ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.9, 7249–7255
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.9.74

# ANTI-VAGUE IDEALS OF SUBTRACTION ALGEBRA

K. PRASAD[1], D. V. RAMALINGA REDDY, Y. PRABHAKARA RAO, S. VENU MADHAVA SARMA, P. RANGASWAMY, AND P. RAMI REDDY

ABSTRACT. In this paper we introduce the concept of anti-vague ideals of subtraction algebra and investigate some results on anti-vague ideals of subtraction algebras.

## 1. INTRODUCTION AND PRELIMINARIES

In this paper we introduced a notion of anti-vague ideal in a subtraction algebra and study some properties of them.

**Definition 1.1.** *By a subtraction algebra, we mean an algebra $(X, -)$ with a single binary operation "-" that satisfies the following identities for any $x, y, z \in X$.*

(1) $x - (y - x) = x$
(2) $x - (x - y) = y - (y - x)$
(3) $(x - y) - z = (x - z) - y$.

The last identity permits us to omit parentheses in expressions of the form $(x - y) - z$. The Subtraction determines an ordered relation on X .

$$a \leq b \Leftrightarrow a - b = 0\,,$$

where $0 = a - a$ is an element that does not dependent on the choice of $a \in X$. The ordered set $(X, \leq)$ is a semi-Boolean algebra. That is ,it is meet semi-lattice

[1]*corresponding author*
2010 *Mathematics Subject Classification.* 06A99.
*Key words and phrases.* Vague set, Vague ideal, Anti-Vague ideal, Subtraction Algebra.

with zero in which every interval $[0, a]$ is a Boolean algebra with respect to the induced order, Here $a \wedge b = a - (a - b)$ the complement of an element b$\in [0, a]$ is $a - b$ and if $b, c \in [0, a]$ then

$$b \vee c = (b' \wedge c')' = a - ((a - b) \wedge (a - c)) = a - ((a - b) - ((a - b) - (a - c))) .$$

**Definition 1.2.** *A non-empty sub-set 'A' of a subtraction algebra X is called an ideal of X, if it satisfies (a-x) $\in$ A for any a$\in$ A and x $\in$ X $\forall$ a, b$\in$ A, whenever a$\vee$ b exists in X then a $\vee$ b $\in$ A.*

**Proposition 1.1.** *Let $X$ be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y then the element $x \vee y = w - ((w - y) - x)$ is a least upper bound for x and y.*

For further reference see [1–7].

## 2. Vague Sets

**Definition 2.1.** *A Vague set 'A' in the universe of discourse 'U' is characterized by two membership functions given by*

(1) *A truth membership function $t_A$:U$\rightarrow$ [0,1] and*
(2) *A false membership function $f_A$:U$\rightarrow$ [0,1]*

*where $t_A(u)$ is a lower bound of the grade of membership of 'u' derived from the evidence for u, and $f_A$(u) is a lower bound on the negation of 'u' derived from the "evidence against u" and $t_A(u) + f_A(u) \leq$ 1.*

Thus the grade of membership of 'u' in the Vague set 'A' is bounded by a sub interval $[t_A(u), 1 - f_A(u)]$ of [0,1] . This indicates that if the actual grade of membership is $\mu(u)$, then $[t_A(u) \leq \mu(u) \leq 1 - f_A(u)]$. The Vague set 'A' is written as $A = \{(u, [t_A(u), f_A(u)])|u \in U\}$ where the interval $[t_A(u), 1 - f_A(u)]$ is called the Vague value of 'u' in $A$ and is denoted by $V_A(u.)$

**Definition 2.2.** *A Vague set 'A' of a set U is called*

(1) *The Zero Vague set of U if $t_A$(u) = 0 and $f_A$(u) = 1 for all u$\in$U.*
(2) *The unit Vague set of U if $t_A$(u) = 1 and $f_A$(u) = 0 for all u $\in$ U.*
(3) *The $\alpha$-Vague set of U if $t_A$(u)= $\alpha$ and $f_A(u) = 1 - \alpha$ for all u$\in$U where $\alpha \in (0, 1)$ For $\alpha, \beta \in [0, 1]$ we now define ($\alpha$,$\beta$) – cut and $\alpha$ – cut of a Vague set.*

**Definition 2.3.** *Let 'A' be a Vague set of a universe X with the true-membership function $t_A$ and the false membership function $f_A$. The $(\alpha,\beta)$ – cut of a Vague set A is a crisp subset $A(\alpha,\beta)$ of the set 'X' given by $A(\alpha,\beta) = \{x \in X \mid V_A(x) \leq [\alpha,\beta]\}$*

**Definition 2.4.** *The $\alpha$ – cut of the Vague set 'A' is a crisp subset $A_\alpha$ of the set X given by $A_\alpha = A(\alpha,\alpha)$ we can define the $\alpha$ – cut as $A_\alpha = \{x \in X \mid t_A(x) \leq \alpha.\}$*

Let $I[0,1]$ denote the family of all closed sub intervals of [0,1]. If I= [a,b] and J=[c,d] be two elements of I[0,1] , We call I≥J if a≥c and b≥d. Similarly we understand the relations I≤J and I=J. Clearly the relation I≥J does not necessarily imply that I⊇J and conversely we define the "imax" to mean the maximum of two intervals as

- imax$(I,J) = [max(a,c), max(b,d)]$,
- imin$(I,J) = [min(a,c), min(b,d)]$.

## 3. Anti-Vague ideals

In what follows let $X$ be a subtraction algebra unless otherwise specified.

**Definition 3.1.** *A Vague set $A$ of $X$ is called a anti-Vague ideal of $X$ if the following conditions are true*

(C1) $V_A(x-y) \leq V_A(x)$ *for all* $x, y \in X$.

(C2) $\exists (x \vee y) \in X \Rightarrow V_A(x \vee y) \leq max\{V_A(x), V_A(y)\}$, *for all* $x, y \in X$.

*That is*

(1) $t_A(x\text{-}y) \leq t_A(x)$, $1\text{-}f_A(x\text{-}y) \leq 1\text{-}f_A(x)$

(2) $t_A(x \vee y) \leq max\{t_A(x), t_A(y)\}$ $1\text{-} f_A(x \vee y) \leq max\{1\text{-}f_A(x), 1\text{-}f_A(y)\}$, *whenever there exists x ∨ y.*

**Example 1.** *Consider a subtraction algebra X = {0, x, y} with the following table*

| - | 0 | x | y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| x | x | 0 | x |
| y | y | y | 0 |

The Vague set A={(0,[0.5,0.5]),(x,[0.5,0.5]),(y,[0.5,0.5])} is a Anti Vague ideal of X.

**Example 2.** *Consider a subtraction algebra X = {0,a,b} with the following table*

| - | 0 | a | b |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| a | a | 0 | a |
| b | b | b | 0 |

Let A be a Vague set in X define as follows. Then the Vague set A={(0,[0.6,0.2]),(a,[0.7,0.2]),(b,[0.6,0.2])} is a Anti Vague ideal of X.

**Example 3.** *Consider a subtraction algebra X= {0,a,b,c,d} with the following table*

| - | 0 | a | b | c | d |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | a | 0 | a |
| b | b | b | 0 | 0 | b |
| c | c | b | a | 0 | c |
| d | d | d | d | d | 0 |

Let A be a Vague set in X define as follows. Then the Vague set A={(0,[0.7,0.2]),(a,[0.7,0.2]),(b,[0.8,0.2]),(c,[0.8,0.2]),(d,[0.7,0.2]} is a Anti Vague ideal of X.

**Proposition 3.1.** *If a Vague set A of X satisfies*

$$\forall x, a, b \in X(V_A(x - ((x - a) - b)) \leq max\{V_A(a), V_b(b)\} .$$

*Then 'A' is anti-Vague ideal of X.*

*Proof.* Let 'A' be a Vague set of $X$ that satisfies the above condition, then

$$\begin{aligned} t_A(x-y) &= t_A((x-y) - (((x-y)-x)-x) \\ &\leq max\{t_A(x), t_A(x)\} = t_A(x) \\ 1 - f_A(x-y) &= 1 - f_A((x-y) - (((x-y)-x)-x)) \\ &\leq max\{1 - f_A(x), 1 - f_A(x)\} == 1 - f_A(x) . \end{aligned}$$

Now suppose $x \vee y$ exists for $x, y \in X$. Putting $w = x \vee y$, we have $x \vee y = w - ((w - x) - y)$ by Proposition 1.1. It follows from example 1 that

$$\begin{aligned} t_A(x \vee y) &= t_A(w - ((w-x)-y)) \leq max\{t_A(x), t_A(y)\} \ and \\ 1 - f_A(x \vee y) &= 1 - f_A(w - ((w-x)-y)) \\ &\leq max\{1 - f_A(x), 1 - f_A(y)\} . \end{aligned}$$

Hence 'A' is anti-Vague ideal of $X$. □

**Proposition 3.2.** *Zero Vague set, unit Vague set and $\alpha$– Vague set of X are trivial anti– Vague ideals of X.*

**Proposition 3.3.** *For every anti–Vague ideal 'A' of $X$, we have the following inequality:* $\forall x \in X, V_A(0) \le V_A(x)$ .

*Proof.* If we take $y = x$ in (C1), then $t_A(0) = t_A(x - x) \le t_A(x)$ and

$$1 - f_A(0) = 1 - f_A(x - x) \le 1 - f_A(x).$$

Hence $V_A(0) \le V_A(x)$ , $\forall x \in X$. □

**Proposition 3.4.** *Let 'A' be a Vague set of $X$ such that*

(K1) $V_A(0) \le V_A(x) \forall x \in X$.

(K2) $V_A(x - z) \le max\{V_A((x - y) - z), V_A(y)\}\ \ \forall x, y, z \in X$.

*Then we have the following implication:* $x \le a \Rightarrow V_A(x) \le V_A(a) \forall a, x \in X$ .

*Proof.* Let $a, x \in X$ be such that $x \le a$ then:

$$\begin{aligned} t_A(x) &= t_A(x - 0) \le max\{t_A(x - a) - 0), t_A(a)\} \\ &= max\{t_A(0), t_A(a)\} = t_A(a) \\ 1 - f_A(x) &= 1 - f_A(x - 0) \\ &\le max\{1 - f_A((x - a) - 0), 1 - f_A(a)\} \\ &= max\{1 - f_A(0), 1 - f_A(a)\} \\ &= 1 - f_A(a) . \end{aligned}$$

Hence $V_A(x) \le V_A(a)$. □

**Proposition 3.5.** *A necessary and sufficient condition for a Vague set $A = (x, t_A, f_A)$ of X to be anti-Vague ideal of $X$ is that $t_A$ and $1 - f_A$ are anti-fuzzy ideals of $X$.*

**Theorem 3.1.** *Let 'A' be a anti-Vague ideal of $X$. Then for $\alpha \in [0, 1]$, the $\alpha$-cut $A_\alpha$ is a crisp ideal of $X$.*

*Proof.* Let $x \in X$ and $a \in A_\alpha$. Then $t_A(a) \le \alpha$, and so $t_A(a - x) \le t_A(a) \le \alpha$. Thus $(a - x) \in A_\alpha$. Let $a, b \in A_\alpha$ and assume that $\exists (a \vee b)$ . Then $t_A(a) \le \alpha$ and $t_A(b) \le \alpha$ which implies from (C2) that $t_A(a \vee b) \le max\{t_A(a), t_b(b)\} \le \alpha$ so that $(a \vee b) \in A_\alpha$ . Therefore $A_\alpha$ is a crisp ideal of X. □

**Theorem 3.2.** *Let 'A' be a anti-Vague ideal of $X$. Then for $(\alpha, \beta) \in [0, 1]$, the Vague-cut $A(\alpha, \beta)$ is a crisp ideal of $X$.*

*Proof.* Let $x \in X$ and $a \in A(\alpha, \beta)$. Then $t_A(a) \leq \alpha$ and $(1 - f_A(a)) \leq \beta$ . Thus $t_A(a - x) \leq t_A(a) \leq \alpha$ and $1 - f_A(a - x) \leq 1 - f_A(a) \leq \beta$. Therefore $(a - x) \in A(\alpha, \beta)$. Now let $a, b \in A(\alpha, \beta)$ and assume that there exists $a \vee b$. Then $t_A(a \vee b) \leq max\{t_A(a), t_A(b)\} \leq \alpha$ , $1 - f_A(a \vee b) \leq max\{1 - f_A(a), 1 - f_A(b)\} \leq \beta$ . Which shows that $(a \vee b) \in A(\alpha, \beta)$. ☐

**Theorem 3.3.** *Any ideal $J$ of $X$ is a Vague-cut ideal of some anti-vague ideal of $X$.*

*Proof.* Consider the vague set $A$ of $X$ is given by $V_A$ (x) = [t,t] if $x \in J[0, 0]$, if $x \notin J$, where $t \in (0, 1)$. It can be proved that $V_A(x - y) \leq V_A(x)$ for all $x, y \in X$ and $V_A(x \vee y) \leq max\{V_A(x), V_A(y)\}$, whenever there exist $x \vee y$ for all $x, y \in X$. Thus 'A' is anti-Vague ideal of $X$ Clearly $J = A_{[t,t]}$. ☐

**Theorem 3.4.** *Let $A$ be a anti-vague ideal of $X$. Then the set $K = \{x \in X | V_A(x) = V_A(0)\}$ is a crisp ideal of $X$.*

*Proof.* Let a $\in$ K and x$\in$X .Then $V_A$(a)= $V_A$(0),and so $V_A$(a-x) $\leq$ $V_A$(a)= $V_A$(0) by (C1). It follows from Proposition 3.2 that $V_A$(a-x)= $V_A$(0). So that (a-x)$\in$ K.Let a,b$\in$ K and assume that there exists a$\vee$b by means of (C2), we know that $V_A$(a$\vee$b) $\leq$ max{$V_A$(a), $V_A$(b)} = $V_A$(0) Thus $V_A$(a$\vee$b) = $V_A$(0) by Proposition 3.2 and so a $\vee$ b $\in$ K. 'K' is a crisp ideal of X. ☐

## REFERENCES


[1] V. B. V. N PRASAD, T. RAMA RAO, T. S. RAO, T. RAMA RAO, K. PRASAD: *Some Basic Principles on Posets, Hasse Diagrams and Lattices*, Test Engineering and Management., **83** (2020), 10771 – 10775.

[2] K. PRASAD, V. B. V. N. PRASAD, T. S. RAO, M. GNANAKIRAN, M. RAMESH: *Anti Fuzzy Gamma Near Algebras Over Anti Fuzzy Fields*, Journal of Critical Reviews., **7** (2020), 313 – 315.

[3] K. PRASAD, V. B. V. N. PRASAD, K. PUSHPALATHA, M. GNANAKIRA: $\alpha_1$ , $\alpha_2$ *Near Subtraction Semigroups*, Journal of Critical Reviews., **7** (2020), 727 – 730.

[4] K. PRASAD, V. B. V. N PRASAD, T.S. RAO, G. BALAJI PRAKASH, RAMA DEVI BURRI: *A Note On Stone Spaces Of Advanced Distributive Lattices*, International Journal of Advanced Science and Technology., **29** (2020), 8494 – 8500.

[5] K. PRASAD, S. VENU MADHAVA SARMA, D. V. RAMALINGA REDDY, Y. PRABHAKARA RAO, P.RANGASWAMY: *A Note on Anti-Fuzzy Ideals in Subtraction Semi Group* , International Journal of Advanced Science and Technology., **29** (2020), 8557 – 8562.

[6] K. PRASAD, S. VENU MADHAVA SARMA, P. RANGASWAMY: *On Subtraction Algebras* , Journal of Xidian University, **14** (2020), 2070 – 2074.

[7] K. PUSHPALATHA, D. RAMESH, S. VENU MADHAVA SARMA, K. PRASAD : *Anti – Fuzzy Ideals (AFI) in Boolean Near Rings (BNR)*, Test Engineering and Management., **83** (2020), 8413 – 8416.

DEPARTMENT OF MATHEMATICS, KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR(DT), A.P., INDIA
*Email address*: prasad.qiscet@gmail.com

DEPARTMENT OF MATHEMATICS, PRAKASAM ENGINEERING COLLEGE
KANDUKURU, PRAKASAM(DT), A.P., INDIA
*Email address*: dvrlreddy@gmail.com

DEPARTMENT OF SCIENCE AND HUMANITIES
VASIREDDY VENKATADRI INSTITUTE OF TECHNOLOGY
NAMBUR, GUNTUR(DT), A.P, INDIA
*Email address*: prabhakargnt@gmail.com

DEPARTMENT OF MATHEMATICS, KONERU LAKSHMAIAH EDUCATION FOUNDATION
VADDESWARAM, GUNTUR(DT), A.P., INDIA
*Email address*: svms_css@kluniversity.in

DEPARTMENT OF MATHEMATICS, RGUKT-AP ONGOLE CAMPUS
ONGOLE, PRAKASAM(DT), A.P., INDIA

DEPARTMENT OF MATHEMATICS
GUDLAVALLERU ENGINEERING COLLEGE
GUDLAVALLERU - 521356 A.P., INDIA