

AN APPROACH OF SUMUDU TRANSFORM TO FRACTIONAL KINETIC EQUATIONS

KOMAL PRASAD SHARMA AND ALOK BHARGAVA¹

ABSTRACT. In this present work, new generalized fractional kinetic equations are proposed which involves generalized Galúe type Struve function and its fractional derivatives. Further, by using the Sumudu transform approach, solutions of these equations are proposed in terms of Mittag-Leffler function. The graphical presentation of the solutions is also given to show the behavior of these solutions with suitable parametric changes.

1. INTRODUCTION

In last few decades, several researchers worked on the study of fractional differential equations (FKEs) and find the importance of these equations in many areas of Astrophysics, Computational and Applied Mathematics. These equations show their robustness due to their significant applications in the fields of science, engineering, and social sciences. Due to the wide importance of kinetic equations and to enhance their uses in Mathematics and Science Haubold and Mathai [4] established a fractional generalization of the kinetic equation involving the rate of change of reaction $N = N(t)$, rate of destruction $d = d(N)$ and rate of production $p = p(N)$ as follows

¹*corresponding author*

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$$(1.1) \quad \frac{dN}{dt} = -d(N_t) + p(N_t),$$

where $N = N_t$ denote the function define by $N_t(t^*) = N(t - t^*)$, $t^* > 0$.

Also, they have studied the special case of the (1.1) for spatial fluctuations or inhomogeneities, in which the quantity N_t are neglected, denoted as

$$(1.2) \quad \frac{dN_i}{dt} = -c_i N_i(t)$$

with the initial condition $N_i(t = 0) = N_0$, the number of densities of a species i at the time $t = 0$, $c_i > 0$. Dropping the index i , and solving (1.2) they get

$$(1.3) \quad N(t) - N_0 = c_0 D_t^{-1} N(t).$$

On replacing Riemann-Liouville fractional integral operator present on the right-hand side of (1.3) by the fractional integral operator ${}_0D_t^{-\nu}$ studied by Miller and Ross [5], a generalized standard fractional kinetic equation (FKE) was obtained as

$$(1.4) \quad N(t) - N_0 = c^\nu D_t^{-\nu} N(t),$$

where

$$(1.5) \quad {}_0D_t^{-\nu} f(x) = \frac{1}{\Gamma(\nu)} \int (t-u)^\nu f(u) du, \quad x > 0, \Re(\nu) > 0.$$

Further, the generalization of the FKE was studied and defined by Saxena and Kalla [8] as

$$(1.6) \quad N(t) - N_0 f(t) = -c_0^\nu D_t^{-\nu} N(t), \quad \Re(\nu) > 0,$$

where $N(t)$ represent the number density of species at the time t , N_0 represent the number of densities at the time $t = 0$, c is the constant and $f(t) \in L(0, \infty)$.

Further, the extension of FKEs pertaining to special functions was studied and established many interesting results of great importance by several researchers like Habenom et al. [3], Haubold and Mathai [4], Nisar et al. [6], are some of them.

In this sequence, here, we present a generalized form of the FKE having generalized Galúe type Struve function and its fractional derivatives and find a solution by the approach of Sumudu transform.

The generalized form of Galúe type Struve function studied by Nisar et al. [6] and presented as

$$(1.7) \quad {}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(z) = \sum_{k=0}^{\infty} \frac{(-\beta)^k}{\Gamma(k\alpha + \mu)\Gamma\left(ak + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{z}{2}\right)^{2k+p+1},$$

where $\alpha > 0, \gamma > 0, a \in \mathbb{N}, p, b, \beta \in \mathbb{C}$ and μ is an arbitrary parameter. For more details one can refer [1, 2].

Following is the well-known result from the literature of fractional calculus [7]

$$(1.8) \quad D^{-\lambda}t^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\lambda+1)}t^{\beta+\lambda}, \quad \Re(\beta) > -1, 0 < \Re(\lambda) < 1, t > 0.$$

So, in view of (1.7) and (1.8) we have

$$(1.9) \quad {}_0D_t^{-\lambda}({}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(t)) = \sum_{n=0}^{\infty} \frac{(-\beta)^n t^{2n+p+\lambda+1} \Gamma(2n+p+2)}{\Gamma(n\alpha + \mu)\Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right) \Gamma(2n+p+\lambda+2)} \left(\frac{1}{2}\right)^{2n+p+1}.$$

The Sumudu Transform, defined by Watugala [10] over the set 'A' of functions as

$$(1.10) \quad S[f(t)] = G(u) = \frac{1}{u} \int_0^\infty f(t) e^{-t/u} dt \quad ; 0 < t < \infty, u \in (-\tau_1, \tau_2),$$

where $A = \{f(t) | M e^{|t|/\tau_j}; if t \in (-1)^j \times [0, \infty)\}$, M is a constant and $\tau_1, \tau_2 > 0$.

Sumudu Transform of Galúe type Struve function

$$(1.11) \quad S[{}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(t)] = \sum_{n=0}^{\infty} \frac{(-\beta)^n u^{2n+p+1} \Gamma(2n+p+2)}{\Gamma(n\alpha + \mu)\Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1}$$

Proof. Using (1.7) and (1.10) we have

$$S[{}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(t)] = \frac{1}{u} \int_0^\infty e^{-t/u} \sum_{n=0}^{\infty} \frac{(-\beta)^n}{\Gamma(n\alpha + \mu)\Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{t}{2}\right)^{2n+p+1} dt,$$

and by changing the order of summation and integral (which is permissible under the given conditions), we get

$$S \left[{}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(t) \right] = \frac{1}{u} \sum_{n=0}^{\infty} \frac{(-\beta)^n}{\Gamma(n\alpha + \mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} \cdot \int_0^{\infty} e^{-t/u} t^{2n+p+1} dt.$$

Solving the integral, we obtain the result. In view of (1.9) and (1.10) we can easily obtain that

$$(1.12) \quad S \left[{}_0 D_t^{-\lambda} \left({}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(t) \right) \right] = \sum_{n=0}^{\infty} \frac{(-\beta)^n u^{2n+p+\lambda+1} \Gamma(2n+p+2)}{\Gamma(n\alpha + \mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1}.$$

□

In the proposed work, we find the results in terms of Mittag-Leffler function [9] defined as

$$(1.13) \quad E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + \beta)}, \quad \Re(\alpha) > 0, \Re(\beta) > 0, \alpha, \beta \in \mathbb{C}.$$

2. MAIN RESULTS

In this section we have taken new generalized forms of FKE by involving Galúe type Struve function and its fractional derivative and find their solution by Sumudu Transform technique. Further by the graphical presentation of the results for suitable parametric values, the results are interpreted.

Theorem 2.1. *If $c > 0, \nu > 0, a \in \mathbb{N}, p, b, \beta \in \mathbb{C}, p > 0, \alpha > 0, \gamma > 0$ and μ is an arbitrary parameter, then the solution of the FKE*

$$(2.1) \quad N(t) - N_0 \left[{}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(t) \right] = -c^\nu {}_0 D_t^{-\nu} N(t)$$

is given by

$$(2.2) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha + \mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{t}{2}\right)^{2n+p+1} E_{\nu,2n+p+2}(-c^\nu t^\nu).$$

Proof. Applying Sumudu Transform on both sides of (2.1), we have

$$\begin{aligned} N(u) &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} u^{2n+p+1} (1+c^\nu u^\nu)^{-1} \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} u^{2n+p+1} \sum_{k=0}^{\infty} (-c^\nu u^\nu)^k \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} \sum_{k=0}^{\infty} (-c^{k\nu} u^{k\nu+2n+p+1}) \end{aligned}$$

Taking inverse Sumudu Transform we have

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{t}{2}\right)^{2n+p+1} \sum_{k=0}^{\infty} \frac{(-c^\nu t^\nu)^k}{\Gamma(k\nu+2n+p+2)}$$

Now, performing simple calculations and using the definition of Mittag - Leffler function from (1.13), we obtain the desired result. $\square$

Graphical Interpretation of Results

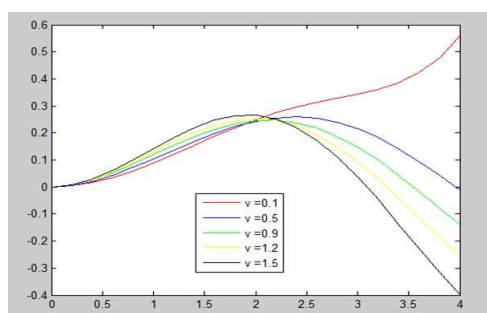


FIGURE 1. for $t=0(0.2)4$

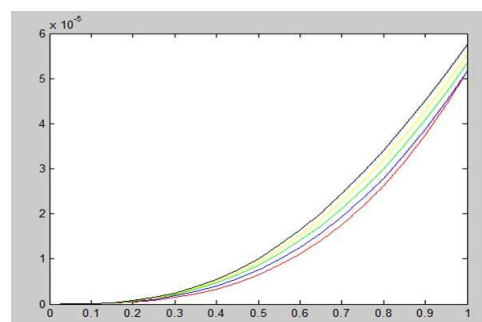


FIGURE 2. for $t=0(0.05)1$

Theorem 2.2. If $c > 0, \nu > 0, a \in \mathbb{N}, p, b, \beta \in \mathbb{C}, p > 0, \alpha > 0, \gamma > 0, d \neq c$ and μ is an arbitrary parameter, then the solution of the FKE

$$(2.3) \quad N(t) - N_0 [{}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(c^\nu t^\nu)] = -d^\nu {}_0 D_t^{-\nu} N(t)$$

is given by

$$(2.4) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu t^\nu}{2}\right)^{2n+p+1} E_{\nu,(2n+p+1)\nu+1}(-d^\nu t^\nu).$$

Proof. Applying Sumudu Transform on both sides of (2.3), we have

$$\begin{aligned} N(u) &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} u^{(2n+p+1)\nu} (1+d^\nu u^\nu)^{-1} \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} u^{(2n+p+1)\nu} \sum_{k=0}^{\infty} (-d^\nu u^\nu)^k \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \sum_{k=0}^{\infty} (-d^{k\nu} u^{(2n+p+k+1)\nu}). \end{aligned}$$

Taking inverse Sumudu Transform we have

$$\begin{aligned} N(t) &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu t^\nu}{2}\right)^{2n+p+1} \\ &\quad \cdot \sum_{k=0}^{\infty} \frac{(-d^{k\nu} t^{k\nu})^k}{\Gamma(k\nu + (2n+p+1)\nu+1)}. \end{aligned}$$

Now, performing simple calculations and using the definition of Mittag - Leffler function from (1.13), we obtain the desired result. $\square$

Graphical Interpretation of Results

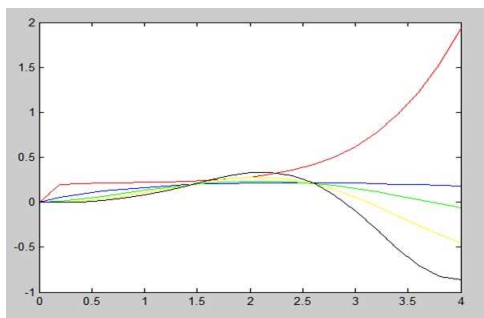


FIGURE 3. for $t=0(0.2)4$

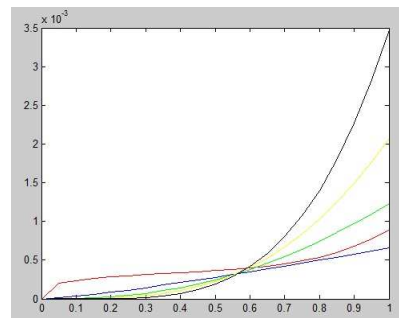


FIGURE 4. for $t=0(0.05)1$

Theorem 2.3. If $c > 0, \nu > 0, a \in \mathbb{N}, p, b, \beta \in \mathbb{C}, p > 0, \alpha > 0, \gamma > 0, \lambda \neq \nu$ and μ is an arbitrary parameter, then the solution of the FKE

$$(2.5) \quad N(t) - N_0 \left[{}_0 D_t^{-\lambda} ({}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(t)) \right] = -c^\nu {}_0 D_t^{-\nu} N(t)$$

is given by

(2.6)

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2) t^{2n+p+\lambda+1}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} E_{\nu, 2n+p+\lambda+2}(-c^\nu t^\nu).$$

Proof. Applying Sumudu Transform on both sides of (2.5), we have

$$\begin{aligned} N(u) &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} u^{2n+p+\lambda+1} (1+c^\nu u^\nu)^{-1} \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} u^{2n+p+\lambda+1} \sum_{k=0}^{\infty} (-c^\nu u^\nu)^k \\ &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} \sum_{k=0}^{\infty} (-c^{k\nu} u^{k\nu+2n+p+\lambda+1}). \end{aligned}$$

Taking inverse Sumudu Transform we have

$$\begin{aligned} N(t) &= N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma(2n+p+2) t^{2n+p+\lambda+1}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} \\ &\quad \cdot \sum_{k=0}^{\infty} \frac{(-c^\nu t^\nu)^k}{\Gamma(k\nu+2n+p+\lambda+2)}. \end{aligned}$$

Now, performing simple calculations and using the definition of Mittag - Leffler function from (1.13), we obtain the desired result. $\square$

Graphical Interpretation of Results

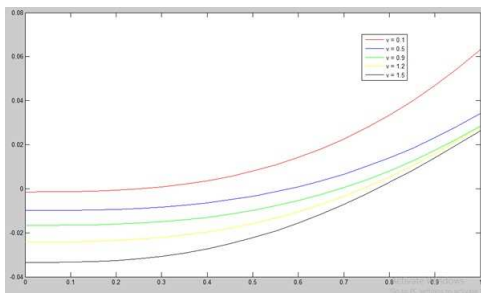


FIGURE 5. for $t=0(0.2)4$

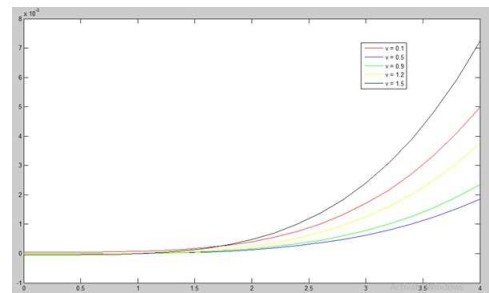


FIGURE 6. for $t=0(0.05)1$

Theorem 2.4. *If $c > 0, \nu > 0, a \in \mathbb{N}, p, b, \beta \in \mathbb{C}, p > 0, \alpha > 0, \gamma > 0, d \neq c$ and μ is an arbitrary parameter, then the solution of the FKE*

$$(2.7) \quad N(t) - N_0 \left[{}_0D_t^{-\lambda} ({}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(c^\nu t^\nu)) \right] = -d^\nu {}_0D_t^{-\nu} N(t)$$

is given by

$$(2.8) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1) t^{(2n+p+1)\nu+\lambda}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \\ \cdot E_{\nu,(2n+p+1)\nu+\lambda+1}(-d^\nu t^\nu).$$

Proof. Applying Sumudu Transform on both sides of (2.7), we have

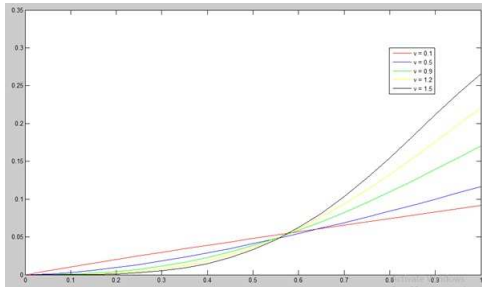
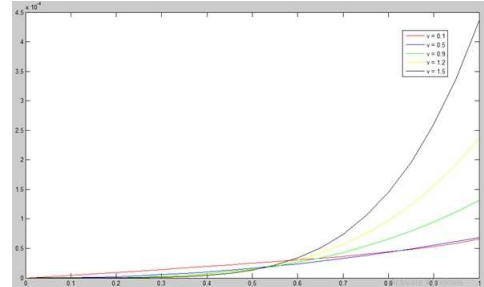
$$N(u) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \\ \cdot u^{(2n+p+1)\nu+\lambda} (1+d^\nu u^\nu)^{-1} \\ = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \\ \cdot u^{(2n+p+1)\nu+\lambda} \sum_{k=0}^{\infty} (-d^\nu u^\nu)^k \\ = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \\ \cdot \sum_{k=0}^{\infty} (-d^{k\nu} u^{(2n+p+k+1)\nu+\lambda}).$$

Taking inverse Sumudu Transform we have

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1) t^{(2n+p+1)\nu+\lambda}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} \\ \times \sum_{k=0}^{\infty} \frac{(-d^\nu t^\nu)^k}{\Gamma(k\nu + (2n+p+1)\nu + \lambda + 1)}$$

Now, performing simple calculations and using the definition of Mittag - Leffler function from (1.13), we obtain the desired result. $\square$

Graphical Interpretation of Results

FIGURE 7. for $t=0(0.2)4$ FIGURE 8. for $t=0(0.05)1$

3. SPECIAL CASES

(i) If we take $d = c$ in Theorem 2.2, then the equation (2.3) reduces in

$$(3.1) \quad N(t) - N_0 [{}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(c^\nu t^\nu)] = -c^\nu {}_0 D_t^{-\nu} N(t)$$

with the solution

$$(3.2) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu t^\nu}{2}\right)^{2n+p+1} \cdot E_{\nu,(2n+p+1)\nu+1}(-c^\nu t^\nu).$$

(ii) If we take $c = 1$ in Theorem 2.2, then the equation (2.3) reduces in

$$(3.3) \quad N(t) - N_0 [{}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(t^\nu)] = -d^\nu {}_0 D_t^{-\nu} N(t)$$

with the solution

$$(3.4) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1)}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{t^\nu}{2}\right)^{2n+p+1} \cdot E_{\nu,(2n+p+1)\nu+1}(-d^\nu t^\nu).$$

(iii) If we take $d = c$ in Theorem 2.4, then the equation (2.7) reduces in

$$(3.5) \quad N(t) - N_0 [{}_0 D_t^{-\lambda} ({}_a W_{p,b,\beta,\gamma}^{\alpha,\mu}(c^\nu t^\nu))] = -c^\nu {}_0 D_t^{-\nu} N(t)$$

with the solution

$$(3.6) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1) t^{(2n+p+1)\nu+\lambda}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{c^\nu}{2}\right)^{2n+p+1} E_{\nu, (2n+p+1)\nu+\lambda+1}(-c^\nu t^\nu).$$

(iv) If we take $c = 1$ in Theorem 2.4, then the equation (2.7) reduces in

$$(3.7) \quad N(t) - N_0 \left[{}_0D_t^{-\lambda} ({}_aW_{p,b,\beta,\gamma}^{\alpha,\mu}(t^\nu)) \right] = -d^\nu {}_0D_t^{-\nu} N(t)$$

with the solution

$$(3.8) \quad N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-\beta)^n \Gamma((2n+p+1)\nu+1) t^{(2n+p+1)\nu+\lambda}}{\Gamma(n\alpha+\mu) \Gamma\left(an + \frac{p}{\gamma} + \frac{(b+2)}{2}\right)} \left(\frac{1}{2}\right)^{2n+p+1} E_{\nu, (2n+p+1)\nu+\lambda+1}(-d^\nu t^\nu).$$

(v) If we put, $a = \alpha = \gamma = 1$ and $\mu = 3/2$ then generalized Galúe type Struve function reduced in generalized Struve function [5] as

$$(3.9) \quad {}_aW_{p,b,\beta,1}^{1,3/2}(t) = \sum_{n=0}^{\infty} \frac{(-\beta)^n}{\Gamma(n+3/2) \Gamma\left(n+p+\frac{(b+2)}{2}\right)} \left(\frac{t}{2}\right)^{2n+p+1} = H_{p,b,\beta}(t).$$

Further, in view of Theorem 2.1, 2.2, 2.3 and 2.4 we readily get results in terms of generalized Struve function, mentioned in (3.9).

(vi) If we put, $a = \alpha = \gamma = 1, \beta = 1, p = m-1, b = -1$ and $\mu = 1$ then generalized Galúe type Struve function reduces in Bessel's function of first kind [1, 2, 5] as

$$(3.10) \quad {}_1W_{m-1,2,1,1}^{1,1}(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1) \Gamma(n+m+1)} \left(\frac{t}{2}\right)^{2n+m} = J_m(t).$$

Further, in view of above-mentioned substitution and Theorem 2.1, 2.2, 2.3 and 2.4 we get results in terms of Bessel's function of first kind.

We can obtain several known and new results from our main results by taking suitable parametric values, but we don't record them here explicitly.

4. CONCLUSION

Here, we propose the solution of generalized fractional kinetic equations in the form of four theorems by using the approach of Sumudu transform. The results of Theorem 2.1 and 2.2 are comparable with the results obtained by Habenom et al. [3] and the results in Theorem 2.3 and 2.4 are believed to be new and have wide applications in science and technology. Further, at the end of proof for each result, the behaviour of these results are interpreted by graphs by taking distinct values of the parameter at different time intervals.

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DEPARTMENT OF MATHEMATICS AND STATISTICS,
MANIPAL UNIVERSITY JAIPUR,
JAIPUR, RAJASTHAN INDIA
Email address: keshav4maths@gmail.com

DEPARTMENT OF MATHEMATICS AND STATISTICS,
MANIPAL UNIVERSITY JAIPUR,
JAIPUR, RAJASTHAN INDIA
Email address: alok.bhargava@jaipur.manipal.edu