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ACCURATE NEIGHBORHOOD RESOLVING NUMBER OF A GRAPH

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ABSTRACT. A neighborhood set of a graph $G(V, E)$ is a subset $S \subseteq V$ such that $G = \cup_{v\in S}\langle N[v]\rangle$, where $N[v]$ is the closed neighborhood of the vertex v. A resolving set of a graph $G(V, E)$ is a subset $S \subseteq V$ such that every pair of distinct vertices of G is resolved by some vertex in S. A neighborhood set of G, which is also a resolving set is called as neighborhood resolving set (nr-set) of G. An nr-set S of G is called an accurate neighborhood resolving set (*anr*-set) of G if $\overline{S}$ has no nr-set of G with cardinality of S. In this paper, we determine the minimum cardinality of nr-sets and anr-sets of total graph of a cycle and a prism graph.

1. INTRODUCTION

The graphs that are considered throughout this paper are finite, simple, connected, nontrivial and undirected. The terms not defined here may be found in [1, 3]. For a graph $G(V, E)$ and a vertex $v \in V$, $N(v)$ denotes the set of all vertices which are adjacent to v and $N[v] = N(v) \cup \{v\}$. A subset S of V is a neighborhood set (n-set) of G if $\cup_{v\in S}\langle N[v]\rangle = G$, where $\langle N[v]\rangle$ is the sub graph of G induced by $N[v]$. The minimum cardinality of an n-set of G is called the neighborhood number of G and is denoted by $ln(G)$. Neighborhood number of a graph was first introduced by E. Sampathkumar and Prabha S. Neeralagi [7].

Given a graph G and a subset S of the vertex set of G, a vertex $s \in S$ resolves a pair of vertices $u, v \in V$, if $d(u, s) \neq d(v, s)$. A resolving set (r-set) S is a subset

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of vertex set of G such that each pair of vertices $u, v \in V(G)$ is resolved by at least one vertex in S. If $S = \{s_1, s_2, \ldots, s_k\}$ is a resolving set of G, then we can associate a unique vector for each $v \in V(G)$ with respect to S as $\Gamma(v/S) = (d(v, s_1), d(v, s_2), \ldots, d(v,s_k))$, where $d(u, v)$ is the distance between the vertices u and v in G. The minimum cardinality of an r-set of G is called the resolving number of G and is denoted by $lr(G)$. The concept of resolving number of a graph was first introduced by P. J. Slater [8] and independently by F. Harary and R. A Melter [2].

A subset S of V is called a neighborhood resolving set (nr-set) of G, if S is both neighborhood set and resolving set of G. The minimum cardinality of an nr-set is called the neighborhood resolving number of G and is denoted $lnr(G)$. An nr-set S of G is called an accurate neighborhood resolving set (anr-set) of G if $\bar{S}$ has no nr-set of G with cardinality of S. The minimum cardinality of an anr-set is called the accurate neighborhood resolving number of G and is denoted by $lnr_a(G)$. The concept of anr-set was first introduced and studied by Reshma et al. in [6]. For similar works we refer [4, 5, 10, 11].

The total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$, if their corresponding elements are either adjacent or incident in G.

We now recall the following results for immediate reference.

Theorem 1.1 (B. Sooryanarayana, Shreedhar K. and Narahari N. [9]). *For a graph G, $lr(T(G)) = 2$ if and only if G is a path P_n, $n \geq 2$.*

Theorem 1.2 (E. Sampathkumar and P. S. Neeralagi [7]). *A set S of vertices of a graph G is an n-set if and only if every edge of $\langle V(G) - S\rangle$ belongs to a triangle one of whose vertices belongs to S.*

If S is an n-set of G, then we say that an edge e is covered by S, if S contains a vertex s such that s is incident with e, or s is adjacent to both the end vertices of e in G. Also, we note that neighborhood property, resolving property, and neighborhood resolving property are all super hereditary.

Corollary 1.1 (E. Sampathkumar and P. S. Neeralagi [7]). *A set S is an n-set of a triangular free graph G if and only if $\overline{S}$ is totally disconnected.*

Observation 1.1. *For any graph G, as every nr-set is also an n-set and an r-set of G, it follows that $lnr(G) \geq ln(G)$ and $lnr(G) \geq lr(G)$.*

Observation 1.2. *For any graph G, as every anr-set is also an nr-set, an r-set and an n-set of G, it follows that $lnr_a(G) \geq lnr(G)$, $lnr_a(G) \geq lr(G)$ and $lnr_a \geq ln(G)$.*

2. TOTAL GRAPH OF A CYCLE

Throughout this section, the vertices $v_0, v_1, v_2, \ldots, v_{n-1}$ of the total graph $T(C_n)$ corresponds to the vertices of the cycle C_n, and the vertices $e_0, e_1, \ldots, e_{n-1}$ of $T(C_n)$ corresponds to the edge of C_n with $e_i = v_i v_{i+1(mod\ n)}$ for each i, $0 \leq i \leq n-1$.

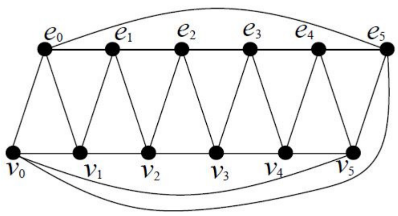

FIGURE 1. The total graph of the cycle C_6.

Theorem 2.1. *For any integer $n \geq 3$, $lnr(T(C_n)) = \begin{cases} 3, & if \quad n = 3. \\ \lceil \frac{2n}{3} \rceil, & if \quad n \geq 4. \end{cases}$*

Proof. Consider the graph $G = T(C_n)$ on $2n$ vertices.

Lower bound: Let S be any nr-set of G and $|S| = k$. Without loss of generality, we take $v_0 \in S$ (due to symmetry). The vertex v_0 covers exactly 7 edges namely, v_0v_1, v_0e_0, v_0v_{n-1}, v_0e_{n-1}, v_1e_0, $v_{n-1}e_{n-1}$ and e_0e_{n-1} as per the criteria of the n-set. While covering these seven edges, to cover the edge e_0e_1, the set S should include at least one of the elements in the set $T = \{e_0, e_1, v_1\}$. However, each single element in $T \cap S$ will cover at the most 6 new edges of G (since one edge is already covered by v_0) and e_1 is the only vertex in $S \cap T$ which covers the maximum of six edges. Further, v_3 is the vertex which covers maximum of 6 edges while covering the next edge v_2v_3. Continuing this way, every vertex in $S - \{v_0\}$ will cover at most 6 edges of G. Hence S will cover at most $7 + 6(k-1) = 6k + 1$ edges of G. Thus, as the graph G is a 4 regular graph $2n$ vertices, (the number of edges in G) $4n \leq 6k + 1$. That is $k \geq \lceil \frac{4n-1}{6} \rceil$. Therefore,

$lnr(G) = \min\{|S| : S$ is an nr-set of $G\} \geq \lceil\frac{4n-1}{6}\rceil = \lceil\frac{2n}{3}\rceil$. But when $n = 3$, by Theorem 1.1, $lr(G) \geq 3$ and hence by Observation 1.1, $lnr(G) \geq 3$.

Upper bound: We show the lower bound obtained above is tight by executing an nr-set S of G.

Case 1: $3 \leq n \leq 6$.

Consider the sets; $S_3 = \{v_0, v_1, v_2\}$, $S_4 = \{v_0, e_1, v_2\}$, $S_5 = \{v_0, e_1, v_3, e_4\}$ and $S_6 = \{v_0, e_1, v_3, e_4\}$. It can be easily verified that S_3, S_4, S_5 and S_6 are nr-sets of G for $n = 3, 4, 5, 6$, respectively.

Case 2: $n \geq 7$.

Consider the set $S = \begin{cases} \{v_0, e_1, v_3, e_4, \dots, v_{n-3}, e_{n-2}\}, & \text{if } n \equiv 0 \pmod 3. \\ \{v_0, e_1, v_3, e_4, \dots, e_{n-3}, v_{n-1}\}, & \text{if } n \equiv 1 \pmod 3. \\ \{v_0, e_1, v_3, e_4, \dots, v_{n-2}, e_{n-1}\}, & \text{if } n \equiv 2 \pmod 3. \end{cases}$

The set S defined above is an n-set of G. In fact,

Subcase 1: $n \equiv 0 \pmod 3$.

In this case, $v_i, e_{j+1} \in S$, for every $i \equiv 0 \pmod 3$ and $0 \leq i \leq n-2$. The edges: $v_{3i}v_{3i\pm1 \pmod n}$ are covered by $v_{3i} \in S$; $v_{3i+1 \pmod n}v_{3i+2 \pmod n}$ are covered by $e_{3i+1} \in S$; $e_{3i+1}e_{3i+2 \pmod n}$ and $e_{3i+1}e_{3i-1 \pmod n}$ are covered by $e_{3i+1} \in S$; $e_{3i-1 \pmod n}e_{3i}$ are covered by $v_{3i} \in S$; $e_{3i}v_{3i+1 \pmod n}$ and $e_{3i-1 \pmod n}v_{3i}$ are covered by $v_{3i} \in S$; $e_{3i+1}v_{3i+1}$ are covered by $e_{3i+1} \in S$; $e_{3i-1 \pmod n}v_{3i-1 \pmod n}$ and $e_{3i}v_{3i}$ are covered by $v_{3i} \in S$; and $e_{3i+1}v_{3i+1}$ are covered by $e_{3i+1} \in S$. Hence, $G = \cup_{v \in S}\langle N[v]\rangle$ and $|S| = |S \cap V(C_n)| + |S \cap E(C_n)| = 2|S \cap V(C_n)| = 2[1 + \frac{n-3}{3}] = \frac{2n}{3} = \lceil\frac{2n}{3}\rceil$.

Subcase 2: $n \equiv 1 \pmod 3$.

In this case, all the edges of G between two vertices are covered by the set $S' = \{v_0, e_1, v_3, e_4, \dots, v_{n-4}, e_{n-3}\}$ as in the above Subcase 1 except the 3 edges, namely $v_{n-2}v_{n-1}$, $e_{n-2}v_{n-1}$ and e_{n-2}, e_{n-1}. These 3 edges are now covered by $v_{n-1} \in S$. Hence S is an n-set with $|S| = |S'| + 1 = 2[1 + \frac{n-4}{3}] + 1 = \frac{2n+1}{3} = \lceil\frac{2n}{3}\rceil$.

Subcase 3: $n \equiv 2 \pmod 3$.

In this case, all the edges of G between two vertices are covered by the set $S' = \{v_0, e_1, v_3, e_4, \dots, e_{n-4}, v_{n-2}\}$ as in Subcase 2 except one edge, namely $e_{n-2}e_{n-1}$.

This extra edge is covered by $e_{n-1} \in S$. Hence S is an n-set and $|S| = 2[1+\frac{n-2}{3}] = \frac{2n+2}{3} = \lceil \frac{2n}{3} \rceil$.

Now to prove $lnr(G) \leq \lceil \frac{2n}{3} \rceil$, it remains to show that the set S defined above is also an r-set. For this, let $S_1 = \{v_0, e_1\}$. Then the vector associated for each vertex of G with respect to S_1 is given by

$$\begin{array}{llll}
\Gamma(v_i/S_1) = (i, 2-i) & \text{and} & \Gamma(e_i/S_1) = (i+1, 1-i) & \text{if} \quad i = 0, 1 \\
\Gamma(v_i/S_1) = (i, i-1) & \text{and} & \Gamma(e_i/S_1) = (i+1, i-1) & \text{if} \quad 2 \leq i \leq \lceil n/2 \rceil - 1 \\
\Gamma(v_i/S_1) = (n-i, i - \lfloor \frac{n-1}{i} \rfloor) & \text{and} & \Gamma(e_i/S_1) = (n-i, i - \lfloor \frac{n-1}{i} \rfloor) & \text{if} \quad \lfloor \frac{n+1}{2} \rfloor \leq i \leq \lceil \frac{n+1}{2} \rceil \\
\Gamma(v_i/S_1) = (n-i, n-i+2) & \text{and} & \Gamma(e_i/S_1) = (n-i, n-i+1) & \text{if} \quad \lceil \frac{n+1}{2} \rceil < i \leq n-1.
\end{array}$$

From the above vector, it is easy to see that $\Gamma(u/S_1) = \Gamma(v/S_1)$ if and only if $(u, v) \in T_1 = \{(v_1, e_0), (v_{\lfloor \frac{n+1}{2} \rfloor}, e_{\lfloor \frac{n+1}{2} \rfloor}), (v_{\lceil \frac{n+1}{2} \rceil}, e_{\lceil \frac{n+1}{2} \rceil})\}$. But, for each pair $(u, v) \in T$, $|d(u, v_3) - d(v, v_3)| \neq 0$ and hence v_3 will resolve u and v. Thus, $S_2 = S_1 \cup \{v_3\}$ is an r-set of G. Further, as $S_2 \subseteq S$ and super hereditary property of resolving sets, the set S is an r-set of G. Hence the proof. □

Theorem 2.2. *For any integer $n \geq 3$,*

$$lnr_a(T(C_n)) = \begin{cases} 4, & \textit{if} \quad n = 3. \\ \lceil \frac{2n}{3} \rceil + 1, & \textit{if} \quad n \equiv 1, 2 \pmod 3. \\ \lceil \frac{2n}{3} \rceil + 2, & \textit{if} \quad n \equiv 0 \pmod 3. \end{cases}$$

Proof. Let $G = T(C_n)$, $n \geq 3$ be the graph with $2n$ vertices.

Lower bound: Let S be an anr-set. Then S is an nr-set and $\bar{S}$ has no nr-set of cardinality $|S|$. Without loss of generality, we assume $v_0 \in S$. We first see that if S contains all the three vertices of a triangle in G, then $\bar{S}$ is not an n-set of G for all $n \geq 4$ and hence $\bar{S}$ has no nr-set of any cardinality. Therefore, if $n = 3, 4$ and $|S| = 3$, then $\langle S \rangle = C_3$ and $n \geq 4$ (else by symmetry $G - S$ has a subgraph H isomorphic to $\langle S \rangle$ and hence $V(H)$ will be an nr-set of G). Further, if $n = 4$, without loss of generality, let $S = \{v_0, e_0, v_1\}$. Then the edge v_2v_3 is not in any triangle of G with one vertex in S, contradiction to Theorem 1.2. So, $lnr_a(G) \geq 4$, for $n = 3, 4$. Let $n \geq 5$ and S' be any minimum nr-set of G. Then, by Theorem 2.1, $|S'| = \lceil \frac{2n}{3} \rceil$. Further, each element of S' is a corner vertex of the shaded triangle which is in maximum number of unshaded triangles behind it (starting with $v_0 \in S'$) as in Figure 2. Thus, in each minimum nr-set S' if $v_i \in S'$ then $e_i \notin S'$. This shows that the set S'' obtained by just interchanging e_i and

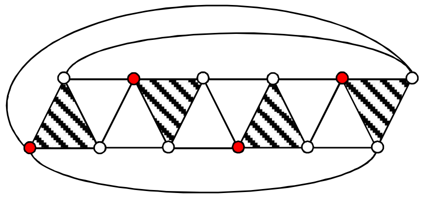

FIGURE 2. Optimal choice of an nr-set.

v_i in S' is also an nr-set of G. Since $S'' \subseteq \bar{S}$, it follows that $lnr_a(G) > lnr(G)$. Hence $lnr_a(G) \geq \lceil \frac{2n}{3} \rceil + 1$. Further, if any nr-set S' contains a pair v_i, e_i for at most one i and no two adjacent pairs of C_n, then it is easy to verify that the set S'' containing x_{i+1} for each x_i in S' is an nr-set of G. Therefore, C_3 should be an induced subgraph of $\langle S \rangle$ for every minimum anr-set S. But then, as these three vertices of a triangle in S will be covering exactly 11 edges, to cover the remaining $4n - 11$ edges we need at least $\frac{4n-11}{6}$ vertices in S other than those three which are in a triangle. That is, $4n-11 \leq 6(|S|-3)$ implies that $|S| \geq \frac{4n+7}{6}$. So, $|S| \geq \lceil \frac{2n}{3} \rceil + 1$ if $n \not\equiv 0 \pmod 3$, and $|S| \geq \lceil \frac{2n}{3} \rceil + 2$ if $n \equiv 0 \pmod 3$.

Upper bound: Here we show that the above lower bound is tight by executing an anr-set of G.

Case 1: $n \equiv 0 \pmod 3$.

When $n = 3$, it is easy to see that the set $S = \{v_0, v_1, v_2, e_1\}$ is an anr-set for G. For $n > 3$, let S_1 be a minimum nr-set of G. Without loss of generality, we assume $v_0 \in S$. Since S_1 is a minimum nr-set, as per the above discussion, e_0, v_1 are not in S. Taking $S = S_1 \cup \{e_0, v_1\}$ and by the super hereditary of nr property, we see that S is an nr-set of G. Further, $\bar{S}$ contains none of the vertices of a triangle of G. Hence $\bar{S}$ is not an nr-set. Thus, S is an anr-set with $|S| = |S_1| + 2 = \lceil \frac{2n}{3} \rceil + 2$.

Case 2: $n \equiv 1 \pmod 3$.

In this case, consider the nr-set $S_1 = \{v_0, e_1, v_3, e_4, \ldots, e_{n-3}, v_{n-1}\}$ (as in the proof of Theorem 2.1). The set $S = S_1 \cup \{e_{n-1}\}$ is then an nr-set and $\langle \{v_0, e_{n-1}, v_{n-1}\} \rangle$ is an induced cycle C_3 of $\langle S \rangle$. Hence, $\bar{S}$ is not an nr-set implies that S is an anr-set with $|S| = |S_1| + 1 = \lceil \frac{2n}{3} \rceil + 1$.

Case 3: $n \equiv 2 \pmod 3$.

In this case, consider the nr-set $S_1 = \{v_0, e_1, v_3, e_4, \ldots, v_{n-2}, e_{n-1}\}$ (as in the proof of Theorem 2.1). The set $S = S_1 \cup \{v_{n-1}\}$ is then an nr-set and $\langle\{v_0, e_{n-1}, v_{n-1}\}\rangle$ is an induced cycle C_3 of $\langle S\rangle$. Hence, $\bar{S}$ is not an nr-set implies that S is an anr-set with $|S| = |S_1| + 1 = \lceil \frac{2n}{3} \rceil + 1$. $\square$

3. Prism graph

The *Cartesian product* of two graphs G and H, denoted by $G\square H$, is a graph whose vertex set is $V(G) \times V(H)$ and two vertices (g, h) and (g', h') are adjacent in $G\square H$ if either $g = g'$ and $hh' \in E(H)$, or $h = h'$ and $gg' \in E(G)$. *Prism graph* is the Cartesian product of C_n and P_2, denoted by $C_n\square P_2$. Throughout this section, we label the vertices of the prism graph, $C_n\square P_2$ as v_0, v_1, v_2, ..., v_{n-1}, u_0, u_1, u_2, ..., u_{n-1} such that v_i is adjacent to $v_{i+1(mod\ n)}$ for all i, $0 \le i \le n-1$, u_i is adjacent to $u_{i+1(mod\ n)}$ for all i, $0 \le i \le n-1$ and v_i is adjacent to u_i for all i, $0 \le i \le n-1$.

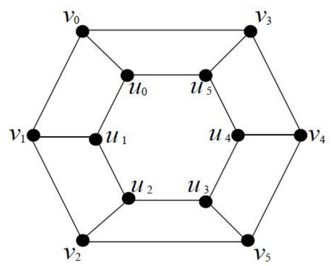

FIGURE 3. Prism graph, $C_6\square P_2$.

Theorem 3.1. *For any integer $n \ge 3$ and a prism graph $C_n\square P_2$,*

$$lnr(C_n\square P_2) = 2\left\lceil \frac{n}{2} \right\rceil.$$

Proof. Let $G = C_n\square P_2$ be a prism graph on $2n$ vertices.

Lower bound: The graph G is triangle free and hence for every n-set S of G, by Corollary 1.1, its complement $\bar{S}$ is totally disconnected and vice versa. Thus, the nr-number of the graph G is equal to $|V(G)| - id(G)$, where $id(G)$ is the independent number of G. Let $S_u = \{u_i : 0 \le i \le n-1\}$ and $S_v = \{v_i : 0 \le i \le n-1\}$. Then S_u and S_v are the partition of $V(G)$. Since $\langle S_u\rangle$ and

$\langle S_v \rangle$ are isomorphic to the cycle C_n, each independent set of G contains at most $\lfloor \frac{n}{2} \rfloor$ vertices from each of these sets. Therefore, $lnr(G) \geq ln(G) \geq 2n - 2\lfloor \frac{n}{2} \rfloor = 2\left(n - \lfloor \frac{n}{2} \rfloor\right) = 2\lceil \frac{n}{2} \rceil$.

Upper bound: We show that lower bound obtained above is tight by exacting an nr-set S. For this, we first consider the set $T = \{v_0, v_3\}$.

Case 1: $3 \leq n \leq 6$.

Consider the sets; $S_3 = \{v_0, u_0, v_1, u_3\}$, $S_4 = \{v_0, u_1, v_2, u_3\}$, $S_5 = \{v_0, u_0, u_1, v_2, u_3, v_4\}$ and $S_6 = \{v_0, u_1, v_2, u_3, v_4, u_5\}$. It can be easily verified that S_3, S_4, S_5 and S_6 are r-sets of G for $n = 3, 4, 5, 6$, respectively. Also, for each of the sets $\bar{S}$ is independent. Hence they are the desired nr-sets.

Case 2: $n \geq 7$.

The vectors associated to each vertex of G with respect to T is as below.

$\Gamma(v_i/T) = (i, 2-i)$	and	$\Gamma(u_i/T) = (i+1, 3-i)$	for	$1 \leq i \leq 2$
$\Gamma(v_i/T) = (i, i-2)$	and	$\Gamma(u_i/T) = (i+1, i-1)$	for	$3 \leq i \leq \lfloor \frac{n}{2} \rfloor$
$\Gamma(v_i/T) = (n-i, i-2)$	and	$\Gamma(u_i/T) = (n+1-i, i-1)$	for	$\lceil \frac{n}{2} \rceil \leq i \leq \lceil \frac{n}{2} \rceil + 1$
$\Gamma(v_i/T) = (n-i, n+2-i)$	and	$\Gamma(u_i/T) = (n+1-i, n+3-i)$	for	$\lceil \frac{n}{2} \rceil + 2 \leq i \leq n-1$

This shows that, T will resolve all the pairs of vertices of G except those in $H = \{(v_i, u_{i-1}) : 3 \leq i \leq \lfloor \frac{n}{2} \rceil\} \cup \{(v_i, u_{i+1 \pmod n}) : \lceil \frac{n}{2} \rceil + 2 \leq i \leq n-1\}$. Now for each pair $(v_i, u_{i+1}) \in H$, $d(v_i, u_1) = d(u_{i+1}, u_1) + 2$ and hence u_1, will resolve all the pairs in H. Thus, the set $S_1 = T \cup \{u_1\}$ is an r-set for G.

Now consider the set $S = \begin{cases} \{v_0, u_1, v_2, u_3, \ldots, v_{n-2}, u_{n-1}\} & \text{if } n \text{ is even.} \\ \{v_0, u_1, v_2, u_3, \ldots, v_{n-1}, u_0\} & \text{if } n \text{ is odd.} \end{cases}$

The set S_1 is a subset of S and hence by super hereditary of the resolving property, S is an r-set. Also, $\bar{S}$ is an independent set of G. Hence, by Corollary 1.1, S is an nr-set and it contains $2\lceil \frac{n}{2} \rceil$ vertices of G. Hence $lnr(G) \leq 2\lceil \frac{n}{2} \rceil$ .Hence the proof. □

We now state the following generalized theorem whose proof follows similar to the above theorem.

Theorem 3.2. *For the integers $m \geq 1$ and $n \geq 3$,*

$$lnr(C_n \Box P_m) = m \left\lceil \frac{n}{2} \right\rceil .$$

For each odd n, every minimum nr-set S of the prism $C_n\Box P_2$ contains $n+1$ elements (by Theorem 3.1). Therefore, $|\bar{S}| = 2n-(n+1) = n-1 < |S|$, implies that $\bar{S}$ can not have any nr-set with cardinality $|S|$. Hence every minimum nr-set of $C_n\Box P_2$ is also an anr-set of G whenever n is odd. But this is not the case when n is even. If n is even, then both S and $\bar{S}$ are independent with $|S| = |\bar{S}| = n$, for every nr-set S of $C_n\Box P_2$. Therefore, every minimum anr-set should contain one more element than in a minimum nr-set. We record these in the form of following theorem.

Corollary 3.1. *For any integer $n \geq 3$ and a prism graph $C_n\Box P_2$,*

$$lnr_a(C_n\Box P_2) = n+1.$$

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