ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7101–7104
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.60

CUBE DIFFERENCE LABELING FOR COMPLETE TRIPARTITE GRAPH

S. VIDYANANDINI

ABSTRACT. Consider an undirected finite connected graph $G = (V, E)$, where V and E are the sets of vertices and edges of G, respectively and $|E| = e$ and $|V| = n$. G possess cube difference labeling if there exits a injection $f : V(G) \longrightarrow \{0, 1, \ldots, p-1\}$ so that the edge set of G has assigned a weight defined by the absolute of cube difference if its end-vertices, the resulting weights are distinct. A graph admitting cube difference labeling is called cube difference graph. In this paper, cube difference labeling for complete tripartite graph are discussed.

1. INTRODUCTION

A cube difference labeling of a graph G of size n exist for function f if f permits an injection from $V(G)$ to the set $\{0, 1, 2, \ldots, n\}$ so that, when each edge uv of G has assigned the weight $|\,[f(u)]^3 - [f(v)]^3\,|$,shows distinct resulting weights [1–3]. The notion of square difference labeling was established by J. Shiama [4, 6, 7]. He proved cube difference labeling admits for paths, cycles, stars, fan graph, wheel graphs, crown graphs, helm graphs, dragon graphs, coconut trees and shell graphs. Graph labeling are widely used in communication network, Mobile telecommunication, military offices [5] .

[1] *corresponding author*

2010 *Mathematics Subject Classification.* 05C78, 05C05.
Key words and phrases. Cube difference labeling, Complete tripartite graph, Cube difference graph.

Definition 1.1. *Consider graph G with vertex set $V(G)$ and edge set $E(G)$. If there exits a injection $f : V(G) \longrightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \longrightarrow N$ given by $f^*(uv) = |\,[f(u)]^3 - [f(v)]^3\,|$ is injective ,Then G is said have cube difference labeling .*

Definition 1.2. *A graph which admits the cube difference labeling is said to be cube difference graph.*

Definition 1.3. *A complete $k - partite$ graph is a $k - partite$ graph if there is an edge between every pair of vertices obtained from different independent sets.*

Definition 1.4. *A set of graph vertices partitioned into three independent sets, such that no two graph vertices within the same set are adjacent. such a graph is said to be complete tripartite graph.*

2. MAIN RESULT

Theorem 2.1. *The tripartite graph $K_{m,n,r}$ for any integer $m, n, r > 0$ admits cubic difference labeling.*

Proof. Consider G as a complete tripartite graph $K_{m,n,r}$ for any position integer m, n, r. Note by the definition of complete tripartite graph $K_{m,n,r}$ has $n+m+r$ vertices and $n\ m\ r$ edges. Without loss of generality, let us consider that $m \leq n \leq r$. Let $|V_1| = m$, $|V_2| = n$ and $|V_3| = r$. Let the vertex subset V_1 has $\{u_0, u_1, u_2, \dots, u_{m-1}\}$. Let the vertex subset V_2 has $\{v_0, v_1, v_2, \dots, v_{n-1}\}$. Let the vertex subset V_3 has $\{w_0, w_1, w_2, \dots, w_{r-1}\}$. Define vertex labeling $f : V_1 \cup V_2 \cup V_3 \longrightarrow \{0, 1, 2, \dots (m+n+r)-1\}$

$$f(u_i) = i, \quad 0 \leq i \leq m-1$$
$$f(v_j) = m+j, \quad 0 \leq j \leq n-1$$
$$f(w_k) = m+n+k, \quad 0 \leq k \leq r-1\,.$$

Define labeling function for Edge f^* as $f^*(uv) = |f(u)^3 - f(v)^3|$ for any edge $vu \in E(G)$ and $f^*(wv) = ||f(w)^3 - f(v)^3||$ for any edge $wv \in E(G)$. It is clear that f is bijective and vertex labels of G are distinct. □

Claim 1. *The edge labels of the edges of G are distinct.*

Let V_j and V_{j+1} for j, $0 \leq j \leq n-2$ be the vertices in V_2 such that their vertex labels are consecutive. Let us assume that $f(V_j) = t$ and $f(V_{j+1}) = t+1$. By the definition of vertex labels of vertices in V_2 it is clear that $t \geq m$. Also, let w_k and w_{k+1} for K, $0 \leq k \leq r-1$ be the vertices in V_3 such that their vertex labels are consecutive. Let us assume that, $f(w_k) = s$ and $f(w_{k+1}) = s+1$. By the definition vertex labels of vertices in V_3, it is clear that $s \geq n$.
Since G is a complete tripartite graph, vertex V_j is adjacent to every vertex u_i in V_1 for $0 \leq i \leq m-1$. Similarly, V_{j+1} is adjacent to vertex u_i in V_1 for $i, 0 \leq i \leq m-1$.

Since the vertex label of u_i and vertex label of V_j are distinct, the induced edge labels of the edges $V_j u_i$ for $i, 0 \leq i \leq m-1$ are distinct. Similarly, edge labels of the edges that was induced, $V_{j+1}u_i$ for $i, 0 \leq j \leq m-1$ are also distinct.

Further, edge labels of the edges that was induced $V_j u_i$ and $V_{j+1}u_i$ form a monotonically increasing sequence as i increases from 0 to $m-1$. Also, Though G is a complete tripartite graph, we have vertex w_k adjoining to every vertex V_j in V_2 for $j, 0 \leq j \leq n-1$. Similarly, w_{k+1} is adjoining∞ to every vertex V_j in V_2 for $j, 0 \leq j \leq n-1$.

Since the vertex label of V_j and vertex label of $w-k$ are distinct, the induced edge label of the edges $w_k V_j$ for $j, 0 \leq j \leq n-1$ are distinct. Similarly, edge labels of the edges that was induced, $w_{k+1}V_j$ for $j, 0 \leq j \leq n-1$ are distinct. Further,edge label of the edges that was induced $w_k V_j$ and $w_{k+1}V_j$ form a monotonically increasing sequence as j increases from 0 to $n-1$. By the definition of f and f^*

$$\begin{aligned} f^*(V_j u_0) &= ||f(V_j)|^2 - |f(u_0)|^2| = |t^2 - 0| = t^2 \\ f^*(V_{j+1}u_{m-1}) &= ||f(V_{j+1})|^2 - |f(u_{m-1})|^2|| = |(t+1)^2 - (m-1)^2| \\ &= |t^2 + 2t - m^2 + 2m| > 0\,, \end{aligned}$$

also,

$$\begin{aligned} f^*(w_k v_0) &= ||f(w_k)|^2 - |f(v_0)|^2| \\ &= |s^2 - 0^2| = s^2 \\ f^*(w_{k+1}v_{n-1}) &= |(s+1)^2 - (n-1)^2| \\ &= |s^2 + 2s + 1 - (n^2 - 2n + 1)| \\ &= |s^2 + 2s - n^2 + 2n| > 0\,. \end{aligned}$$

Hence, the edge labels of G are distinct. Hence the theorem

Example 1.

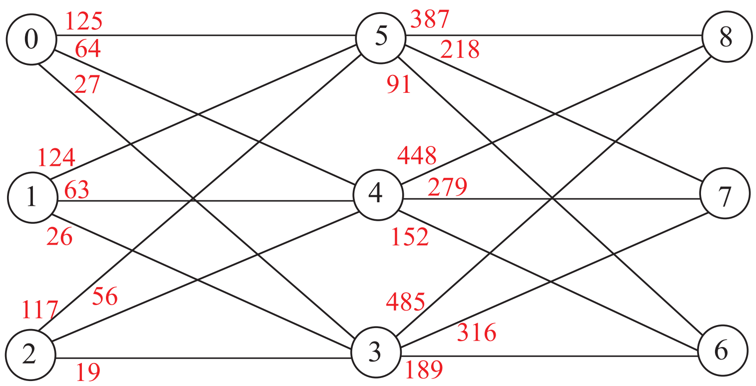

FIGURE 1. Y-Tree

REFERENCES

[1] V. AJITHA, S. ARUMUGAM: *Geemina KA. On Square sum graph*. AKCE J.Graphs, Comin., **6** (2006), 1–10.

[2] L. BEINEKE, S. M. HEGDE: *Strongly multipicative graphs*, Discuss. Math.Graph theory, **21** (2000) 21, 63–75.

[3] J. A. GALLIAN: *A dynamic survey of graph labeling*, The Electronics journal of Combinatories, **7**, 2010.

[4] J. SHIAMA: *Permutation sum labeling for some shadow graph*, International Journal of Computer Application, **40**(6) (2012), 31–35.

[5] D. B. WEST: it Introduction to Graph Theory, Prentice-Hall, 2001.

[6] J. SHIAMA: *Square sum labeling for some middle and total graphs*, International Journal of Computer Application, **37**(4) (2012), 6–8.

[7] J. SHIAMA: *Square difference labeling for some Graphs*, International Journal of Computer Application, **44**(4) (2012), 30–33.

DEPARTMENT OF MATHEMATICS, FACULTY OF ENGINEERING AND TECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
KATTANKULATHUR -603 203, KANCHEEPURAM DISTRICT, TAMILNADU, INDIA
Email address: vidhyanandhini.maths@gmail.com