Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7223–7233
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.71

DETERMINATION OF EARTH'S MASS DENSITY DISTRIBUTION BASED ON SATELLITE DATA

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ABSTRACT. Determination of mass anomalies in the Earth's interior is one of the important topics in Geoscience research. This can be achieved by inversion of Newton's gravitational potential. This inversion is known as inverse gravimetric problem. Regularized Functional Matching Pursuit (RFMP) method is one of the available methods to solve inverse gravimetric problem approximately (see [4]). In this paper, RFMP algorithm is implemented to calculate mass density distribution in the area of German Saarland. Gravitational potential of Saarland region is a key input data for this reconstruction. This gravitational potential is computed from 'Gravity field and steady-state Ocean Circulation Explorer (GOCE)' data. GOCE level-2 data analysing tool 'GOCE User Toolbox (GUT)' is used for this gravitational potential computation.

1. INTRODUCTION

Gravity field on the Earth's exterior reflects the mass density distribution of Earth's interior. This gravity field varies from place to place on the Earth. Because, materials within the Earth's interior are not uniformly distributed. Discovery of mass anomalies in the Earth's interior is one of the important tasks in field of Geosciences. There is no direct method to find mass density distribution of Earth's interior. Reconstruction of mass density distribution from gravitational

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2010 *Mathematics Subject Classification.* 83C99.

Key words and phrases. Inverse gravimetric problem, Gravitational potential, Mass density distribution, Mathematical model.

potential is one of the indirect methods. Newton's law of gravitation states the connection between gravitational potential V of the Earth's exterior and mass density ρ of the Earth's interior as follows:

$$V(x) = \gamma \int_{\mathcal{B}} \frac{\rho(y)}{|x-y|} dy, x \in \mathbb{R}^3 \setminus \mathcal{B}$$

where γ is a gravitational constant.

Mass density distribution of Earth's interior is obtained by inverting the Newton's law of gravitation. It means that mass density of Earth's interior is determined from gravitational potential on the Earth's surface. This inverting process is known as inverse gravimetric problem. There are some methods available in the literature to solve inverse gravimetric problem ([8]). Regularized Functional Matching Pursuit (RFMP) algorithm is one of them ([4]). This algorithm is already used in the literature to determine mass anomalies in some parts of the Earth such as South America, India and Himalayas. In [5], RFMP algorithm is used for reconstruction of mass density distribution of South America. Data from 'Earth Gravitational Model 2008 (EGM2008)' is used for this computation. EGM2008 is a geopotential model of the Earth. It is developed by National Geospatial Intelligence Agency (NGA). In the same paper, mass transport in the Amazon area for the year 2008 is also examined by using RFMP algorithm. 'Gravity Recovery and Climate Experiment (GRACE)' satellite data is used for this computation.

In [6], RFMP algorithm is used to locate and analyse extreme weather events of the late summer months of six successive years starting from 2005 in the Amazon area. Data from GRACE satellite is considered for this computation. In [7], mass density distribution in the area of Himalayas and India are reconstructed from EGM2008 data. RFMP algorithm is used for this computation. RFMP has been considered in the following research articles: [9], [13], [12] and [10].

In this paper, RFMP algorithm is used to reconstruct mass density distribution of the German Saarland region. Fourth generation data of 'Gravity field and steady-state Ocean Circulation Explorer(GOCE)' is used for this computation. In the first part of numerical work, gravitational potential of Saarland region is computed form GOCE data. GOCE User Toolbox (GUT) is used for this gravitational potential computation. Finally, RFMP algorithm is used to reconstruct mass density distribution from gravitational potential.

2. PRELIMINARIES

2.1. Mathematical Formulation of Inverse Problem. An inverse problem is a general framework that is used to convert observed measurements into an information about a physical object. A mathematical model is a mapping A from the set (of causes) X to the set (of effects) Y, i.e.,

$$A : X \to Y \ .$$

A direct problem amounts to description and evaluation of A. An inverse problem is a task to find for a given element $y \in Y$ an element $x \in X$ such that $A(x) = y$.

2.2. Gravity field and steady-state Ocean Circulation Explorer(GOCE) level-2 Product. 'GO_CONS_GCF_2_DIR_R4' is a fourth generation GOCE level-2 data product. Direct (DIR) gravity field modelling approach is used in the computation of 'GO_CONS_GCF_2_DIR_R4' model. DIR starts with an a priori gravity field model 'GO_CONS_GCF_2_DIR_R3' and adds GOCE information to improve it. 'GO_CONS_GCF_2_DIR_R3' is a third generation GOCE level-2 data product.
'GO_CONS_GCF_2_DIR_R3' coefficients are given up to spherical harmonic of degree 240 and order 240. 'GO_CONS_GCF_2_DIR_R4' coefficients are given up to spherical harmonic of degree 260 and order 260. Further details about 'GO_CONS_GCF_2_DIR_R4' model refer [2].

3. COMPUTATION OF GRAVITATIONAL POTENTIAL

Computation of gravitational potential is a initial step to estimate the mass density distribution. GOCE User Toolbox(GUT) computes gravitational potential of Saarland region from 'GO_CONS_GCF_2_DIR_R4'.

GUT input arguments for computation of gravitational potential are defined in the following:

- workflow, input data file, longitude bound of equiangular output grid, latitude bound of equiangular output grid, latitude & longitude grid spacing, reference ellipsoid, tide system and output file name.

A pre-build work flow *surfacegravitationalpot_gf* is one of the most important input arguments for computation of gravitational potential. Because, it extracts spherical harmonic coefficients from an input file and computes gravitational potential of a chosen grid. Computation results are stored in a *netCDF* output file. All other input arguments of GUT are inputs for *surfacegravitationalpot_gf*.

Longitude range is given by west border(w) and east border(e). Latitude range is given by south border(s) and north border(n). The GUT syntax for longitude and latitude bound inputs is '`-R w:e,s:n`'. West border(w), east border(e), south border(s) and north border(n) should be in degrees(°). Longitude range should be with in interval $[-360, 360]$. Latitude range should be with in interval $[-90, 90]$. Longitude grid spacing is denoted by 'de'. Latitude grid spacing is denoted by 'dn'. Longitude grid spacing(de) and latitude grid(dn) should be in degrees(°). The GUT syntax for grid spacing is '`-I de:dn`'. Moreover, Longitude range must be an integral multiple of grid spacing(de) and latitude range must be an integral multiple of grid spacing(dn). More details about GUT refer [3].

3.1. **Gravitational potential of the Earth.** Gravitational potential of the whole Earth is computed by GUT. Input file 'GO_CONS_GCF_2_DIR_R4' and workflow '*surfacegravitationalpot_gf*' are command line inputs for the whole Earth's gravitational potential computation. All other inputs values are assigned to its default values. Defaults values of remaining inputs are given in Table 1. BRAT-

Input Arguments	Default Values
Longitude range(w:e) Latitude range(s:n)	.50:359.50 -89.50:89.50
Longitude grid spacing(de) Latitude grid spacing(dn)	1.0 1.0
Reference Ellipsoid	GRS80
Tide-system	tide-free

TABLE 1. Default input arguments for the computation of gravitational potential of whole Earth

Display Tool is used for visualization of GUT output file. It is visualization tool of Basic Radar Altimetry Toolbox [refer [1]]. It is a command line processor. Gravitational potential map of whole Earth is in Figure 1.

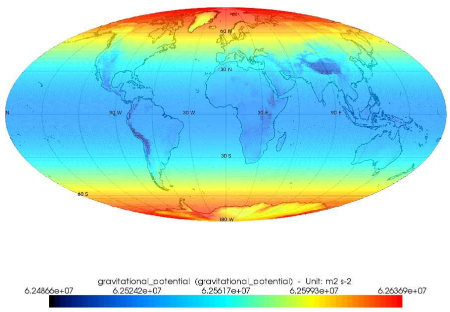

FIGURE 1. Gravitational potential of the whole earth is computed by GUT

3.2. **Gravitational potential of Saarland region.** In this section, gravitational potential of Saarland region is computed from GOCE data. GUT is used for this computation.

Input file 'GO_CONS_GCF_2_DIR_R4' and workflow '*surfacegravitationalpot_gf*' are important inputs of GUT for Saarland's gravitational potential computation. Other inputs of GUT are given in Table 2. Gravitational potential map of Saar-

Input Arguments	Input Values
Longitude range(w:e) Latitude range(s:n)	6:8 49:50
Longitude grid spacing(de) Latitude grid spacing(dn)	0.1 0.1
Reference Ellipsoid	WGS84
Tide-system	tide-free
Output file	saarland.nc

TABLE 2. Input arguments for computation of gravitational potential of Saarland region

land region is generated from GUT output file '*saarland.nc*'. 3D figure of this gravitational potential is given in Figure 2.

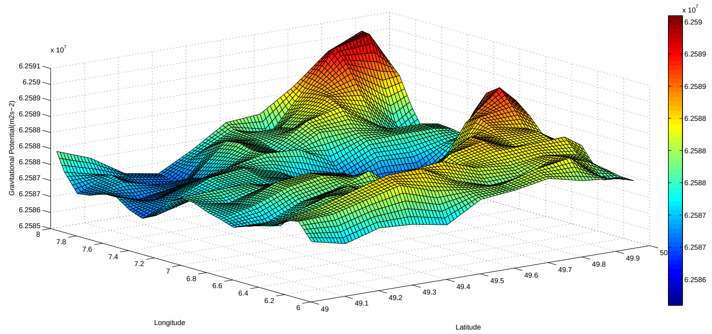

FIGURE 2. Gravitational potential of Saarland region

4. RECONSTRUCTION OF MASS DENSITY DISTRIBUTION OF SAARLAND

In this section, mass density distribution of Saarland region is reconstructed from gravitational potential of Saarland region. Gravitational potential is computed from GOCE level-2 data in Section 3. Regularized Functional Matching Pursuit (RFMP) algorithm takes this gravitational potential as input and it computes mass density distribution.

In this paper, only harmonic part of the mass density distribution is reconstructed from gravitational potential. Mass density distribution of Earth's deep interior cannot be recovered from gravity data ([11]). Therefore, only density close to the surface is reconstructed for Saarland region. Following dictionary is used in RFMP algorithm for computation of mass density distribution:

$$\begin{aligned} \mathcal{D} &= \{G_{0,n,j} | n = 3, ..., 11, j = -n, ..., n\} \\ &\cup \{K_h^I(x, .) | h = 0.97, x \in grid(\mathcal{B})\}, \end{aligned}$$

where $grid(\mathcal{B})$ is nearly quadratic grid, which is equiangular each in longitude and latitude. 231 grid points are taken for computation over Saarland region. The dictionary $\mathcal{D}$ contains 135 complete orthonormal elements and 231 localized kernel functions. Total dictionary elements used in this computation is 366.

RFMP algorithm is executed for 1000 iterations, i.e., 1000 dictionary functions are chosen from dictionary $\mathcal{D}$. These choices are not necessarily pairwise distinct. Because, same dictionary elements may be picked in more than one iteration. Since inverse gravimetric problem is ill-posed, regularization parameter λ should be used to solve inverse gravimetric problem. Number of iterations, number of grid points and number of dictionary elements should be increased to find better regularization parameter λ.

In this work, mass density distribution of Saarland region for regularization parameter $\lambda = 0$, $\lambda = 10$ and $\lambda = 50$ are computed by repeating RFMP algorithm for 1000 times. 3D figure of these results are shown in Figure 3, 4, and 5.

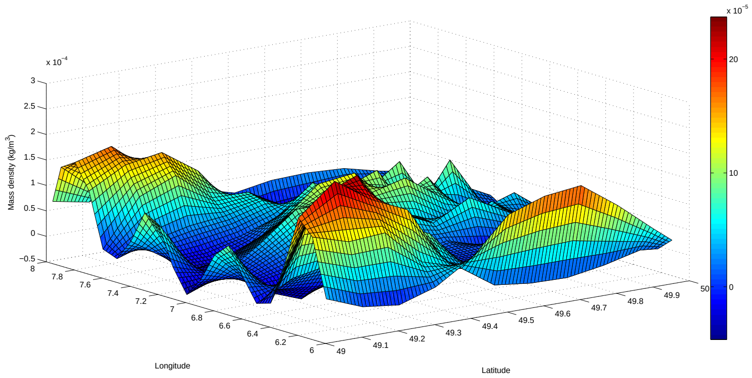

FIGURE 3. Mass density distribution of Saarland region for regularization parameter $\lambda = 0$

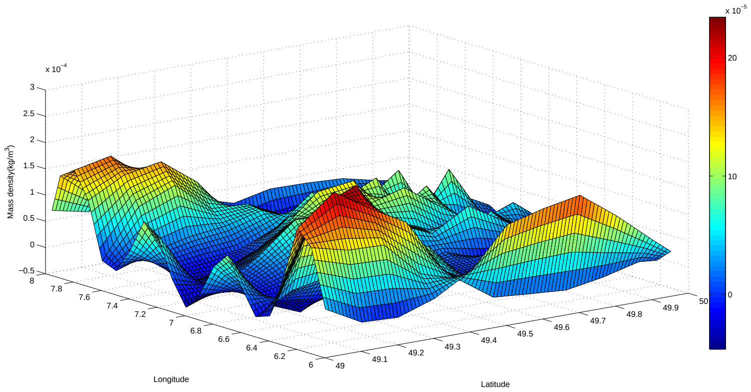

FIGURE 4. Mass density distribution of Saarland region for regularization parameter $\lambda = 10$

Gravitational potential of Saarland region is computed by GUT from GOCE level-2 data. 2D figure of this result is in Figure 6.

Mass density distribution of Saarland region for regularization parameter $\lambda = 10$ is computed by repeating RFMP algorithm for 1000 times. 2D figure of this result is in Figure 7.

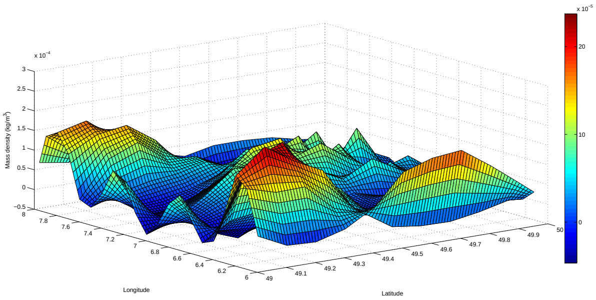

FIGURE 5. Mass density distribution of Saarland region of regularization parameter $\lambda = 50$

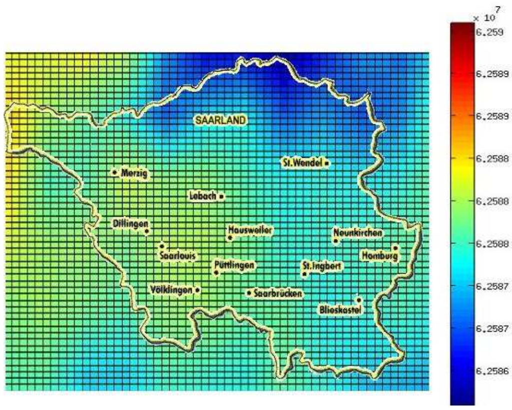

FIGURE 6. Gravitational potential of Saarland region

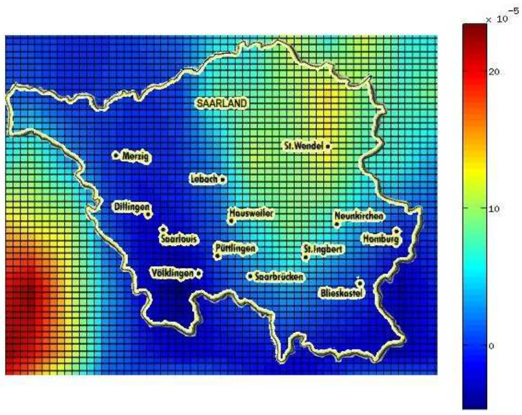

FIGURE 7. Mass density distribution of Saarland region for regularization parameter $\lambda = 10$

Mass density distribution of Saarland region for regularization parameter $\lambda = 50$ is computed by repeating RFMP algorithm for 1000 times. 2D figure of this result is in Figure 8.

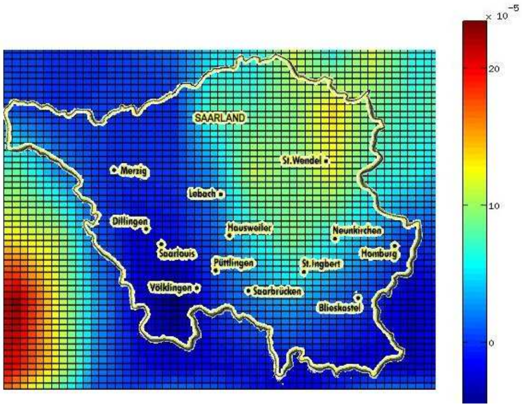

FIGURE 8. Mass density distribution of Saarland region for regularization parameter $\lambda = 50$

5. CONCLUSION

Mineral mining companies and oil companies are spending more money and time for mineral and oil exploration projects. Currently, drilling process is used in most of this projects. Here also, drilling process is used for exploration of geothermal resource. In general, cost for drilling process is too expensive. This process takes more time to discover new resources. If mass density distribution of the Earth's interior is known, cost and time to discover new resources can be reduced by analysing this density distribution results. Mass density distribution of the Earth's interior is obtained by solving inverse gravimetric problem. In this paper, mass density distribution of the German Saarland region is reconstructed from gravitational potential of Saarland region by using Regularized Functional Matching Pursuit (RFMP) algorithm. GOCE User Toolbox (GUT) is used to compute gravitational potential of Saarland region from GOCE level-2 data. Results of this paper can be used to locate new resources in the area of German Saarland after improving the accuracy these results. Accuracy of these results can be improved by increasing number of data points, choosing best regularization parameter, choosing various type of dictionary elements and repeating algorithm for more times.

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