is not necessary that every vertex of a graph has pacifying edges. One trivial example is the complete graph where every vertex has eccentricity one. There are other non trivial examples. Take the complete bipartite graph $K_{m,n}$ where $m,n > 2$. Each vertex of this graph has eccentricity two. Since $m,n > 2$ by adding an edge between any single pair of non-adjacent vertices the eccentricity of none of the vertices reduces. In other words no vertex of $K_{m,n}$ has a pacifying edge. C_5 is another example of a graph in which no vertex has a pacifying edge.

The following is an example of a graph in which some vertices have pacifying edges while some others do not have any pacifying edge. Here, xw, uy and zv

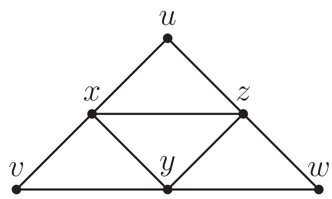

FIGURE 1.

are the pacifying edges of x,y and z respectively as they reduce the eccentricity of these vertices from 2 to 1. But, the vertices u,v and w do not have any pacifying edge.

An even graph G is $symmetric$ if for every $u \in V(G)$ there exists a $v \in V(G)$ such that $I(u,v)=G$, see Gobel et.al [4]. Hypercubes, even cycles etc are well known examples of symmetric even graphs. From the definition of symmetric even graphs it is clear that they are unique eccentric vertex graphs and every vertex of such a graph is an eccentric vertex. For more examples of symmetric even graphs and one way of constructing such graphs, see, Gobel et.al [4].

Before we state the next theorem we introduce the following notations.
For integers i and j,

$$\begin{aligned} i +_n j &= i + j \text{ if } i + j \leq n \\ &= i + j - n \text{ if } i + j > n \end{aligned}$$

$$\begin{aligned} i -_n j &= i - j \text{ if } i - j > 0 \\ &= n + i - j \text{ if } i - j \leq 0 \end{aligned}$$

Theorem 2.1. *Consider the odd cycle $C_{2n+1} (n > 2)$ with vertex set $\{v_1, \ldots, v_{2n+1}\}$.*

(1) If n is even the pacifying edges of a vertex v_i are
   (a) $v_i v_{i+_{2n+1} n}$
   (b) $v_i v_{i+_{2n+1}(n+1)}$
   (c) $v_i v_{i+_{2n+1}(n+2)}$
   (d) $v_i v_{i+_{2n+1}(n-1)}$
   (e) $v_{i+_{2n+1}1} v_{i+_{2n+1}(n+1)}$
   (f) $v_{i-_{2n+1}1} v_{i+_{2n+1}n}$
   (g) $v_{i+_{2n+1}1} v_{i+_{2n+1}n}$
   (h) $v_{i-_{2n+1}1} v_{i+_{2n+1}(n+1)}$

(2) If n is odd the pacifying edges v_i are $v_i v_{i+_{2n+1} n}$ and $v_i v_{i+_{2n+1}(n+1)}$.

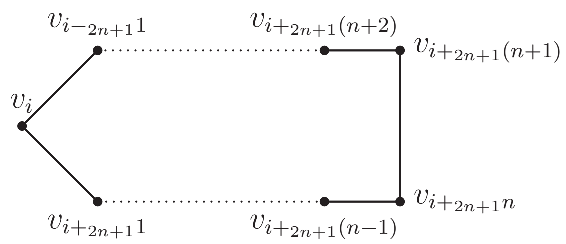

FIGURE 2.

Proof.

(1) Suppose n is even. Add the edge $v_i v_{i+_{2n+1} n}$. Then we get two cycles, say C_1' and C_2', both containing v_i and having $n + 2$ and $n + 1$ edges. $n + 2$ is even and v_i has eccentricity $\frac{n}{2} + 1$ in C_1' and hence in the graph

$G+v_iv_{i+_{2n+1}n}$. Similarly by adding the edge $v_iv_{i+_{2n+1}(n+1)}$ the eccentricity of v_i reduces to $\frac{n}{2}+1$.

Adding the edge $v_iv_{i+_{2n+1}(n+2)}$ we get cycles C_1' and C_2' where C_1' has $n+3$ edges and C_2' has n edges and both contain the vertex v_i. C_1' has radius $\frac{n}{2}+1$ and C_2' has radius $\frac{n}{2}$. Therefore v_i has eccentricity $\frac{n}{2}+1$ in the new graph. Similarly adding the edge $v_iv_{i+_{2n+1}(n-1)}$ reduces the eccentricity of v_i to $\frac{n}{2}+1$.

Adding an edge between v_i and a vertex other than $v_{i+_{2n+1}n}$, $v_{i+_{2n+1}(n+1)}$, $v_{i+_{2n+1}(n+2)}$, $v_{i+_{2n+1}(n-1)}$ we get two cycles C_1' and C_2', both containing v_i, and one of them having radius greater than $\frac{n}{2}+1$. Therefore eccentricity of v_i in such a graph is greater than $\frac{n}{2}+1$. Now we add an edge between v_j and v_k such that $j,k\neq i$. Let C_1' and C_2' be the resulting two cycles. Take two cases.

Case-I: $v_i\in C_1'$ where $|E(C_1')|<|E(C_2')|$. That is, C_2' has atleast $n+2$ edges or radius of C_2' is atleast $\frac{n}{2}+1$. Assume $d(v_i,v_j)\leq d(v_i,v_k)$. Let $\bar{v_j}$ be the eccentric vertex of v_j in C_2'. That is $d(v_j,\bar{v_j})\geq\frac{n}{2}+1$. Therefore $d(v_k,\bar{v_j})\geq\frac{n}{2}$. Since $n>2$, $\frac{n}{2}>1$.

$$\begin{aligned}d(v_i,\bar{v_j})&=min\{d(v_i,v_j)+d(v_j,\bar{v_j}),d(v_i,v_k)+d(v_k,\bar{v_j})\}\\&\geq min\{d(v_i,v_j)+\frac{n}{2}+1,d(v_i,v_k)+\frac{n}{2}\},\end{aligned}$$

$d(v_i,\bar{v_j})=\frac{n}{2}+1$ only when $d(v_i,v_k)=d(v_i,v_j)=1$ and this implies $n=2$. Since $n>2$ we have $d(v_i,\bar{v_j})>\frac{n}{2}+1$. Hence v_jv_k is not a pacifying edge of v_i.

Case-II: $v_i\in V(C_2')$ where $E(C_2')>E(C_1')$. Here we shall consider two sub cases.

SubCase-I:$|E(C_2')|=n+2$ and $|E(C_1')|=n+1$. Assume $d(v_i,v_j)\leq d(v_i,v_k)$ Let $\bar{v_j}$ be the vertex that is eccentric to both v_j and v_k in C_1'. Then $d(v_i,\bar{v_j})=d(v_i,v_j)+\frac{n}{2}$. But $d(v_i,\bar{v_j})=\frac{n}{2}+1$ when v_i is adjacent to v_j. In this case we have that the eccentricity of v_i is $\frac{n}{2}+1$. In other words, for the vertex v_i, the edge v_jv_k such that v_j is adjacent to v_i and $d_{C_{2n+1}}(v_j,v_k)=d_{C_{2n+1}}(v_i,v_k)=n$ is a pacifying edge of v_i. Thus the edges $v_{i+_{2n+1}1}v_{i+_{2n+1}(n+1)}$ and $v_{i-_{2n+1}1}v_{i+_{2n+1}n}$ are pacifying edges of the vertex v_i.

Subcase-II: $|E(C_2')| = n+3$ and $|E(C_1')| = n$. Let $\bar{v_j}$ be the vertex eccentric to v_j in C_1'. Then

$$d(v_i, \bar{v_j}) = \begin{cases} d(v_i, v_j) + \frac{n}{2} \text{ if } d(v_i, v_j) < d(v_i, v_k) \\ d(v_i, v_k) + d(v_j, \bar{v_j}) - 1 \\ \qquad = d(v_i, v_j) + \frac{n}{2} - 1 \text{ if } d(v_i, v_j) = d(v_i, v_k) \end{cases}$$

and

$$d(v_i, v_j) = d(v_i, v_k) = 2 \implies n = 2.$$

But we have $n > 2$. Therefore $d(v_i, v_j) = d(v_i, v_k) \implies$ both are greater than 2. Hence $d(v_i, \bar{v_j}) \geq \frac{n}{2} + 2$. Therefore $v_j v_k$ is not a pacifying edge. Hence we assume that $d(v_i, v_j) < d(v_i, v_k)$. Then $d(v_i, \bar{v_j}) = d(v_i, v_j) + \frac{n}{2}$. Thus $d(v_i, \bar{v_j}) = \frac{n}{2} + 1$ if and if only if v_i is adjacent to v_j. In other words, for the vertex v_i, the edge $v_j v_k$ such that v_j is adjacent to v_i, $d_{C_{2n+1}}(v_j, v_k) = n - 1$ and $d(v_i, v_k) = n$ is a pacifying edge of v_i. Thus the edges $v_{i+_{2n+1}1} v_{i+_{2n+1}n}$ and $v_{i-_{2n+1}1} v_{i+_{2n+1}(n+1)}$ are pacifying edges of the vertex v_i.

Subcase-III: $|E(C_2')| > n+3$. In this case $e_{C_2'}(v_i) \geq \frac{n}{2} + 2$ or $e_{C_{2n+1}}(v_i) \geq \frac{n}{2} + 2$.

Thus we get that the pacifying edges of v_i are precisely

(a) $v_i v_{i+_{2n+1}n}$
(b) $v_i v_{i+_{2n+1}(n+1)}$
(c) $v_i v_{i+_{2n+1}(n+2)}$
(d) $v_i v_{i+_{2n+1}(n-1)}$
(e) $v_{i+_{2n+1}1} v_{i+_{2n+1}(n+1)}$
(f) $v_{i-_{2n+1}1} v_{i+_{2n+1}n}$
(g) $v_{i+_{2n+1}1} v_{i+_{2n+1}n}$
(h) $v_{i-_{2n+1}1} v_{i+_{2n+1}(n+1)}$

(2) Assume n is odd. Joining v_i to $v_{i+_{2n+1}n}$ we get two cycles C_1' and C_2' having $n+1$ and $n+2$ edges respectively. Then C_1' and C_2' have radii $\frac{n+1}{2}$. Therefore the eccentricity of v_i in both C_1' and C_2' is $\frac{n+1}{2}$ or eccentricity of v_i in the $G + v_i v_{i+_{2n+1}n}$ is $\frac{n+1}{2}$. Similarly by adding the edge $v_i v_{i+_{2n+1}(n+1)}$ the eccentricity of v_i reduces to $\frac{n+1}{2}$.

Now, let v_i be joined to any vertex other than $v_{i+_{2n+1}n}$ and $v_{i+_{2n+1}(n+1)}$. Then one of the cycles formed contains atleast $n+3$ edges. That is the radius of that cycle is $\frac{n+3}{2}$ or eccentricity of v_i in the new graph is atleast $\frac{n+3}{2}$. Hence any such edge cannot be a pacifying edge of v_i. Suppose we join v_j and v_k where $j, k \neq i$. Let C_1' and C_2' be the two cycles formed where $|E(C_1')| \leq |E(C_2')|$. Here we shall consider two cases.

Case-I: Suppose $v_i \in V(C_1')$. Take the following subcases.

Subcase-I: $|E(C_1')| = n+1$ and $|E(C_2')| = n+2$. Then C_1' is an even cycle. Let $d(v_i, v_j) < d(v_i, v_k)$. Let $\bar{v_j}$ be the eccentric vertex of v_j in C_2'. $d(v_i, \bar{v_j}) = d(v_i, v_j) + d(v_j, \bar{v_j}) = d(v_i, v_j) + \frac{n+1}{2} > \frac{n+1}{2}$. Hence $e_{G+v_jv_k} < \frac{n+1}{2}$. That is, v_jv_k is not a pacifying edge.

Subcase-II: If $|E(C_2')| \geq n+3$ then the radius of $C_2' \geq \frac{n+1}{2} + 1$. Then $d(v_i, \bar{v_j}) \geq \frac{n+1}{2} + 1$ where $\bar{v_j}$ is the eccentric vertex of v_j in C_2'. That is, the eccentricity of v_i in $G + v_iv_j$ is atleast $\frac{n+1}{2} + 1$. That is v_jv_k is not a pacifying edge.

Case-II: Suppose $v_i \in V(C_2')$. We have that $|E(C_2')| \geq n+2$. Again we consider two subcases cases.

Subcase-I: $|E(C_2')| = n+2$. Let $\bar{v_j}$ be the eccentric vertex of v_j in C_1'.

$$d(v_i, \bar{v_j}) = \begin{cases} d(v_i, v_j) + d(v_j, \bar{v_j}) \text{ if } d(v_i, v_k) > d(v_i, v_j) \\ d(v_i, v_j) + d(v_j, \bar{v_j}) - 1 \text{ if } d(v_i, v_k) = d(v_i, v_j) \end{cases}$$

$d(v_i, v_k) = d(v_i, v_j) \implies d(v_i, \bar{v_j}) = d(v_i, v_j) + \frac{n+1}{2} - 1.$

$d(v_i, v_k) = d(v_i, v_j) = 1 \implies$ our cycle is C_3 which is not the case.

Hence $d(v_i, v_j) > 1$ or $d(v_i, \bar{v_j}) < \frac{n+1}{2}$.

So v_jv_k cannot be a pacifying edge of v_i.

If $d(v_i, v_k) > d(v_i, v_j)$ then

$$\begin{aligned} d(v_i, \bar{v_j}) &= d(v_i, v_j) + d(v_j, \bar{v_j}) \\ &= d(v_i, v_j) + \frac{n+1}{2} \\ &> \frac{n+1}{2}. \end{aligned}$$

Hence $e_{G+v_jv_k}(v_i) > \frac{n+1}{2}$. That is, v_jv_k is not a pacifying edge.

Subcase-II $|E(C_2')| \geq n+3$. This implies $d(v_i, \bar{v}_i) \geq \frac{n+3}{2}$ where $\bar{v}_i$ is the eccentric vertex of v_i in $|C_2'|$.

In other words $v_i v_{i+2n+1n}$ and $v_i v_{i+2n+1(n+1)}$ are the only pacifying edges of v_i.

□

Now we shall find the pacifying edges of vertices of a symmetric even graph.

Theorem 2.2. *Let G be a Symmetric Even graph having diameter d. Then*

(1) If d is even then the only pacifying edge of a vertex v is $v\bar{v}$.

(2) If d is odd the pacifying edges of v are

(a) All edges vy such that y is either $\bar{v}$ or a vertex adjacent to $\bar{v}$.

(b) All edges $x\bar{v}$ such that x is either v or a vertex adjacent to v.

Proof. Let v_1 and v_2 be vertices such that $d(v_1, v) = r_1$, $d(v_2, \bar{v}) = r_2$ and $d(v_1, v) \leq d(v_2, v)$. Now consider the graph $G+v_1v_2$. If $d(v_1, v) = d(v_2, v)$, then $d_{G+v_1v_2}(v, \bar{v}) = d$ and hence the eccentricity of v does not decrease. So we can assume that $d(v_1, v) < d(v_2, v)$. Let u be a vertex belonging to a shortest $v - v_2$ path. If $d(u, v) = m$, then, since G is symmetric even, $d(\bar{u}, \bar{v}) = m$. Therefore $d(v_2, \bar{u}) \leq r_2 + m$. $d(v_2, \bar{u}) = r_2 + m - \ell$ implies $d(u, \bar{u}) = d - r_2 - m + r_2 + m - \ell = d - \ell$, a contradiction to fact that G is self centered. Therefore $d(v_2, \bar{u}) = r_2 + m$. That is the length of the shortest path from v to $\bar{u}$ in $G + v_1v_2$ passing through the edge v_1v_2 is $r_1 + 1 + r_2 + m$. Hence $d_{G+v_1v_2}(v, \bar{u}) = min\{d - m, r_1 + 1 + r_2 + m\}$. Let w be a vertex in the shortest $v - v_2$ path such that $d(w, v) = k$(ie $d(\bar{w}, \bar{v}) = k$) and $r_1 + 1 + r_2 + k = d - k$ or $d - k - 1$ according to the parity of $r_1 + r_2 + 1$ and d. For any vertex x such that $d(\bar{v}, x) < k$ we have that $d_{G+v_1v_2}(v, x) < r_1 + r_2 + 1 + k$ and for any vertex x such that $d(\bar{v}, x) > k$ we have $d_{G+v_1v_2}(v, x) < d - k$. That is $\bar{w}$ is an eccentric vertex of v in $G + v_1v_2$. Hence the eccentricity of v is $d_{G+v_1v_2}(\bar{w}, v)$. Now we shall consider two cases.

(1) Assume d is even. When $r_1 + r_2$ is odd $r_1 + r_2 + 1$ is even and hence $r_1 + r_2 + 1 + k = d - k$ or $k = \frac{d}{2} - \frac{r_1+r_2+1}{2}$ and therefore $e_{G+v_1v_2}(v) = r_1 + r_2 + 1 + k = \frac{d}{2} + \frac{r_1+r_2+1}{2}$.

When r_1+r_2 is even r_1+r_2+1 is odd and hence $r_1+r_2+1+k = d-k-1$ or $k = \frac{d}{2} - \frac{r_1+r_2+2}{2}$ and therefore $e_{G+v_1v_2}(v) = r_1 + r_2 + 1 + d = r_1 + r_2 + 1 + \frac{d}{2} - \frac{r_1+r_2+2}{2} = \frac{d}{2} + \frac{r_1+r_2}{2}$.

Thus $e_{G+v_jv_k}(v) = \frac{d}{2} + \lceil \frac{r_1+r_2}{2} \rceil$. This is a minimum when $r_1 = r_2 = 0$. That is, the only pacifying edge is $v\bar{v}$.

(2) Assume d is odd. When $r_1 + r_2$ is odd, $r_1 + r_2 + 1$ is even and hence $r_1+r_2+1+k = n-k-1$ or $x = \frac{d-1}{2} - \frac{r_1+r_2+1}{2}$ and therefore $e_{G+v_1v_2}(v_i) = r_1 + r_2 + 1 + d = \frac{d-1}{2} + \frac{r_1+r_2+1}{2}$. When $r_1 + r_2$ is even $r_1 + r_2 + 1$ is odd and hence $r_1 + r_2 + 1 + k = n - k$ or $x = \frac{d-1}{2} - \frac{r_1+r_2}{2}$ and therefore $e_{G+v_1v_2}(v) = r_1 + r_2 + 1 + k = \frac{d-1}{2} + \frac{r_1+r_2+2}{2}$.

Thus $e_{G+v_1v_2}(v) = \frac{d-1}{2} + \lfloor \frac{r_1+r_2+2}{2} \rfloor$. This is a minimum when $r_1 = r_2 = 0$ or $r_1 = 1, r_2 = 0$ or $r_1 = 0, r_2 = 1$. That is, the pacifying edges are

(a) All edges vy such that y is either $\bar{v}$ or a vertex adjacent to $\bar{v}$.

(b) All edges $x\bar{v}$ such that x is either v or a vertex adjacent to v.

□

3. Shrinking Edges

Definition 3.1. *For a graph G, an edge $uv \in E(G^c)$ is called a Shrinking Edge if $rad(G + uv) \leq rad(G + xy)$ for every $xy \in E(G^c)$.*

The following corollary identify the shrinking edges of an odd cycle.

Corollary 3.1. *(to theorem 2.1) Consider the cycle C_{2n+1} having vertex set $\{v_1, \dots, v_{2n+1}\}$. An edge v_iv_j in C^c_{2n+1} is a shrinking edge if and only if it is the pacifying edge of some vertex v_i.*

Proof. Let n be even. If v_iv_j, an edge of C^c_{2n+1}, is a pacifying edge of a vertex v_k then $e_{G+v_iv_j}(v_k) = \frac{n}{2} + 1$ and also for all $v_\ell \neq v_k$, we have $e_{G+v_iv_j}(v_\ell) \geq \frac{n}{2} + 1$. Therefore $rad(G + v_iv_j) = \frac{n}{2} + 1$. By adding a single edge(any of the pacifying edges) the eccentricity of every vertex can be reduced exactly to $\frac{n}{2}+1$. Therefore an edge is a shrinking edge if and only if it is a pacifying edge of some vertex. Similarly the case when n is odd. Here instead of $\frac{n}{2} + 1$ we have $\frac{n+1}{2}$. □

We have a similar result for symmetric even graphs and the proof is also the same.

Corollary 3.2. *(to theorem 2.2) Consider the symmetric even graph G. An edge uv in G^c is a shrinking edge if and only if it is the pacifying edge of some vertex v.*

4. CONCLUSION

Here we introduced the concept of pacifying edges and shrinking edges of the vertices of a graph and the same has been identified for odd cycles and symmetric even graphs. For these classes of graphs, the pacifying edges of any vertex depends on the parity of the radius of the graph. For odd cycles and symmetric even graphs, any edge that is a pacifying edge of some vertex is shown to be a shrinking edge of the graph.

5. ACKNOWLEDGEMENT

The work is a part of the thesis submitted by first author to Cochin University of Science and Technology (CUSAT) for the award of the degree of Doctor of Philosophy (Ph.D) and has not been submitted elsewhere.

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