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SPLITTANCE OF CYCLES ARE ANTI-MAGIC

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ABSTRACT. An anti-magic labeling of a graph G is a bijective function $f : E(G) \longrightarrow \{1, 2, \ldots, |E|\}$ such that the vertex-sum for distinct vertices are different. Vertex-sum of a vertex is defined as the sum of the labels of the edges that are incident to the vertex. A graph that admits anti-magic labeling is called anti-magic. It was conjectured by Hartsfield and Ringel that every connected graph except the complete graph on two vertices has an anti-magic labeling. In this paper, we prove that the splittance of cycles are anti-magic.

1. INTRODUCTION

In this paper, we consider simple, finite and undirected graphs. For graph theoretic terms, we refer from the book [6]. Suppose $G = (V, E)$ is a graph and $f : E \longrightarrow \{1, 2, \ldots, |E|\}$ is a bijective mapping. An anti-magic labeling of a graph G is a bijective function $f : E(G) \longrightarrow \{1, 2, \ldots, |E|\}$ such that the vertex-sum for distinct vertices are different. Vertex-sum of a vertex is defined as the sum of the labels of the edges that are incident to the vertex. A graph that admits anti-magic labeling is called anti-magic. Anti-magic labeling was introduced by Hartsfield and Ringel [3]. In the literature, many graph classes such as paths, cycles, wheels and complete graphs are proved to be anti-magic. In [3], Hartsfield and Ringel conjectured that every connected graph other than

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complete graph on two vertices is anti-magic and every tree other than path on two vertices is anti-magic.

In spite of many articles published related to anti-magic graphs, these conjectures are still unsolved. Alon et. al [1] proved that there is a constant C such that $\delta \geq Clog|V(G)|$ are anti-magic. Liang and Zhu [5] proved that cubic graphs are anti-magic. Kaplan et al. [4] proved that trees without vertices of degree 2 are anti-magic. Liang et al. [7] proved a restricted class of trees with many degree 2 vertices to be anti-magic. For an exhaustive survey on anti-magic graphs, we refer Dynamic survey by Gallian [2].

2. SPLITTANCE OF A GRAPH

In this section, let us define the basic definitions required to prove our main result. Splitting graph of any graph was introduced by E. Sampathkumar and Walikar [6] in the year 1980. Let G be a graph. Add a new vertex $u^{'}$ for every vertex u of G. Add edges between $u^{'}$ and all the vertices of G that are adjacent to vertex u. The graph obtained in this way is called splitting graph of G and we denote it as $S(G)$. The splitting graph of cycle $C_5$is shown in below

Example 1.

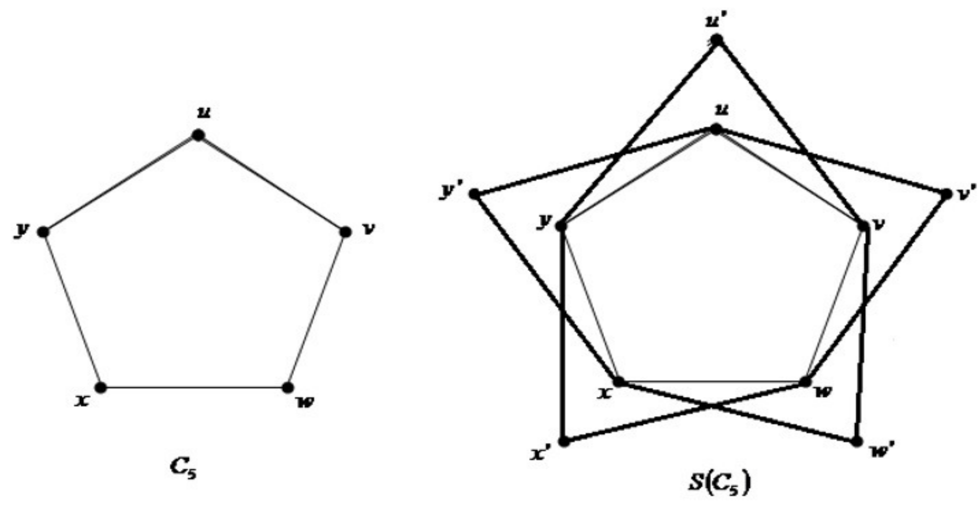

FIGURE 1. ANti-magic labeling ofC_5 and $S(C_5)$

One can easily observe that if G is $a(p,q)$ graph, then $S(G)$ is a $(2q,3q)$ graph. For any vertex u in G, $\overset{deg}{S(G)}\ u = 2 \overset{deg}{G}\ u$and $\overset{deg}{S(G)}\ u' = \overset{deg}{G}\ u$, where u' is the newly added vertex in $S(G)$. In [6] , Sampathkumar and Walikar proved the following characterization result on splittance of a graph.

Theorem 2.1. *A graph G is a splitting graph if and only if $V(G)$can be partitioned into two sets $V_1 \cup V_2$ such that (i) there exists a bijective mapping $V_1 \to V_2$ and (ii) $N(v_2) = N(v_1) \cap V_1$, where $N(v) = \{u : uv \in E(G)\}$.*

3. Main Result

In [3], Hartsfield and Ringel proved that all cycles are anti-magic. In this section, we prove our main result that the splittance of cycles are anti-magic.

Theorem 3.1. *Splittance of cycles are anti-magic.*

Proof. Consider a cycle $C_n, n \geq 3$ along with its anti-magic labeling φ. For the convenience, let us name the edges of C_n as $\{e_1, 2_2, \ldots, e_n\}$ in such a way that $\varphi(e_i) = i$. That is, arrange the edges of C_n as per the increasing order as defined by the anti-magic labeling φ. From the definition of anti-magic labeling, the vertex label of a vertex $u \in V(C_n)$ is defined as the sum of the edge labels of edges that are incident with vertex u. Let us arrange the vertices of C_n as $\{v_1, v_2, \ldots, v_n\}$ as per the increasing order of their vertex labels. Since the cycles C_n are anti-magic, this arr angement of vertices and edges are possible. During the operation of splittance of cycles, let $v_1', v_2', v_3', \ldots, v_n'$ are the newly added vertices corresponding to the vertices $v_1, v_2, v_3, \ldots, v_n$ respectively. Let $e_{i,1}$ and $e_{i,2}$, for $1 \leq i \leq n$, are the newly added edges that are incident with vertex v_i' and the adjacent vertices of v_i. In view of the above notations, the vertex set of the splittance of cycle C_n is $V(S(C_n)) = \{v_1, v_2, \ldots, v_n\} \cup \{v_1', v_2', \ldots, v_n'\} = V_1 \cup V_2$ and the edge set of the splittance of cycle C_n is $E(S(C_n)) = \{e_1, e_2, \ldots, e_n\} \cup \{e_{1,1}, e_{1,2}, \ldots, e_{n,1}, e_{n,2}\} = E_1 \cup E_2$. Note that $|V(S(C_n))| = 3n$. Now, let us define the function $f : E(S(C_n)) \longrightarrow \{1, 2, 3, \ldots, 3n\}$ as follows:
For each edge $e_i \in E_1, 1 \leq i \leq n$ define:

$$f(e_i) = 2n + i \,. \tag{3.1}$$

For each edge $ei, j \in E_2, 1 \leq i \leq n$ and $1 \leq j \leq 2$, define:

$$f(n) = \begin{cases} 2i - 1 & \text{if } j = 1 \\ 2i & \text{if } j = 2 \,. \end{cases} \tag{3.2}$$

For each vertex $v_1 \in V_1, 1 \leq i \leq n$, the vertex sum

$$\psi_f(v_i) = \sum_{e \in E(v_i)} f(e) \,,$$

where $E(v_i)$ is the set of edges that are incident with vertex v_i. For each vertex $v_i^{'} \in V_2, 1 \leq i \leq n$, the vertex sum

$$\psi_f(v_i^{'}) = \sum_{e \in E(v_i^{'})} f(e) = 4i - 1 \,,$$

where $E(v_i^{'}$ is the set of edges that are incident with vertex $v_i^{'}$.

From the definition of edge labels defined in equations (3.1) and (3.2), it is clear that the edge labels of the edges in E_1 are from the set $\{2n+1, 2n+2, \ldots, 3n\}$ and the edge labels of the edges in E_2 are from the set $\{1, 2, \ldots, 2n\}$. Therefore, the function f is a bijective function. □

Remark 3.1. *Vertex sum of vertices of $S(C_n)$ defined by ψ_f are distinct.*

By the definition of splittance of graph, it is clear that $deg(v_i) = 4$ for every vertex $v_i \in V_1$, for $1 \leq i \leq n$. Similarly, $deg(v_i^{'} = 2$ for every vertex $v_i^{'} \in V_2$, for $1 \leq i \leq n$. From the definition, it is clear that the set of vertex sum of vertices in V_1 and the set of vertex sum of vertices in V_2 are disjoint. Further, the labels in the set of vertex sum of vertices in V_1 are distinct. Similarly, the labels in the set of vertex sum of vertices in V_2 are distinct. Therefore, vertex sum of vertices of $S(C_n)$ defined by ψ_f are distinct. Hence, we proved that splittance of cycles $S(C_n)$ for $n \geq 3$ are anti-magic

4. Conclusion and discussions

In this paper, we proved that the splittance of cycles $S(C_n)$ for $n \geq 3$ are anti-magic. In this direction, it is natural to ask whether for what classes of graphs C, its splittance admit anti-magic. Is it necessary that a graph G is anti-magic, to admit the anti-magicness of its splittance graph $S(G)$? In this point of view, we

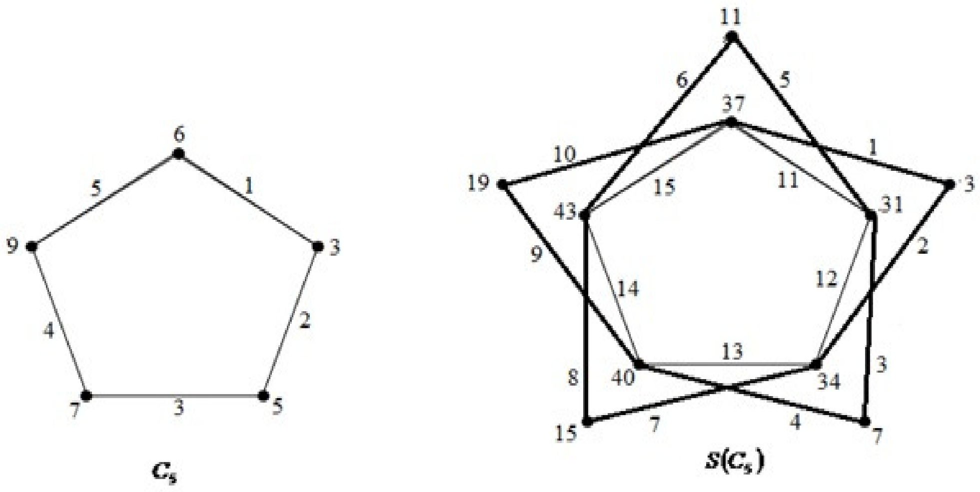

FIGURE 2. Anti-magic labeling ofC_5 and $S(C_5)$

provided the result to support the conjecture that every connected graph other than K_2 is anti-magic.

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