ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7065–7073
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.56 Spec. Issue on MEM-2020

EFFECT OF VISCOUS DISSIPATION ON MHD UNSTEADY FLOW THROUGH VERTICAL POROUS MEDIUM WITH CONSTANT SUCTION

S.P. AGARWAL AND ANIL KUMAR[1]

ABSTRACT. Two-dimensional heat and mass transfer MHD unsteady and mixed convective flow of an electrically conducting viscous incompressible fluid passes through a semi-infinite vertical porous plate with variable thermal conductivity is investigated. The suction velocity perpendicular to the plate is considered as a constant. The Perturbation technique is used to solve the governing equations. The effects of the fluid flow parameters on the velocity and concentration profile are discussed and explained with the help of graphs.

1. INTRODUCTION

Unsteady MHD flow plays a vital role in industries and nature. Heat and mass transfer used in the chemical process such as food processing and polymer production. Jha and Oni (2018) depict mixed convection flow past a vertical channel with flow reversal and temperature dependent viscosity by exact solution. Sharma et al. (2007) investigated the viscous conducting MHD forced flow past an impervious rotating disk filled with porous medium. Vijayaragavan et al. (2019) investigate the heat and mass transfer on mixed convective MHD unsteady casson fluid flow passes through a moving semi - vertical porous plate with effects of chemical reaction and Dufour. Reddy and Raju (2019) investigated the chemical radiation and radiation effects on unsteady MHD and mixed

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 03C65, 60H20, 60H40, 60H35.

Key words and phrases. viscous , MHD, Fluid flow, Perturbation technique, variable thermal conductivity, mixed convective flow.

convection flow over a radiative infinite vertical porous plate with radiation and absorption. Reddy and Raju (2018) depicts the free convective MHD flow passes through a porous plate. Dey, D. (2019) investigated the MHD micro-polar and Mixed Convective flow past in a porous medium with radiation and absorption. Subhanna et al. (2018) depicts the free convective flow through infinite vertical porous plate in the influence of magnetohydrodynamics. Shakthikala et al. (2018) investigated the effect of chemical reaction, oscillating temperature and radiation in the porous medium on MHD unsteady flow of a viscoelastic fluid. Moniem and Hassanin (2013) investigated the solution of magnetohydrodynamics flow through a porous medium bounded by infinite vertical porous plate with oscillatory suction. Ahmad, S. (2009) analyzed the heat and mass transfer on three dimensional unsteady and free convection flows over a porous infinite vertical plate.

2. FORMULATION OF THE PROBLEM

The two dimensional mixed convective MHD unsteady boundary layer flow of an electrically conducting viscous incompressible fluid passes through semi infinite vertical porous plate with constant suction velocity has been considered. The flow is assumed to be in the x* - axis, which is vertically upward direction and y* - axis perpendicular to it. The velocity components along x*and y* axis are u* and v* respectively. The governing equations under the approximation of Boussinesq and boundary layer can be form as:

$$\frac{\partial v^*}{\partial y*} = 0 \tag{2.1}$$

$$\begin{aligned} \frac{\partial u^*}{\partial t*} + v * \frac{\partial u^*}{\partial y*} = & -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \frac{1}{\rho}\frac{\partial}{\partial y^*}(\mu\frac{\partial u^*}{\partial y*}) + g\beta(T^* - T*_\infty) \\ & + g\beta(C^* - C^*_\infty) + g\beta^*(C^* - C^*_\infty) - v\frac{u^*}{k^*} - \frac{\sigma B_0^2 u^*}{\rho} \end{aligned} \tag{2.2}$$

$$\frac{\partial T^*}{\partial t*} + v^*\frac{\partial T^*}{\partial y*} = \frac{\partial}{\partial y*}(\alpha\frac{\partial T^*}{\partial y*}) \tag{2.3}$$

$$\frac{\partial c^*}{\partial t*} + v^*\frac{\partial C^*}{\partial y*} = D\frac{\partial^2 C^*}{\partial y^{*2}}. \tag{2.4}$$

Here, ρ the fluid density, P^* the pressure, μ the fluid viscosity, g the acceleration due to gravity, β the thermal expansion, β^* the concentration expansion,

T^* the temperature of the fluid inside the boundary layer, T^*_∞ the fluid temperature away from the plate, C^* the concentration in the boundary layer, C^*_∞ the concentration in the fluid away from the plate, v the kinematic viscosity, K^* the permeability of the porous medium, σ the electrical conductivity of the fluid, t^* the time, B_0 the magnetic induction, D the mass diffusivity, T^*_w the plate temperature, α_0 the thermal conductivity at temperature T^*_w and α the thermal diffusivity of the fluid Consider the thermal conductivity in the following form: $\alpha = \alpha_0\{1 + a(T^*_w - T^*_\infty)\} = \alpha_0(1 + \beta\theta); \beta = a(T^*_w - T^*_\infty)$ The initial and boundary conditions are:

$$\begin{cases} u^* = U^*_p, \quad T^* = T^*\infty + (T^*_w - T^*_\infty e^{n^*t*}), \\ C^* = C^*_\infty + (C^*_w - C^*_\infty \quad \text{at} \quad y^* + 0) \\ u^* \to U^*_\infty(1 + e^{n^*t^*}), \quad T^* \to T^*_\infty, \\ C^* \to C^*_\infty \quad \text{as} \quad y^* \to \infty, \end{cases} \tag{2.5}$$

where C^*_w is the concentration of plate, U^*_p is the plate velocity, U^*_∞ is free stream velocity, U_0 and n^* are constants.

Equation (2.1) implies that

$$v^* = -V_0, \tag{2.6}$$

where V_0 be a a positive constant and minus sign is taken since the suction is along the porous plate.

The momentum equation (2.2) outside the boundary layer becomes:

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = \frac{U^*_\infty}{\partial t^*} + \frac{v}{k^*}U^*_\infty. \tag{2.7}$$

Introducing some non-dimensional numbers as follows:

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{V_0}, t = \frac{t^*V_0^2}{\nu}, n = \frac{n^*\nu}{v_0^2}, y = \frac{V_0 y^*}{\nu},$$
$$U_\infty = \frac{U^*_\infty}{U}, U_p = \frac{U^*_p}{U_0}, \theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}, C = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty},$$
$$K = \frac{K^*V_0^2}{\nu^2}, P_r = \frac{\nu}{\alpha_0}, S_c = \frac{\nu}{D}, G_r = \frac{\nu_{\beta g(Tw^* - T^8_\infty)}}{U_0 v_0^2},$$
$$G_m = \frac{\nu_{\beta^* g(C^*_w - C^*_\infty)}}{U_0^2 V_0^2} \text{ and } M = \frac{\sigma\nu_{B^2}}{\rho V_0^2},$$

where, M is Hartmann number, Gm is modified Grashof number, Gr is Grashof number, Sc is Schmidt number, Pr is Prandtl number and K is porosity.

The corresponding boundary conditions reduce to

$$\begin{cases} y = 0 \ \ , \theta = e^{nt}, \ \ c = e^{nt}, \ \ u = U_p \\ y \to \infty, \ \ \theta \to 0, \ \ , u \to U_\infty = 1 + e^{nt}. \end{cases} \tag{2.8}$$

The equations (2.2) to (2.4) becomes

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = 2\frac{\partial U_\infty}{\partial t} + \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m C + N(U_{\infty - u}) \tag{2.9}$$

$$\frac{\partial v}{\partial t} - \frac{\partial v}{\partial y} = 2\frac{1}{P_r}[(1 + \beta\theta)\frac{\partial^2 \theta}{\partial y^2} + \beta(\frac{\partial \theta}{\partial y})^2] \tag{2.10}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{2.11}$$

where $N + M + \frac{1}{K}$.

3. SOLUTION OF THE PROBLEM

Consider the velocity, temperature and concentration of the fluid near the porous plate as follows

$$u = u_0 e^{nt} \tag{3.1}$$

$$\theta = \theta_0 e^{nt} \tag{3.2}$$

$$C = C_0 e^{nt}. \tag{3.3}$$

Equations (2.8) to (2.10) give:

$$u_0^{''} - A_3^2 u_0 = A_4\theta_0 - G_m C_0 \tag{3.4}$$

$$(\theta_0^{''})^2 + P_r\theta_0^{'}\theta_0^{''} + nP_r(\theta_0^{'})^2 = 0 \tag{3.5}$$

$$C_0^{''} + S_c C_0^{'} - nS_c C_0 = 0. \tag{3.6}$$

The corresponding boundary conditions become as follows:

$$\begin{cases} y = 0 : \ \ u_0 = U_p, \ \ \theta_0 \to 1, \ \ C_0 \to 1 \\ y = \infty : \ \ u_0 = 1, \ \ \theta_0 \to 1, \ \ C_0 \to 0. \end{cases} \tag{3.7}$$

By solving equations (3.4) to (3.6) under the boundary condition (3.8), we have

$$\begin{cases} u = [A_7 e^{-A_3 y} + A_5 e^{-A_2 y} - A_6 e^{-A_1 y}]e^{nt} \\ \theta = e^{-A_2} y e^{nt} \\ C = e^{-A_1 y} e^{nt}, \end{cases} \tag{3.8}$$

where

$$A_1 = \frac{S_c + \sqrt{S_c^2 + 4nS_c}}{2}, \quad A_2 = \frac{P_r + \sqrt{P_r^2 + 4nP_r}}{2}, \quad A_3 = \sqrt{N + n - 1}, \quad A_4 = \frac{N}{\theta_r} - G_r,$$

$$A_5 = \frac{A_4}{A_2^2 + A_2 A_3^2}, \quad A_6 = \frac{G_m}{A_1^2 + A_2 A_3^2} \quad \text{and} \quad A_7 = 1 + A_6 - A_5.$$

4. RESULT AND DISCUSSION

The effect of flow parameters like Magnetic parameter M, Prandtl number Pr, Grashof number Gr, Schmidt number Sc and modified Grashof Number Gm has been discussed. Figures 1 to 2 show that the velocity profile increase as modified Grashof Number Gm and Grashof number Gr increase. Figures 3 to 5 show that velocity profile decrease as increase to Magnetic parameter M, Prandtl number Pr and Schmidt number Sc. Figure 6 show that temperature profile decreases as Sc increase. Figure 7 show that concentration profile decreases as Schmidt number Sc increase.

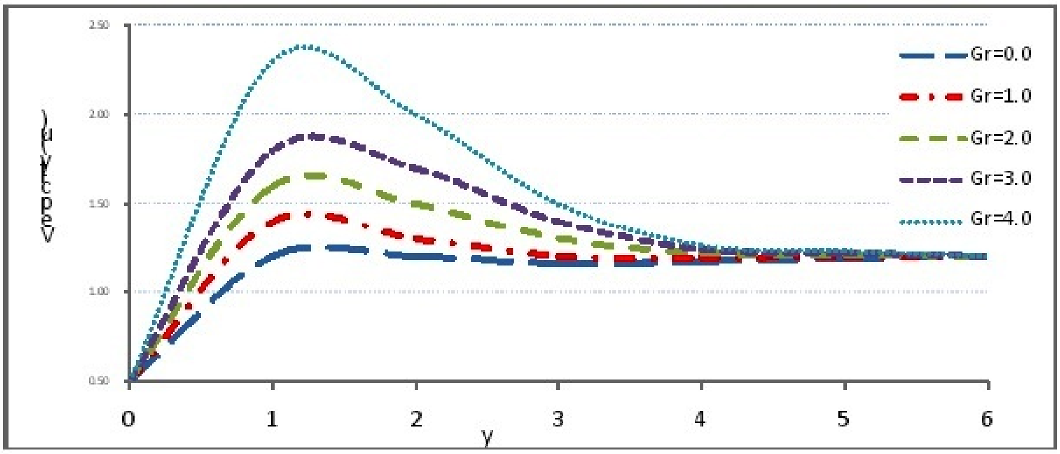

FIGURE 1. The effect of Gr on Velocity profile.

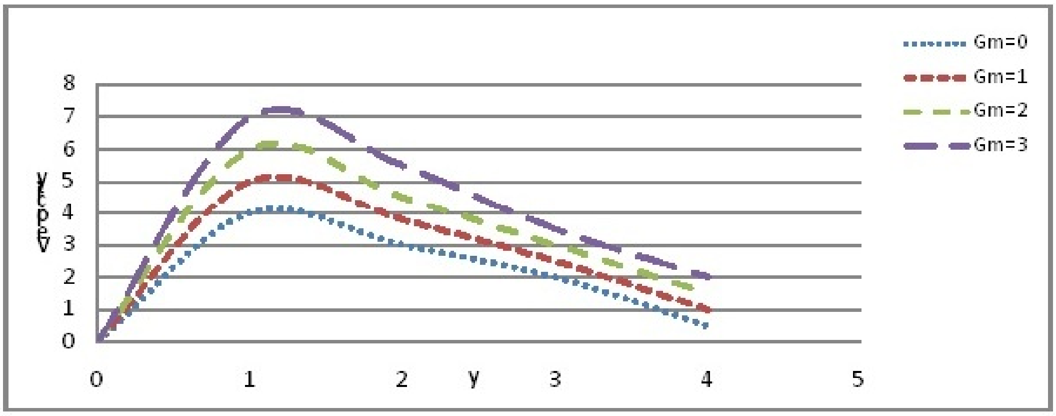

FIGURE 2. The effect of Gm on Velocity profile.

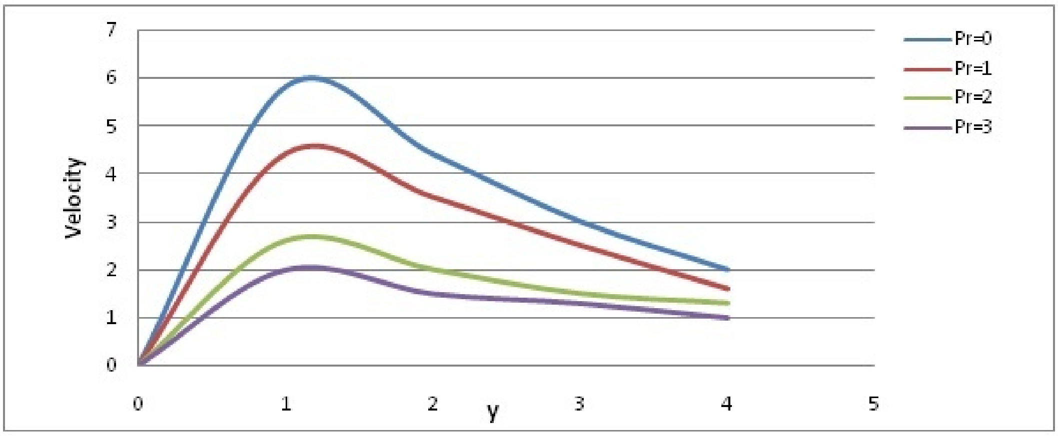

FIGURE 3. The effect of Pr on Velocity profile.

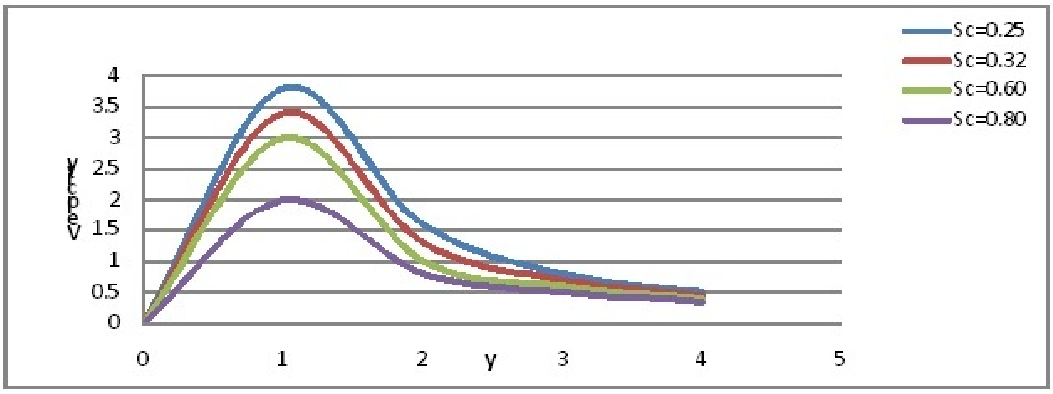

FIGURE 4. The effect of Gc on Velocity profile.

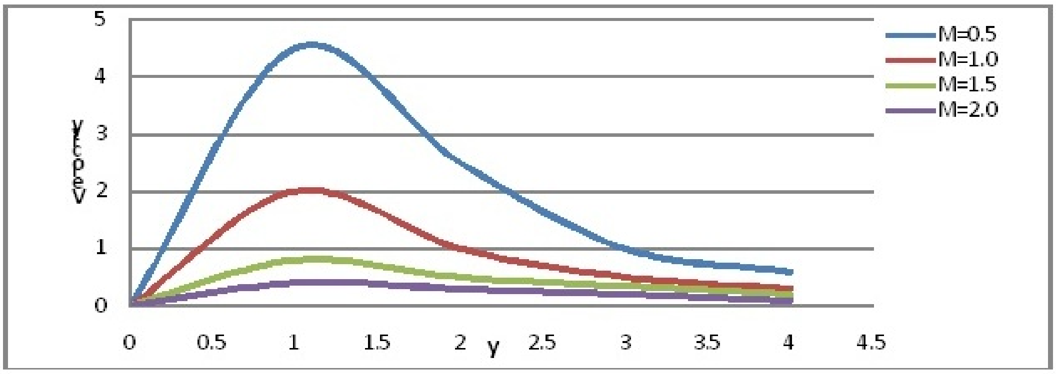

FIGURE 5. The effect of M on Velocity profile.

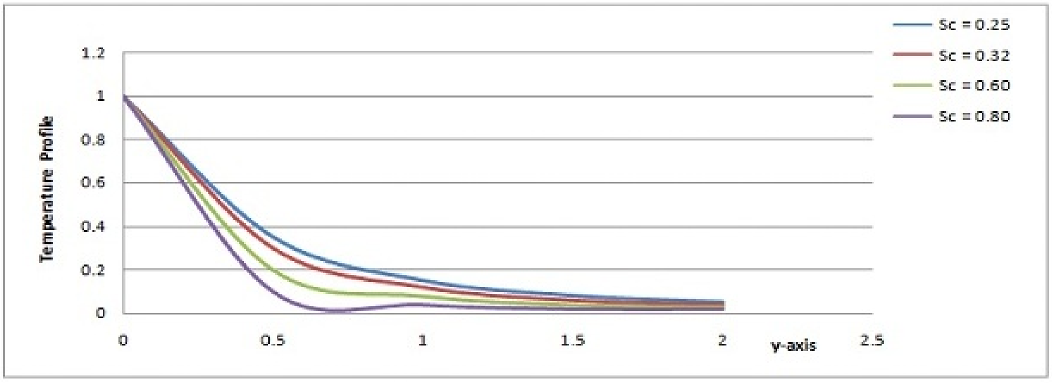

FIGURE 6. The effect of Sc on Temperature profile.

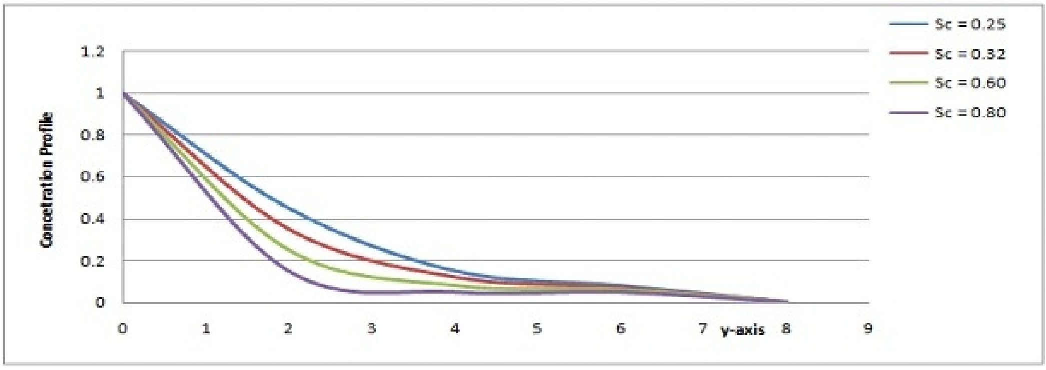

FIGURE 7. The effect of Gr on Concentration profile.

5. Conclusion

The present work described two-dimensional mixed convective MHD unsteady flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with variable thermal conductivity. We conclude that the velocity profile increases when Gr and Gm increase while it is decreases when Pr, Sc and M increase. The temperature and concentration profiles decrease with increasing value of Sc.

References

[1] B.K. Jha, M.O. On: *Mixed convection flow in a vertical channel with temperature dependent viscosity and flow reversal*, An exact solution, International Journal of Heat and Technology, **36**(2) (2018), 607–613.

[2] B.K. Sharma, A.K. Jha, R.C. Chaudhary: *MHD forced flow of a conducting viscous fluid through a porous medium induced by an impervious rotating disk*, Rom. Journ. Phys., **52**(1-2) (2007), 73–84.

[3] R. Vijayaragavan, S. Karthikeyan, P. Premalatha : *Heat and Mass Transfer on an unsteady MHD mixed convective casson fluid flow past a moving vertical porous plate with effects of Dufour and chemical reaction*, International Journal of Mathematics Trends and Technology, **65**(2) (2019), 117–129.

[4] D.B. Reddy, G.S.S. Raju: *Radiation and chemical radiation effects on unsteady MHD Mixed convection flow over a Radiative Vertical Porous Plate with Radiation Absorption*, Global Journal of Pure an Applied Mathematics, **13**(8) (2017), 4015–4034.

[5] P.R.K. Reddy, M.C. Raju: *MHD free convective flow past a porous plate*, International Journal of Pure and Applied Mathematics, **118**(5) (2018), 507–529.

[6] D. Dey: *Mixed Convective MHD micro-polar fluid flow in a porous medium with radiation absorption*, International Journal of Mathematical, Engineering and Management Sciences, **4**(2) (2019), 387–399.

[7] K. Subhanna, S.G. Mohiddin, R.B. Vijaya: *MHD free convective flow through porous medium past an infinite vertical porous plate*, Annals of Pure and Applied Mathematics, **16**(2) (2018), 419–428.

[8] R. Shakthikala, S. Prahala, S. Ajuja: *Unsteady MHD flow of a viscoelastic fluid with oscillating temperature, chemical reaction and radiation effect in porous medium*, International Journal of Research in Advent Technology, **6**(12) (2018), 3777–3783.

[9] A.A. Moniem, W.S. Hassanin: *Solution of MHD Flow past a vertical Porous plate through a porous medium under oscillatory suction*, Applied Mathematics, **4** (2013), 694–702.

[10] S. AHMAD: *The study of heat and mass transfer on free convection three dimensional unsteady flows over a porous vertical plate*, Journal of Energy, Heat and Mass Transfer, **31** (2009), 89–110.

PROFESSOR, DEPARTMENT OF CIVIL ENGINEERING
SAI NATH UNIVERSITY
RANCHI, JHARKHAND, INDIA

PROFESSOR, DEPARTMENT OF APPLIED MATHEMATICS
SUBHARTI INSTITUTE OF TECHNOLOGY & ENGINEERING,
MEERUT (U.P.), INDIA
Email address: dranilkumar06@yahoo.co.in