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HYPERGRAPH NEAR-RING GROUPS WITH A.C.C. ON ANNIHILATORS

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ABSTRACT. Here we investigate some properties of hypergraphic aspects of near-rings and near-ring groups with a.c.c. on annihilators. We observe that the hyper continuous character of the binary operations available in a near-ring group leads towards the componentwise hyper continuous character in most natural way. In contrast, it needs some tough work for sufficient conditions so as to impose on such hyper continuity in the whole system. The so-called pseudo character on nilpotency and strongly semi-primeness, leads satisfactorily towards our goal, giving elegant results on N-groups rather than the near-ring structure. Hypergraphic structure on such N-groups gives some noticeable results. It is interesting to note the relevance and elegancy of the results obtained, as the same may be determined with accomodating justification on such structures. Moreover, all these lead us to some radical character of N-groups. Together with all these, we delve into some structure theory, related to these radical structures in connection with hyper compactness etc. of the groups in discussion.

1. INTRODUCTION

Some recent development in hypergraph theory linking commutative rings is on annihilating ideals and Betti numbers. Some of these spring up as a natural generalisation of graphs containing lines and cycles that are worth mentioning [4]. Selvakumar and Ramanathan [18, 19] recently extended the notion

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of annihilating ideal graph of a commutative ring introduced by Behboodi and Rakeei [20, 21]. Huy Tai Ha and Adam Van Tuyl [6] used monomial ideals, edge ideal hypergraphs and their graded Betti numbers to understand how the combinatorial structure appears within the resolutions of its edge ideals i.e. the hyperedges each of which element is an ideal.

On the otherhand, the results due to A. W. Goldie on the structure of semiprime rings [2, 3] seem to be still relevant due to its elegancy. Here we try to investigate some properties of hypergraphic structure on some generalizations of what have been achieved on near-rings with a.c.c. on annihilators [1] by Chowdhury, Saikia and Masum [13, 15]. The so-called pseudo character on nilpotency and strongly semi-primeness, leads us satisfactorily towards our goal giving elegant results on N-groups rather than the near-ring structure. Hypergraphic structure on N-groups gives some interesting fruitful results so far space biased vigour is concerned.

An N-group E with Goldie character [12] is well behaved so far the pseudo quality on nilpotency as well as strongly semi-primesness are involved for the proper development of such a make-up.

Extending the idea of boundedness of Beidleman and Cox [10], so called hyper E-boundedness etc. together with the notion of hyper nilpotent sets are playing some important role on an N-group E with zero annihilators $[Ann(e) = 0]$, which occur as a necessity of the N-group E in the above context. It is interesting to note the relevance and elegancy of the results obtained, as the same may be determined with accomodating justification on such hypergraph N-groups that their discrete character is in association with the hyper E-boundedness with zero radical or hyperopenness of the same.

For the sake of completeness of the idea we are dealing with, once more we like to mention that any near-ring (near-ring group) with ascending chain condition(a.c.c.) on its left near-ring subgroups obviously satisfies what Oswald [1] has chosen(viz., no infinite direct sum of left ideals and satisfies the acc on left annihilators). But rings like $Z[X_i | i = 1, 2, ..., X_i X_j = X_j X_i]$ satisfy the a.c.c. on left annihilators having no infinite direct sum of its left ideals, though it may have a strictly ascending infinite chain of ideals viz.,

$(X_1) \subset (X_1, X_2) \subset ...$

Thus near-rings described in [1], need not satisfy the a.c.c. on its sub-algebraic structures. This has lead Chowdhury and Saikia towards the idea of the so-called strictly Artinian radical [13].

With N, a (right) hypergraph near-ring, a hypergraph N-group is a pair (E, μ), where E is a hypergraph N- group under addition and μ is a hypercontinuous map from $N \times E$ to E such that $\mu(a + b, e) = \mu(a, e) + \mu(b, e)$ and $\mu(ab, e) = \mu(a, \mu(b, e))$, for all $a, b \in N$ and $e \in E$ [5, 11, 17]. As seen [17], in case of the topological group $\mathbb{R}$ of real numbers under addition and $\mathbb{Z}$, the discrete group of integers under addition, $T = \mathbb{R}/\mathbb{Z}$ is the one-dimensional torus. On the other hand, $\mathbb{R}^n$ denotes the Euclidean N-group and T^n the n-dimensional torus.

As insisted by Kaplansky [7] regarding continuity of addition and multiplication on the product space, in case of a ring, it is noticeable that the coordinate wise hypercontinuity is all that is necessary.

To be more explicit, our proposed hypercontinuity regarding addition and multiplication is as follows: We stick to the definition of a hypergraph (right) near-ring N where coordinate wise hypercontinuities of the respective mappings are imposed. Keeping this in note, we define hyper N-group E as an N-group E in which a hypergraph is given with four hypercontinuous mappings $\mu_1, \mu_2, \mu_3, \mu_4$ such that for given $e \in E$ (i) $\mu_1(x) = x + e$, (ii) $\mu_2(x) = e + x$, for all $x \in E$, (iii) $\mu_3(n) = ne$, for all $n \in N$, and given $m \in N$, (iv) $\mu_4(e) = me$, for all $e \in E$.

Note. It is clear that if $1 \in N$, then for a given $e \in E$, as $-e = (-1)e$, the map $x \to e - x$ is hypercontinuous, for all $x \in E$. Hence if V is hyperopen in E and $e \in E$, then each of $V + e, e + V$ and $-V$ is hyperopen in E.

1.1. **Definitions and notations.**

1.1.1. *Essentiality in near-rings and near-ring groups.* If L and B are two N-subgroups of E such that $L \subseteq B$, then L is a strictly **essential N-subgroup of** B in E, denoted $L \subseteq_e B$, if for any non zero N-subgroup C of B has a non zero intersection with L.

A **strictly essential left N-subgroup** L **of** N is a strictly essential N-subgroup of ${}_N N$, (denoted $L \subseteq_e {}_N N$). Moreover for $L \subseteq B \subseteq E$, $L \subseteq_e E$ if and only if $L \subseteq_e B \subseteq_e E$.

An ideal I of E is an **essential ideal** of E when it has a non zero intersection with any non zero ideal of E. If a left ideal I of N is an essential ideal of $_NN$, then I is an essential left ideal of N.

It is clear that a strictly essential N-subgroup of E is always essential as an ideal of it. We note that in the symmetric group $N(= S_3)$,[[5](37),342]. $\{0,a\}$,$\{0,b\}$, $\{0,c\}$ and $\{0,x,y\}$ are proper non zero N-subgroups of $_NN$ where $\{0,x,y\}$ is an ideal of $_NN$ and is not strictly essential as an N-subgroup of it though it is an essential ideal. Thus an essential ideal of E need not be strictly essential as an N-subgroup of it. In an N-group E we define the set

$Z_l(E) = \{x \in E|Lx = 0, for\ some\ strictly\ essential\ N - subgroup\ L\ of_N N\}$.

An annihilator of $S(\subseteq E)$ in N is the set $\{n \in N|nS = 0\}$ denoted by Ann(S) or $Ann_N(S)$, if $S \subseteq N$, it is denoted by $l(S)$ or $l_N(S)$, the **left annihilator of** S **in** N. If $x \in E(orN)$ then $Ann(\{x\})$ and $l(\{x\})$ are denoted by $Ann(x)$ and $l(x)$ respectively.

Near-ring N is a duo near-ring if $a,b \in N$, $ab = bc = da$ for some c,d [22]. An element $q \in N$ is quasi-regular [8] if there is an element $n \in N$ such that $n(1-q) = 1$. The set Q will denote the set of all quasi-regular elements of N.

The intersection of all maximal N-subgroups of $_NN$ is **the radical subgroup** of N [8] and is denoted by A.

The radical $J(E)$ of E [9] is the intersection of all ideals of E that are maximal as N-subgroups of E. Similarly, we define the radical $J(N)$ of N.

A subset L of E is **pseudo(ps)-nilpotent** w.r.t a proper left N-subset B of N, if there exists a least positive integer(nilpotency)$\sharp n$ such that $B^nL = 0$. Also a subset C of E is ***strictly*** **pseudo(ps)-nilpotent** w.r.t a proper left N-subset B of N, if there exists a least positive integer(nilpotency) $n(\geq 2)$ such that $B^nC = 0$. Subset L of E is ***strictly*** **pseudo(ps)-nilpotent** if it is *strictly* ps-nilpotent w.r.t some proper left N-subset B of N.

An element $b \in E$ is **pseudo(ps)-nilpotent** if the singleton subset $\{b\}$ of E is a ps-nilpotent subset of E. An element $a \in N$ is **self nilpotent** if it is ps-nilpotent w.r.t the proper left N-subset Na of N. Similarly, an element $b \in E$ is **strictly pseudo(ps)-nilpotent** if the singleton subset $\{b\}$ of E is a strictly ps-nilpotent subset of E.

An element $x \in E$ is N_u**-nilpotent**(where N_u is the set of all non units of N), if there exists a least positive integer (nilpotency) t such that $N_u^t x = 0$ but $N_u^t \neq 0$.

An N-group E is **pseudo(ps)-strongly semiprime** if E has no non-zero ps-nilpotent subset. Again an N group E is **pseudo(ps)-strictly semiprime** if E has no non-zero strictly ps-nilpotent subset.

Note(A):

(i) A self nilpotent element is clearly nilpotent–the source of the notion of pseudo nilpotency.

(ii) A ps-strictly semiprime N-group is ps-strongly semiprime.

The Table 1 below leads us to the necessary justifications of what have been discussed.

1.1.2. *Discussion.* A good number of examples (as appeared in [16])justifies what we are going to present from a comprehensive point of view .

Example 1. *The non-zero symmetric near-ring $N(= D_s)$ [[5](24, p345)] without unity has non-zero proper left N-subsets viz.* $\{0, b\}$, $\{0, b, a + b\}$, $\{0, b, 2a + b\}$, $\{0, b, 2a, 2a + b\}$, $\{0, 2a, b, 2a + b, 3a + b\}$ *etc.*

Example 2. *In the near-ring $N(= \mathbb{Z}_8)$ [[5](22), p 343] without unity, N has proper left N-subsets viz.* $\{0, 1\}, \{0, 2\}, \{0, 4\}, \{0, 4, 5\}, \{0, 4, 6\}, \{0, 2, 4, 6\}, \{0, 4, 6, 7\}, \{0, 2, 3, 4, 6\}$ *etc.*

Example 3. *In the near-ring $N(= \mathbb{Z}_8)$ without unity [[5](84), p343], the only left N-subset is $\{0, 2\}$.*

Example 4. *The near-ring $N(= \mathbb{Z}_8)$ with unity [[5](46), p343] has $\{0, 4\}$ and $\{0, 2, 4, 6\}$ as proper left N-subsets.*

Example 5. *In the near-ring $N(= \mathbb{Z}_8)$ [[5](22), p343] without unity, the only proper left N-subset is $\{0, 4, 5\}$.*

Example 6. *In the Klein's 4-group, $N = \{0, a, b, c\}$ [[5](4), p340] is a near-ring with unity having a left N-subset $\{0, a\}$.*

Example 7. *We have $N(= \mathbb{Z}_8)$ [[5](46), p343] as a near-ring with unity having a left N-subset $N2$.*

Example 8. *In a near-ring $N(= \mathbb{Z}_8)$ [[5](22), p343] without unity, $N3$ is a left N-subset.*

Example	Observation	Comment
1.2.1	For any $L \subseteq N$, $X^n L \neq 0$, for any $n \in \mathbb{Z}^+$	N has no strictly ps-nilpotent subset of N
1.2.2	$\{0,4,5\}\{0,5\} = \{0,4\} \neq 0$ and $\{0,4,5\}^2\{0,5\} = 0$	$\{0,5\}$ is a strictly ps-nilpotent subset of N
1.2.3	For any subset $L(\neq \{0,2\})$, we have $\{0,2\}L \neq 0$ but $\{0,2\}^2 L = 0$	Any subset L $(\neq \{0,2\})$of N is strictly ps-nilpotent
1.2.4	$\{0,2,4,6\}\{0,2\} \neq 0$ and $\{0,2,4,6\}^2\{0,2\} = 0$	$\{0,2\}$ is a strictly ps-nilpotent subset of N
1.2.5	$\{0,4,5\}\{3\} \neq 0$ and $\{0,4,5\}^n\{3\} = 0$, $(n > 1)$	3 is a strictly ps-nilpotent element of N
1.2.6	$\{0,a\}b \neq 0$ and $\{0,a\}^2 b = 0$	b is a strictly ps-nilpotent element of N
1.2.7	$(N2)2 = \{0,4\}$ but $(N2)^2 2 = 0$	2 is a self nilpotent element of N
1.2.8	$N3 = \{0,2,4,6\}, (N3)3 = \{0,4\}$ but $(N3)^2 3 = 0$	3 is a self nilpotent element of N
1.2.9	$\{0,a\}^n L \neq 0$, $L(\neq 0, \{0,b\})$, $\{0,b\}$ and $\{0,b\}^n B \neq 0$, $B(\neq 0, \{0,a\})$ where $n \in \mathbb{Z}^+$	${}_N N$ is ps-strictly semi-prime
1.2.10	$L(\neq 0)\{0,a\}^n L \neq 0, \{0,a,c\}^n L \neq 0$ for any $n \in \mathbb{Z}$ and $\{0,b\}L = 0$	do
1.2.11	$N_u 2 = \{0,4\}, N_u^2 2 = 0$ but $N_u^2 \neq 0$	2 is a N_u-nilpotent of N

TABLE 1. Observation and Comment

Example 9. *The near-ring N [[5](7), p340] of Klein 4-group with unity has the non-zero proper left N-subsets $\{0,a\}, \{0,b\}$ and $\{0,a,b\}$ such that $\{0,a\}^n L \neq 0$, for any subset $L(\neq 0)$ except $\{0,b\}$ and $\{0,b\}^n B \neq 0$, for any subset $B(\neq 0)$ and any $n \in \mathbb{Z}^+$.*

Example 10. *In the near-ring $N(= \mathbb{Z}_8)$ [[5](46), p343] with unity, the units are $1,3,5,7$ and N_u is the set of all non-units of N.*

We say, a subset B of E is **strictly pseudo(ps)-nil** if each element of B is strictly ps-nilpotent.

N	E		
	0	1	2
0	0	0	0
i	0	1	2
a	0	0	1
b	0	0	2
c	0	1	0
d	0	1	1
e	0	2	0
f	0	2	1
g	0	2	2

TABLE 2. Product in E over N

Now it is noticeable that an element $\sum_{i=1}^{n} x_i \in \bigoplus_{i=1}^{n} E_i$ (where $\bigoplus_{i=1}^{n} E_i$ is the direct sum of N-groups E_i [5] is N_u-nilpotent if each $x_i \in E_i$ is N_u-nilpotent, as $\forall i, N_u^{t_i} x_i = 0$ but $N_u^{t_i} \neq 0$, for some t_i(least)$\in \mathbb{Z}^+$ and hence $\bigoplus_{i=1}^{n} N_u^m x_i = 0$ but $N_u^m \neq 0$ where m=max $(t_1, t_2, ..., t_n)$.

In obvious sense, it is assumed that 0 (zero of E) is a strictly ps-nilpotent element of E. Also, if N has no proper left N-subset, then no element of E is strictly ps-nilpotent.

Throughout our discussion, Q' will denote the subset of E consisting of all N_u-nilpotent elements of E. A non-empty proper N-subgroup of E is $\mathbf{N_u}$**-nil** if it is contained in Q'.

Note B:

(i) It is clear that any proper left N-subset of N with nilpotency greater than 2 is strictly ps-nilpotent.

(ii) If $B(\subseteq_N N)$ is strictly ps-nilpotent w.r.t. a proper left N-subset L of N, then LB is nilpotent.

(iii) It is obvious that a N_u-nilpotent element of E with nilpotency greater than one is strictly ps-nilpotent.

The following example clarifies what we have proposed to carry out regarding radical of E.

In Table 2 above and $E = \{0, 1, 2\}$ it is seen that $N = \{0, i, a, b, c, d, e, f, g\}$ is a right near-ring with unity i w.r.t. the operations $(j + k)(x) = j(x) + k(x)$; $(j.k)(x) = j(k(x))$, for all $j, k \in N$ and $x \in E$.

Also E is a near-ring group over N w.r.t. the operation $N \times E$ described in the Table 2 above.

Here we see that N has proper left N-subgroups $A_1 = \{0, a, b\}, A_2 = \{0, c, e\}, A_3 = \{0, d, g\}$ where A_1 and A_2 are left ideals. But $J(N) = A_1 \cap A_2 = 0$ and $J(E) = 0$. Hence, $J(N)E = J(E)$. We shall consider near-ring groups in the above sense termed as **fully radical character**.

We are now in position to discuss what we have attempted for so far our two sided as well as unbalanced (non-symmetric) so called continuity problems are concerned.

1.1.3. *Hypergraphs, hyper continuity, hyper open, hyperclosure, hyper interior.* At the very outset we begin with the notion of a hypergraph. For a non-empty set X, a **hypergraph** is a pair $(X, \mathcal{E})$ where $\mathcal{E} \subseteq P(X) \backslash \{\phi\}$ and $P(X)$ is the power set of X. Each element of $\mathcal{E}$ is termed as a **hyper edge** and each such hyper edge is a **hyper open** set in X. Unless otherwise specified throughout this paper N will mean a zero symmetric (right) near-ring with unity 1; E (or $_N E$) will denote the left N-group, a left N-subgroup L of N will mean an N-subgroup of $_N N$ and a left ideal of N will mean an ideal of $_N N$. For near-ring and near-ring group preliminaries readers may go through Pilz [5].

Now, we begin with two arbitrary non-empty sets X and X' where $\mathcal{E} \subseteq P(X)$ and $\mathcal{E}' \subseteq P(X')$. A map $f : X \to X'$ is **hyper continuous** at **a point** say, a $(\in X)$ provided for each hyper edge $S' \in \mathcal{E}'$ containing $f(a)$ there exists $S \in \mathcal{E}$ containing $a (\in X)$ such that $f(S) \subseteq S'$.

If f is hyper continuous at any point of X, then $f : X \to X'$ is **hyper continuous**.

In the symmetric group $(S_3, +)$ [[5], p.341], $(S_3, \mathcal{E})$ with $\mathcal{E} = \{\{0, x, y\}, \{a, b, c\}\}$, is a hypergraph and $\{0, x, y\}$ and $\{a, b, c\}$ are hyper edges and the sets $\{0, x, y\}$ and $\{a, b, c\}$ are hyper open.

A subset, A of X is **hyper closed** if A^c(complement of A) is hyper open. In the above example hyper closed sets are $\{0, x, y\}$ and $\{a, b, c\}$.

The intersection of two hyper closed sets need not be again hyper closed as it is observed here.

The intersection of all hyper closed subsets of X containing a subset B of X is the **hyper closure** of B and is denoted by $\overline{B}$.

And hence in contrast to what we have in case of a topological space the hyper closure of a set need not exist.

To study some characteristics of minimal hyper closed sets, we consider the hypergraph $(D_8, \mathcal{E})$, where D_8 [[5](4), p. 345] is the near-ring with the edge set $\mathcal{E} = \{\{0, 2a\}, \{a, 3a\}, \{b, 2a+b\}, \{3a+b, a+b\}\}$, for $B = \{0, 2a+b\}$, the hyper closed subsets containing B are $\{0, 2a, a, 3a, b, 2a+b\}$ and $\{0, 2a, b, 2a+b, 3a+b, a+b\}\}$ which are minimal as hyper closed subsets of D_8 containing B. Now, $\{0, 2a, b, 2a+b\}$ is the intersection of all such minimal hyper closed subsets of D_8 containing B which we term as **minimal hyper closure** of B and is denoted by $\overline{B}^m$. Thus, the intersection of all minimal hyper closed subsets of E containing a subset B of E is the minimal hyper closure of B and is denoted by $\overline{B}^m$.

In general, $\overline{B}^m = \cap\{F|F \textit{ is a minimal hyper closed set containing } \mathrm{B}(\subseteq E)\}$. It is to be noted that in case $\mathcal{E}$ is a topology, then we get only one minimal hyper closed set containing $B(\subseteq E)$ which is $\overline{B}$[a **hyper-graphical** form what is available in topology.]

It is noted that in case $\mathcal{E}$ is a topology then we get only one minimal $\mathcal{E}$-closed set containing $B(\subseteq E)$ which is $\bar{B}$[a **hyper-graphical** difference from what is available in topology].

In the group $(Z_8, +)$[[5], p342], consider the hyper graph $(Z_8, \mathcal{E})$ with $\mathcal{E} = \{\{0, 4\}, \{1, 5\}, \{2, 6\}, \{3, 7\}\}$ and here the hyper closure of $\{0, 2\}$ is $\{0, 2, 4, 6\}$. It is observed above that the hyper closure of $\{0, 2, 4, 6\}$ is $\{0, 2, 4, 6\}$.

And this leads us to define what we now tempt for, such as, a subset B of X is a **pseudo(ps)-closed** when $\overline{B} = B$ and therefore $\{0, 1, 4, 5\}$ is ps-closed.

As a note we say that hyper closedness of a set implies the ps-closedness of the same but the converse need not be true.

As the hyper closure we say, the **hyper interior** of a subset A of X (denoted by A^o) is the union of all hyper open subsets of X contained in A.

In the dihedral group $(D_8, +)$ [[5], p.344] consider the hypergraph $(D_8, \mathcal{E})$ with $\mathcal{E} = \{\{0, 2a\}, \{a, 3a\}, \{b, 2a+b\}, \{3a+b, a+b\}\}$, the hyper interior of a set $\{0, a, 2a, 3a, b\}$ is $\{0, a, 2a, 3a\}$ which is not hyper open. But in case of a topology T, the T-interior of a set is T-open.

In the above example if we take $A = \{0, a, 2a, 3a\}$, then $A^o = A$ and the like cases lead us to define that a subset A of X is **pseudo(ps)-open** when $A^o = A$.

1.1.4. *Observations.*

Example 11. *Consider two hypergraphs* $(X, \mathcal{E})$ *and* $(Y, \mathcal{E}')$ *where* $X = \{a, b, c\}$ *and* $Y = \{\alpha, \beta, \gamma, \delta\}$ *are two sets and* $\mathcal{E} = \{\{a, b\}, \{c\}\}$ *(*$\subseteq P(X)$*) and* $\mathcal{E}' = \{\{\alpha\}, \{\beta, \gamma, \delta\}\}$*(*$\subseteq P(Y)$*). Also, consider a mapping* $f : X \to Y$ *defined by* $f(a) = \alpha, f(b) = \alpha, f(c) = \beta$*. Now, we have* $\{\alpha\}$*(*$\in \mathcal{E}'$*) containing* $f(a)$ *and* $\{a, b\}$*(*$\in \mathcal{E}$*) containing* $a(\in X)$ *such that* $f(\{a, b\}) = \{\alpha\}$*.*

Here we note that for any $W(\in \mathcal{E}')$ *containing* $f(a)$ *there exists* $V(\in \mathcal{E})$ *containing* $a(\in X)$ *such that* $f(V) \subseteq W$*. [hyper continuous]*

Example 12. *For a set* $X = \{a, b, c\}$ *having* $\mathcal{E} = \{\{a, b\}, \{b, c\}\}$*, consider a mapping* $f : X \times X \to X$ *defined by* $f(a, a) = b, f(a, b) = c, f(a, c) = a, f(b, a) = c, f(b, b) = b, f(b, c) = b, f(c, a) = a, f(c, b) = b, f(c, c) = b$*.*

We notice here $\{b, c\}(\in \mathcal{E})$ *contains* $f(a, b)$ *and* $\{a, b\} \times \{a, b\}(\in \mathcal{E} \times \mathcal{E})$ *contains* (a, b) *such that* $f(\{a, b\} \times \{a, b\}) = \{b, c\}$*. Thus, for any* $W(\in \mathcal{E})$ *containing* $f(a, b)$ *there exists an* $U \times V(\in \mathcal{E} \times \mathcal{E})$ *containing* (a, b) *such that* $f(U \times V) \subseteq W$*.*

If $(X, \mathcal{E})$ is a hypergraph and $(X \times X, \mathcal{E} \times \mathcal{E})$ is corresponding product hyper graph then a map $f : X \times X \to X$ is hyper continuous when considered the class of hyperedges as $\mathcal{E} \times \mathcal{E}$ and $\mathcal{E}$ in the sense mentioned already.

If X is equipped with an (or, more than one)(a binary) operation like a group, a ring or a near-ring, near-ring group etc. and the corresponding operation(s) is (are) hyper continuous as is already defined, then the corresponding algebraic structures would be hypergraph algebraic structures like **hypergraph group, hypergraph ring, hypergraph near-ring, hypergraph near-ring group** etc.

Note C:

(i) For $B \subseteq X$, $x \in \overline{B}$ and if and only if $V \cap B \neq \phi$, for all ps-open subset V of X containing $x(\in X)$.

(ii) If V is a hyper open subset of a (additive) hypergraph group X, then $B + V$ is ps-open for any subset B of X.

(iii) If W is a ps-open subset of X, then $D + W$ is ps-open, for any subset D of X.

(iv) If V is a hyper open subset of hypergraph near-ring group E, $e \in E$ then $e + V$, $V + e$ are ps-open.

Similarly, if U is a hyper open subset of hypergraph near-ring N, $n \in N$, then $n + U$, $U + n$ are ps-open.

(v) It can be seen that if B is an N-subgroup$\sharp$(ideal) of E, then the hyper closure $\overline{B}$ of B is also an N-subgroup(ideal) of E.

Similarly, if B is an N-subgroup$\sharp$(ideal) of ${}_NN$, then the hyper closure $\overline{B}$ of B is also an N-subgroup$\sharp$(ideal) of ${}_NN$.

A subset B of X is hyper dense in a subset, say A of X when $\overline{B} = A$.

If $\overline{B} = X$, then B is **hyperdense** in X.

In the hypergraph $(\mathbb{Z}_8, \mathcal{E})$ where $(\mathbb{Z}_8, +)$ [[5], p.342] is the group and $\mathcal{E} = \{\{0,4\}, \{1,5\}, \{2,6\}, \{3,7\}\}$, the set $\{0,2\}$ is hyper dense in $\{0,2,4,6\}$.

X is **ps-discrete** if every non empty subset of X is **ps-open**.

Then, X is ps-discrete when every nonempty subset of X is hyper open. If we consider the hypergraph $(S_3, \mathcal{E})$ in the symmetric group $(S_3, +)$[[5], p.341] with $\mathcal{E} = \{\{0\}, \{a\}, \{b\}, \{c\}, \{x\}, \{y\}\}$ then S_3 is ps-discrete.

If for any class $C = \{A_i\}_{i \in \Delta}$ of ps-open subsets of X with $X = \cup_{i\in\Delta} A_i$ there exist $A_{\lambda_1}, A_{\lambda_2}, A_{\lambda_3}, ..., A_{\lambda_n} \in C$ such that $X = A_{\lambda_1} \cup A_{\lambda_2} \cup A_{\lambda_3} \cup ... \cup A_{\lambda_n}$, then X is **ps-compact**.

If the members of C are hyper open, then X is **hyper compact**.

It is noticeable that the two notions of hyper open and ps-open are compact invariant.

We are now at a position to describe our results in terms of so-called topological sense that X is **ps-disconnected** if it has a non-empty proper subset, which is both ps-open and ps-closed.

In the near-ring N of Klein's four group [[5](12), p. 340] we consider the hypergraph $(N, \mathcal{E})$ with $\mathcal{E} = \{\{0,a\}, \{b,c\}\}$. We note that for any subset $L(\neq 0)$ of N, $L\{b,c\} = \{0,a\}$ a hyper open subset of N containing 0 (zero of N). In view of this, a subset B of N is **hyper bounded**, if for any hyper open subset U of E containing 0 there is a hyper open subset V of E such that $BV \subseteq U$. If $B = N$ then N is itself hyper bounded. If $\mathcal{E}$ is a topology then the idea of topological boundedness coincides with that of hyper boundedness. This may be termed as a generalization of what Beidleman and Cox have dealt with in the discussion of boundedness in case of a topological near-ring [10]. It is important to note for our purpose that for obvious reason, we follow the subsequent extension of what has been described above as follows:

A subset B of N is E-**hyper bounded** w.r.t $E = \bigoplus_{i=1}^{n} E_i$, each E_i being an N-group, if for any hyper open subset $\bigoplus_{i=1}^{n} V_i$ of $\bigoplus_{i=1}^{n} E_i$ containing 0(zero of

E), there exists a hyper open subset $\bigoplus_{i=1}^{n} U_i$ of $\bigoplus_{i=1}^{n} E_i$ such that $\bigoplus_{i=1}^{n} BU_i \subseteq \bigoplus_{i=1}^{n} V_i$.

It is clear that if a subset B of N is hyper bounded w.r.t. $E = \bigoplus_{i=1}^{n} E_i$, then B is hyper bounded for each i.

We observe that the hypergraph $(N, \mathcal{E})$ with near-ring $N(= \mathbb{Z}_8)$[[5](46), p.343] and $\mathcal{E} = \{\{0,4\},\{1,5\},\{2,6\},\{3,7\}\}$, where the left N-subsets of the near-ring N are $\{0,4\}$ and $\{0,2,4,6\}$.

Now for the subset $L(= \{0,2\})$ we have $\{0,2,4,6\}L = \{0,4\}$ and $\{0,2,4,6\}^2L = 0$, which belongs to every hyper open subset of N(containing 0).

The notion of topologically nilpotent subset of a near-ring N due to Beidleman and Cox is extended to that of hyper nilpotent subset of the N-group E as a direct sum with the help of E-hyper open set, the so called E-hyper nilpotent set as follows: A subset $\bigoplus_{i=1}^{n} D_i$ of E where $E = \bigoplus_{i=1}^{n} E_i$ is **hyper nilpotent** if for any hyper open subset $\bigoplus_{i=1}^{n} U_i$ of E containing 0, there exists a left N-subset $C_1 \cup C_2 \cup ... \cup C_n$ of N such that $\bigoplus_{i=1}^{n} C_i^t D_i \subseteq \bigoplus_{i=1}^{n} U_i$, for some $t \in \mathbb{Z}^+$.

2. Preliminaries

In the following lemmas, we assume that N is a duo near-ring with the acc on annihilators of subsets of E in N and thus we call the N-group E as a $_{duo}$ **acc** N**-group** E.

Lemma 2.1. *If E is a ps-strictly semi-prime $_{duo}$acc N-group, then E has no non-zero strictly ps-nil N-subset.*

Proof. Let $B(\neq 0)$ be an N-subset of E and $L(\neq 0)$ be a left N-subset of N with $LB \neq 0$. So, $aB \neq 0$ for some $a(\neq 0) \in L$ as $LB \neq 0$. We have $aNaB \neq 0$ and a non-zero $ab \in aB(b \in B)$ with Ann(ab) as large as possible. And therefore, $aNab \neq 0$(if not, the set $\{b\}$ is a strictly ps-nilpotent subset of E).

So, $axab \neq 0$ for some $x \in N$ and hence, $xab \neq 0$ if not, $axab = 0$ giving thereby $x \notin Ann(ab)$. But Ann(ab) is maximal and Ann(ab)$\subseteq$ Ann($axab$), we get Ann(ab)=Ann($axab$). So $(xa)^2b \neq 0$ or $(xax)ab \neq 0$ or $xax \notin$ Ann(ab)= Ann($axab$) or, $(xa)^3b \neq 0$ and so on.

Thus, $(xa)^tb \neq 0$ for any $t \in \mathbb{Z}^+$. So B is not strictly ps-nil. $\square$

We note that for $b(\neq 0) \in B$, we have $a \in L$ such that ab is a non-zero non strictly ps-nilpotent element of B.

Lemma 2.2. *The set $Z_l(E)$ is a strictly ps-nil N-subset of the $_{duo}$ acc N-group E.*

Proof. By Lemma 2.10 of [14], it is clear that $Z_l(E)$ as an N-subset of E. If $e \in Z_l(E)$ and $Ne = 0$, then $e = 0$, which is strictly ps-nilpotent.

And $Ne \neq 0$, gives $xe \neq 0$, for some $x(\neq 0) \in N$.

So, $B(= Nx)$ is a left N-subset of N such that $Be \neq 0$, as $1 \in N$. Moreover, N is a duo near-ring, we have

$$Ann(e) \subseteq Ann(b_1e) \subseteq Ann(b_1b_2e) \subseteq ... \tag{2.1}$$

(for any $b_i \in B$). If (2.1) is not strictly ascending, let

$$Ann(b_1b_2...b_te) = Ann(b_1b_2...b_{t+1}e)$$

(for some $t \in \mathbb{Z}^+$). Thus, we get

$$Ann(b_1b_2...b_te) = Ann(b_1b_2...b_{t+n}e)$$

(for all $n \in \mathbb{Z}^+$)

Clearly, $Le = 0$ as $e \in Z_l(E)$ for some $L \subseteq_e {}_NN$. And so L is contained in $Ann(e)$ and hence $Ann(e)$ is essential in $_NN$(as $L \subseteq_e {}_NN$) giving thereby $Ann(b_1b_2...b_{t+1}e) \subseteq_e {}_NN$. Now, $(Nb_1b_2...b_t) \cap Ann(b_1b_2...b_{t+1}e)e$ is non-zero.

If $nb_1b_2...b_te \neq 0$, for $n \in N$ then
$nb_1b_2...b_tb_1b_2...b_te = 0$, as $nb_1b_2...b_t \in Ann(b_1b_2...b_{t+1}e)(= Ann(b_1b_2, , , b_te))$, which gives $n \in Ann((b_1b_2...b_t)^2e)(= Ann(b_1b_2...b_te))$, and so $nb_1b_2...b_te = 0$, which contradicts the choice of $nb_1b_2...b_te$. Hence (1) is a strictly ascending chain, which violates the hypothesis. So we get $nb_1b_2...b_te = 0$, for all $b_i \in B$ and $n \in N$, which gives $B^te = 0$, for some $t(\geq 2) \in \mathbb{Z}^+$. Again if $B = Nx = N$, then $B^t = N^t = N$ gives $Ne = 0$ and is not true and therefore, B is a proper left N-subset of N. So, $e(\in Z_l(E))$ is a strictly ps-nilpotent element and thus $Z_l(E)$ is a strictly ps-nil N-subset of E. □

From above we get the following

Lemma 2.3. *If E is a ps-strictly semi-prime $_{duo}$acc N-group, then $Z_l(E) = 0$*

We have the following by lemma 2.11 of [14].

Lemma 2.4. *If E is a ps-strictly semi-prime $_{duo}$acc N-group such that N has no infinite direct sum left ideals and an essential left ideal of N is a strictly essential as N-subgroup of $_NN$ too, then the annihilators of subsets of E in N satisfy the dcc.*

Note D: We consider in case of N-group E, two hypergraphs viz. $(E, \mathcal{E})$ and $(N, \aleph)$

(1) Here the hypergraph $(E, \mathcal{E})$ is such that E_1 is an Artinian N-subgroup of E and $\mathcal{E} = \{A^c : A\ is\ an\ N - subgroup\ of\ E_1\}$ or,
(2) In this case E satisfies the acc on annihilators of subsets of E in N and $\mathcal{E} = \{Ann(S)^c : S \subseteq N\}$ or,
(3) Here E is as in case of Lemma 2.4 and $\mathcal{E} = \{Ann(S)^c : S \in \aleph\}$ where $\aleph = \{(S =)Ann(E_1) : E_1 \subseteq E\}$, then we have $\bar{B} = \bar{B}^m$ (here, each of the collections of the hyper closed sets containing B is either empty or only a singleton set from the collection $\mathcal{E}$.) Hence in the first case $\bar{B} = \bar{B}^m = E$ and in the other cases we have $\bar{B} = \bar{B}^m$=the only set E(in the collection of hyper closed sets).

Lemma 2.5. *Let I be a left N-subgroup of N and B be an N-subgroup of E with distributively generated annihilators of subsets of E in N. If i_1a_1 is a non strictly ps-nilpotent element of IB with $Ann(i_1a_1)$ maximal and $i_2a_2 \in (Ann(i_1a_1) \cap I)B$ with the same characteristic as i_1a_1, then $Ann(i_1a_1 + i_2a_2) = Ann(i_1a_1) \cap Ann(i_2a_2)$.*

Proof. If $x \in Ann(i_1a_1) \cap Ann(i_2a_2)$, then $x = \sum_{fin} \pm s_j$ where $s_j \in S_1$ (set of distributive elements). And distributively generated character of $Ann(i_1a_1)$ gives, $Ann(i_1a_1) \cap Ann(i_2a_2) \subseteq Ann(i_1a_1 + i_2a_2)$.

Conversely, let $y = (\sum_{fin} \pm t_j) \in Ann(i_1a_1 + i_2a_2) =< S_2 >$, where $t_j \in S_2$, a set of distributive elements. Then $t_j \in Ann(i_1a_1 + i_2a_2)$ which gives $t_j(i_1a_1 + i_2a_2) = 0$ or, $t_ji_1a_1 + t_ji_2a_2 = 0$ or, $(\sum_{fin} \pm s_k)(t_ji_1a_1 + t_ji_2a_2) = 0$, as $i_2(= \sum_{fin} \pm s_k) \in Ann(i_1a_1) =< S_3 >$, where $s_k \in S_3$ (a set of distributive elements), $i_2t_ji_2a_2 = 0$ or, $t_jqi_2a_2 = 0$, for some $q \in N$, being duo near-ring and so $t_j \in Ann(qi_2a_2)(\supseteq Ann(i_2a_2))$. Now $qi_2a_2(\in (Ann(i_1a_1)) \cap I)B$ is a non strictly ps-nilpotent element. If not, then $Sqi_2a_2 \neq 0$ such that $S^nqi_2a_2 = 0$, for some proper left N-subset S of N and some $n(\geq 2) \in \mathbb{Z}^+$. By taking $T = Sq$ we get $Ti_2a_2 \neq 0$ such that $T^ni_2a_2 = 0$, giving that i_2a_2 is strictly ps-nilpotent element, which is not true. Thus, $Ann(qi_2a_2) = Ann(i_2a_2)$, as $Ann(i_2a_2)$ is maximal. So, $t_j \in Ann(i_2a_2)$ for each j, which gives $t_ji_2a_2 = 0$, for each j; this gives $t_ji_1a_1 = 0$ and hence $(\sum_{fin} \pm t_j)i_1a_1 = 0$ and $(\sum_{fin} \pm t_j)i_2a_2 = 0$, which imply $y(= \sum_{fin} \pm t_j) \in Ann(i_1a_1) \cap Ann(i_2a_2)$. Thus, $Ann(i_1a_1 + i_2a_2) \subseteq Ann(i_1a_1) \cap Ann(i_2a_2)$.

In the near-ring $N(=\mathbb{Z}_8)$[[5](22), p.343], all the proper left N-subsets are $\{0,1\}, \{0,2\}, \{0,4\}, \{0,4,5\}, \{0,2,4,6\}, \{0,4,6\}, \{0,2,4,6,7\}, \{0,2,3,4,6\}$ where $\{0,4\}$ is distributively generated left annihilator and $\{0,3\}$ is a strictly ps-nilpotent subset of N, as $\{0,2\}\{0,3\} \neq 0$ and $\{0,2\}^2\{0,3\} = 0$. So, ${}_NN$ is not ps-strictly semi-prime. Moreover, $\{0,4\}$ and $\{0,2,4,6\}$ are only two left N-subgroups as well as ideals. So each of them is essential left ideal as well as strictly essential as an N-subgroups of ${}_NN$. But ${}_NN$ contains no element e such that $Ann(e) = 0$. Thus, we see how ps-strictly semi-prime character together with distributively generated annihilator and coincidence of essential left ideals and strictly essential N-subgroups of ${}_NN$[[1], Th.(7)] play key role for the existence of an element e of ${}_NN$ such that $Ann(e) = 0$. And we note the following (may be considered as one major result on so far the Goldie character of an N-group is concerned). □

Lemma 2.6. *Let N-group E be as in Lemma 2.4 and the annihilators of subsets of E in N are distributively generated, then there exists $e \in E$ such that $Ann(e) = 0$.*

Proof. Let B be a non-zero N-subgroup of E and let $I \subseteq_e {}_NN$.

From Lemma 2.1, B is not strictly ps-nil. So we have a non strictly ps-nilpotent element of the form ia, $(i \in I, a \in B)$. By hypothesis, we consider $i_1a_1 \in IB$, $(i_1 \in I, a_1 \in B)$, with i_1a_1 non strictly ps-nilpotent such that $Ann(i_1a_1)$ is as large as possible. If $Ann(i_1a_1) = 0$, we stop. If not then $Ann(i_1a_1) \cap I \neq 0$ as $I \subseteq_e {}_NN$. As above, a non strictly ps-nilpotent element $i_2a_2 \in (Ann(i_1a_1) \cap I)B$, $(i_2 \in Ann(i_1a_1) \cap I, a_2 \in B)$ with $Ann(i_2a_2)$ as large as possible. Using Lemma 2.4, we get that there exists $n \in \mathbb{Z}^+$ such that

$Ann(i_1a_1 + i_2a_2 + ... + i_na_n) = Ann(i_1a_1 + i_2a_2 + ... + i_{n+1}a_{n+1}) = ...$

Now, $Ann(i_1a_1+i_2a_2+...+i_na_n) = Ann(i_1a_1+i_2a_2+...+i_{n+1}a_{n+1}) = Ann(i_1a_1+ ...+i_na_n) \cap Ann(i_{n+1}a_{n+1})$, giving thereby $Ann(i_1a_1+...+i_na_n) \subseteq Ann(i_{n+1}a_{n+1})$.

But, by our choice, with $i_{n+1}a_{n+1}$ non strictly ps-nilpotent, $Ann(i_{n+1}a_{n+1})$ is as large as possible. Also $i_{n+1} \in Ann(i_1a_1 + ... + i_na_n)(\subseteq Ann(i_{n+1}a_{n+1})$ which implies $i_{n+1}i_{n+1}a_{n+1} = 0$.

Alternately, strictly ps-nilpotency of $i_{n+1}i_{n+1}a_{n+1}$ leads to $i_{n+1}(= 0) \in Ann(i_1a_1 + ... + i_na_n) \cap I \implies Ann(i_1a_1 + ... + i_na_n) = 0$ as $I \subseteq_e {}_NN$ or $Ann(e) = 0$ where $e = i_1a_1 + ... + i_na_n$. □

For $e \in E$ with $Ann(e) = 0$ we get easily the following

Lemma 2.7.

(i) *An ideal B (N-subgroup) of ${}_NN$ is maximal if and only if Be is a maximal ideal (N-subgroup) of Ne.*

(ii) $J(N)e = J(Ne)$.

Note E:

(i) If S is an N-subgroup (ideal) of E then it is easy to see that $\bar{S}$ is also an N-subgroup(ideal) of E.

Now, if $E = Ne$ with $Ann(e) = 0$ and Ce (an N_u-nil N-subgroup of Ne) is not contained in $J(Ne)$, then there exists an ideal Be of Ne which is maximal as N-subgroup of Ne such that $Ne = Be + Ce$ which gives $N = B + C$ [Lemma 2.7(i)]. Hence $1 = b + c$, for some $b \in B$, $c \in C$.

Now, $N_u^t ce = 0$ but $N_u^t \neq 0$ for some t (least) $\in \mathbb{Z}^+$ [Ce being N_u-nil] and $ce \in Ce$, which gives $N_u^t c = 0$, as $Ann(e) = 0$. Now, for $n_1n_2...n_t \in N_u^t$, we get $n_1n_2...n_t = n_1n_2...n_t(b+c) - n_1n_2...n_tc + n_1n_2...n_tc (\in B)$ as B is an ideal of ${}_NN$ and $n_1n_2...n_tc = 0$ which gives $N_u^t \subseteq B$. Next suppose, $t > 1$, then $n_1n_2...n_{t-1} = n_1n_2...n_{t-1}(b+c) - n_1n_2...n_{t-1}c + n_1n_2...n_{t-1}c (\in B)$ as $n_1n_2...n_{t-1}c \in N_u^t \subseteq B$. Thus by induction, we get $N_u \subseteq B$, which is a contradiction. Hence it follows that $J(E)$ contains all the N_u-nil N-subgroups of E.

(ii) In case of a near-ring group $E = \bigoplus_{i=1}^n Ne_i$ we have $J(E)$ contains all the N_u-nil N-subgroups of E.

3. Main results

Now we want to get regarding two sided and asymmetric hyper continuity, separately in case of mappings from $(X \times X, \mathcal{E} \times \mathcal{E})$ to $(X, \mathcal{E})$.

3.1. **Observations:**

3.1.1. We observe that $(G, +)$ is a hypergraph group, A is the binary operation and $\mathcal{E}$ is a hypergraph such that $A : G \times G \to G$ is hyper continuous at (a, b). Then the maps ${}_aA : G \to G$ where ${}_aA(x) = a + x$ and $A_b : G \to G$ where $A_b(x) = x + b$, for all $x \in G$ are both hyper continuous at b and a respectively.

But the converse may not be true. In other words if ${}_aA$ and A_b are both hyper continuous, then $A : G \times G \to G$ need not be hyper continuous.

3.1.2. As mentioned above a *hypergraph group is a pair* $(G, \mathcal{E})$ with $(G, +)$, a group and $\mathcal{E}$, the class of hyperedges from G together with the hyper continuous binary operation map $f(=+) : G \times G \to G$.

3.1.3. At first, it is noticed that the converse of the above appears as false at least when $\mathcal{E}$ is a topology.

In each of the following examples we observe some characteristics of the binary operation of a group w.r.t the respective subclasses of the power set of the group.

Example 13. *In the symmetric group* $(S_3, +)$ *[[5], p.341], consider a hypergraph* $(S_3, \mathcal{E})$ *with* $\mathcal{E} = \{\{a, c\}, \{b, c\}, \{x, y\}\}$. *Here for* $\{x, y\}(\in \mathcal{E})$ *containing* $y(= a + b)$ *we have* $\{b, c\}(\in \mathcal{E})$ *containing* b *and* $\{a, c\}(\in \mathcal{E})$ *containing* a *such that* $a+\{b, c\} = \{x, y\}$ *and* $\{a, c\} + b(= \{y\}) \subset \{x, y\}$*, but* $\{a, c\} + \{b, c\}(= \{0, x, y\}) \not\subseteq \{x, y\}$.

Example 14. *In the group* $(\mathbb{Z}_8, +)$ *[[5], p.342], consider a hypergraph* $(\mathbb{Z}_8, \mathcal{E})$ *with* $\mathcal{E} = \{\{2, 3\}, \{2, 4\}, \{5, 6, 7\}\}$. *Now for* $\{5, 6, 7\}(\in \mathcal{E})$ *containing* $7(= 3 + 4)(\in \mathbb{Z}_8)$ *there exists* $\{2, 4\}(\in \mathcal{E})$ *containing* $4(\in \mathbb{Z}_8)$ *such that* $3 + \{2, 4\}(= \{5, 7\}) \subset \{5, 6, 7\}$ *and also there exists* $\{2, 3\}(\in \mathcal{E})$*containing* 3 *such that* $\{2, 3\} + 4(= \{6, 7\} \subset \{5, 6, 7\})$ *but* $\{2, 3\} + \{2, 4\}(= \{4, 5, 6, 7\}) \not\subseteq \{5, 6, 7\}$.

Example 15. *In the dihedral group* $(D_8, +)$ *[[5], p.344], consider a hypergraph* $(D_8, \mathcal{E})$ *with* $\mathcal{E} = \{\{a, 2a\}, \{b, 3a+b\}, \{a+b, 2a+b\}\}$ *we see that for* $\{a+b, 2a+b\}(\in \mathcal{E})$ *containing* $2a + b$ $(\in D_8)$ *there exists* $\{b, 3a + b\}$ $(\in \mathcal{E})$ *containing* $b(\in D_8)$ *such that* $2a + \{b, 3a + b\} = \{a + b, 2a + b\}$ *and also there is* $\{a, 2a\}(\in \mathcal{E})$ *containing* $2a$ $(\in D_8)$ *such that* $\{a, 2a\} + b = \{a + b, 2a + b\}$ *but* $\{a, 2a\} + \{b, 3a + b\}(= \{a + b, b, 2a + b\}) \not\subseteq \{a + b, 2a + b\}$.

The above examples reveal that we require some condition(s) that may lead us to what we are looking for.

Example 16. *In the symmetric group* $(S_3, +)$ *[[5], p. 341], we have a hypergraph* $(S_3, \mathcal{E})$ *with* $\mathcal{E} = \{\{0, b\}, \{a, y\}, \{b, c, x, y\}\}$. *Here we note that* $a + \{0, b\}(= \{a, y\}) \in \mathcal{E}$ *but* $\{0, b\}+c(= \{c, y\}) \notin \mathcal{E}$; $\{0, b\}+\{0, b\} = \{0, b\}$; *for* $\{b, c, x, y\}(\in \mathcal{E})$ *containing* $c(= 0 + c)(\in S_3)$ *there exists* $\{0, b\}(\in \mathcal{E})$ *containing* $0(\in S_3)$ *such that* $\{0, b\} + c(= \{c, y\}) \subset \{b, c, x, y\}$; *also for* $\{b, c, x, y\}(\in \mathcal{E})$ *containing* $x(= a + c)(\in S_3)$ *we get* $a + \{b, c, x, y\} = \{b, c, x, y\}$ *but* $\{a, y\} + \{b, c, x, y\} \not\subseteq \{b, c, x, y\}$.

Example 17. *Let us consider a hypergraph* $(\mathbb{Z}_8, \mathcal{E})$ *where* $\mathcal{E} = \{\{0,6\}, \{0,7\}, \{2,4\}, \{3,5\}, \{3,4\}, \{4,5\}, \{1,7\}\}$ *and the group* $(\mathbb{Z}_8, +)$ *[[5], p. 342].*

Here for any $W(\in \mathcal{E})$ *containing* $0(= 0 + 0)(\in \mathbb{Z}_8)$*, we get* $4 + W, W + 5 \in \mathcal{E}$*; but no* U *and* $V(\in \mathcal{E})$ *containing* $0(\in \mathbb{Z}_8)$ *such that* $U + V \subseteq W$*. Again for* $\{3,5\}$ *and* $\{4,5\}(\in \mathcal{E})$ *containing* $5(= 0 + 5)(\in \mathbb{Z}_8)$ *there exists* $\{0,6\}$*(or,* $\{0,7\}$*)*$(\in \mathcal{E})$ *containing* $0(\in \mathbb{Z}_8)$ *such that* $\{0,6\} + 5 = \{3,5\}$ *and* $\{0,7\} + 5 = \{4,5\}$*. And we have for* $\{1,7\}(\in \mathcal{E})$ *containing* $1(= 4 + 5)(\in \mathbb{Z}_8)$ *there exists* $\{3,5\}(\in \mathcal{E})$ *containing* $5(\in \mathbb{Z}_8)$ *such that* $4 + \{3,5\} = \{1,7\}$ *but no* U *and* $V(\in \mathcal{E})$ *containing* 4 *and* $5(\in \mathbb{Z}_8)$ *respectively such that* $U + V \subseteq \{1,7\}$*.*

Example 18. *In the dihedral group* $(D_8, +)$ *[[5], p.344], consider a hypergraph* $(D_8, \mathcal{E})$ *with* $\mathcal{E} = \{\{0,b\}, \{a, 3a+b\}, \{2a, 2a+b\}, \{3a, 3a+b\}\}$*. It is clear that* $3a + \{0,b\}(= \{3a, 3a+b\}); \{0,b\} + 3a + b(= \{a, 3a+b\}) \in \mathcal{E}; \{0,b\} + \{0,b\} = \{0,b\}$*; for* $\{3a, 3a+b\}(\in \mathcal{E})$ *containing* $3a + b(= 0 + 3a + b)(\in D_8)$ *there exists no* $V(\in \mathcal{E})$ *containing* $0(\in D_8)$ *such that* $V + 3a + b \subseteq \{3a, 3a+b\}$*. Again for* $\{2a, 2a+b\}(\in \mathcal{E})$ *containing* $2a + b(= 3a + 3a + b)(\in D_8)$ *there exists* $\{3a, 3a+b\}(\in \mathcal{E})$ *containing* $3a + b(\in D_8)$ *such that* $3a + \{3a, 3a+b\} = \{2a, 2a+b\}$ *but no* U *and* $V(\in \mathcal{E})$ *containing* $3a$ *and* $3a + b(\in D_8)$ *respectively such that* $U + V \subseteq \{2a, 2a+b\}$*.*

Example 19. *In the symmetric group* $(S_3, +)$ *[[5], p.341] we have a hypergraph* $(S_3, \mathcal{E})$ *with* $\mathcal{E} = \{\{0,x\}, \{a,c\}, \{b,c\}, \{x,y\}\}$*, we see that* $a + \{0,x\}(= \{a,c\}), \{0,x\} + b(= \{b,c\}) \in \mathcal{E}$*; for* $\{0,x\}(\in \mathcal{E})$ *containing* $0(= 0 + 0)(\in S_3)$ *there exist no* U *and* $V(\in \mathcal{E})$ *containing* $0(\in S_3)$ *such that* $U + V \subseteq \{0,x\}$*; for* $\{b,c\}(\in \mathcal{E})$ *containing* $b(= 0 + b)$ *there exists* $\{0,x\}(\in \mathcal{E})$ *containing* $0(\in S_3)$ *such that* $\{0,x\} + b = \{b,c\}$*. Again for* $\{x,y\}(\in \mathcal{E})$ *containing* $y(= a + b)(\in S_3)$ *there exists* $\{b,c\}(\in \mathcal{E})$ *containing* $b(\in S_3)$ *such that* $a + \{b,c\} = \{x,y\}$ *but* $\{a,c\} + \{b,c\}(= \{0,x,y\}) \nsubseteq \{x,y\}$*.*

Example 20. *In the dihedral group* $(D_8, +)$ *[[5], p.344], consider a hypergraph* $(D_8, \mathcal{E})$ *with* $\mathcal{E} = \{\{0, 2a+b\}, \{3a, a+b\}, \{a, 3a+b\}\}$*. It is seen here that* $3a + \{0, 2a+b\}(= \{3a, a+b\}) \in \mathcal{E}$*, but* $\{0, 2a+b\} + 3a + b(= \{3a, 3a+b\}) \notin \mathcal{E}$*;* $\{0, 2a+b\} + \{0, 2a+b\} = \{0, 2a+b\}$*; for* $\{a, 3a+b\}(\in \mathcal{E})$ *containing* $3a + b(= 0 + 3a + b)(\in D_8)$*, there exists no* $V(\in \mathcal{E})$ *containing* $0(\in D_8)$ *such that* $V + 3a + b \subseteq \{a, 3a+b\}$*.*

Again, for $\{0, 2a+b\}(\in \mathcal{E})$ *containing* $2a + b(= 3a + 3a + b)(\in D_8)$ *such that* $3a + \{a, 3a+b\} = \{0, 2a+b\}$ *but* $\{3a, a+b\} + \{a, 3a+b\}(= \{0, b, 2a, 2a+b\}) \nsubseteq \{0, 2a+b\}$*.*

It is observed here that in each of the above examples the binary operation of a group, say X is not hyper continuous at a point of $X \times X$.

Theorem 3.1. *Consider two hypergraphs $(X, \mathcal{E})$ and $(Y, \mathcal{E}')$ with two mappings $f : X \times X \to X$ and $g : X \times Y \to Y$, where X is equipped with a binary operation having an identity e(which is also scalar multiplicative identity with respect to Y) and $g(f(n,m),y) = g(n, g(m,y))$ for all $n, m \in X$ and $y \in Y$ satisfy the following conditions **(a two sided hypergraph system)** at $(x,y) \in X \times Y$.*

(i) *for any $V(\in \mathcal{E})$ containing e we have ${}_xf(V) \in \mathcal{E}, g_y(V) \in \mathcal{E}'$.*
(ii) *f is hyper continuous at (e,e).*
(iii) *g_y is hyper continuous at e.*
(iv) *${}_xg$ is hyper continuous at y, then g is hyper continuous at (x,y).*

Proof. Now, for any $W(\in \mathcal{E}')$ containing $g(x,y)(= {}_xg(y))$ we get, $V(\in \mathcal{E}')$ containing y such that ${}_xg(V) \subseteq W$. Again, because of two sided hypergraph system character, there exists $U(\in \mathcal{E})$ containing $e(\in X)$ such that $g_y(U) \subseteq V$ and therefore we get, $A \times B(\in \mathcal{E} \times \mathcal{E})$ containing $(e,e)(\in X \times X)$ such that $f(A \times B) \subseteq U$ finally leading to

$$\begin{aligned} g({}_xf(A) \times g_y(B)) &= g(f(\{x\} \times A) \times g(B \times \{y\})) \\ &= g(\{x\} \times g(A \times g(B \times \{y\}))) \\ &= g(\{x\} \times g(f(A \times B) \times \{y\})) \subseteq g(\{x\} \times g(U \times \{y\})) \\ &= g(\{x\} \times g_y(U)) \subseteq g(\{x\} \times V) \\ &= {}_xg(V) \subseteq W \end{aligned}$$

which gives us g as hyper continuous at (x,y). □

From above we get the following:

Corollary 3.1. *Consider the hypergraph $(X, \mathcal{E})$ with a mapping $f : X \times X \to X$ satisfying the conditions*

(a) *for some $e(\in X), f(e,x) = x = f(x,e)$*
(b) *$f(x, f(y,z)) = f(f(x,y),z)$ for all $x, y, z, \in X$ along with the following conditions at $(a,b)(\in X \times X)$.*
(i) *for any $V(\in \mathcal{E})$ containing e we have ${}_af(V)$, $f_b(V) \in \mathcal{E}$.*
(ii) *f is hyper continuous at (e,e).*

(iii) f_b *is hyper continuous at* e.

(iv) ${}_af$ *is hyper continuous at* b*, then* f *is hyper continuous at* (a,b).

It is obvious that in the above examples, if f is a binary operation then, the condition (i) does not hold good in Ex. 3. 7 and 3. 11; condition (ii) does not hold good in Ex. 3. 8 and 3. 10; also Ex. 3. 9 and 3. 11 do not follow the condition (iii). Thus, we arrive to conclude that to become f as a hyper continuous map at $(a,b)(\in X\times X)$; (i), (ii), (iii) together with (iv) are the conditions to be satisfied.

From the following examples it is clear that, in case of Klein's four group a two sided hypergraph system is a sufficient condition for the said make up, because absence of any one of them nullifies the same.

Let $G=\{0,a,b,c\}$ be the Klein's four group [[5], p.339] and consider the mappings on G defined by Table 3. It follows that $N=\{f_0,f_1,f_2,f_3,f_4,f_5,f_6,f_7,f_8,f_9,f_{10},f_{11},f_{12},f_{13},f_{14},f_{15}\}$ is a right near-ring with unity f_1 w.r.t. the following operations:

$(f+g)(x)=f(x)+g(x)\quad\longrightarrow(*)$

and $(f.g)(x)=f(g(x))$ for all $f,g\in N,x\in G$

Here we observe the near-ring group structure of G over N w.r.t. the operation $N\times G\to G$; $(f_p,e)\to f_p(e)\quad\longrightarrow(**)$

Example 21. *Consider the hypergraphs* $(N,\mathcal{E})$ *and* $(G,\mathcal{E}')$ *where* $\mathcal{E}=\{\{f_9,f_{10}\}\}$ *and* $\mathcal{E}'=\{\{a,b\},\{b,c\}\}$*. Here for* $\{b,c\}(\in\mathcal{E}')$ *containing* $c(=f_{10}(a))(\in G)$ *we have* $\{a,b\}(\in\mathcal{E}')$ *containing* $a(\in G)$ *and* $\{f_9,f_{10}\}(\in\mathcal{E})$ *containing* $f_{10}(\in N)$ *such that* $f_{10}(\{a,b\})=\{b,c\}$ *and* $\{f_9,f_{10}\}\{a\}(=\{c\})\subset\{b,c\}$ *but* $\{f_9,f_{10}\}\{a,b\}$ $(=\{0,b,c\})\nsubseteq\{b,c\}$.

Example 22. *Consider the hypergraphs* $(N,\mathcal{E})$ *and* $(G,\mathcal{E}')$ *where* $\mathcal{E}=\{\{f_8,f_{12}\}\}$ *and* $\mathcal{E}'=\{\{a,b\},\{b,c\}\}$*. Here for* $\{b,c\}(\in\mathcal{E}')$ *containing* $c(=f_{12}(a))(\in G)$ *we have* $\{a,b\}(\in\mathcal{E}')$ *containing* $a(\in G)$ *and* $\{f_8,f_{12}\}(\in\mathcal{E})$ *containing* $f_{12}(\in N)$ *such that* $f_{12}(\{a,b\})=\{b,c\}$ *and* $\{f_8,f_{12}\}\{a\}=\{b,c\}$ *but* $\{f_8,f_{12}\}\{a,b\}$ $(=\{0,b,c\})\nsubseteq\{b,c\}$.

Example 23. *Consider the hypergraphs* $(N,\mathcal{E})$ *and* $(G,\mathcal{E}')$ *where* $\mathcal{E}=\{\{f_9,f_{10}\},\{f_1,f_4\}\}$ *and* $\mathcal{E}'=\{\{a,b\},\{b,c\}\}$*. Here we note that* $f_{10}\{f_1,f_4\}(=\{f_9,f_{10}\})\in\mathcal{E}$ *but* $\{f_1,f_4\}\{a\}(=\{a\})\notin\mathcal{E}'$*;* $\{f_1,f_4\}\{f_1,f_4\}=\{f_1,f_4\}$*; for* $\{a,b\}(\in\mathcal{E}')$ *containing* $a(=f_1(a))(\in G)$ *there exists* $\{f_1,f_4\}(\in\mathcal{E})$ *containing* $f_1(\in N)$ *such*

$G \to G$	0	a	b	c
f_0	0	0	0	0
f_1	0	a	b	c
f_2	0	a	b	0
f_3	0	0	b	0
f_4	0	a	0	0
f_5	0	0	b	c
f_6	0	0	0	c
f_7	0	a	0	c
f_8	0	b	0	0
f_9	0	c	0	0
f_{10}	0	c	b	0
f_{11}	0	b	b	0
f_{12}	0	c	b	c
f_{13}	0	b	b	c
f_{14}	0	b	0	c
f_{15}	0	c	0	c

TABLE 3. Operation Table

that $\{f_1, f_4\}\{a\}(=\{a\}) \subset \{a, b\}$*; also for* $\{b, c\}(\in \mathcal{E}^{'})$ *containing* $c(= f_{10}(a))(\in G)$ *we get* $f_{10}(\{a, b\}) = \{b, c\}$ *but* $\{f_9, f_{10}\}\{a, b\}(= \{0, b, c\}) \nsubseteq \{b, c\}$.

Example 24. *Let us consider the hypergraphs* $(N, \mathcal{E})$ *and* $(G, \mathcal{E}^{'})$ *where* $\mathcal{E} = \{\{f_1, f_8\}, \{f_8, f_{12}\}\}$ *and* $\mathcal{E}^{'} = \{\{a, b\}, \{b, c\}\}$. *Here we note that* $f_{12}\{f_1, f_8\}(=\{f_8, f_{12}\}) \in \mathcal{E}$ *and* $\{f_1, f_8\}\{a\}(= \{a, b\}) \in \mathcal{E}^{'}$. *Now for any* $W(\in \mathcal{E})$ *containing* $f_1(= f_1.f_1)(\in N)$ *we get no* U *and* $V(\in \mathcal{E})$ *containing* $f_1(\in N)$ *such that* $UV \subseteq W$. *Again for* $\{a, b\}(\in \mathcal{E}^{'})$ *containing* $a(= f_1(a))(\in G)$ *there exists* $\{f_1, f_8\}(\in \mathcal{E})$ *containing* $f_1(\in N)$ *such that* $\{f_1, f_8\}\{a\} = \{a, b\}$. *And we have for* $\{b, c\}(\in \mathcal{E}^{'})$ *containing* $c(= f_{12}(a))(\in G)$ *there exists* $\{a, b\}(\in \mathcal{E}^{'})$ *containing* $a(\in G)$ *such that* $f_{12}(\{a, b\}) = \{b, c\}$ *but* $\{f_8, f_{12}\}\{a, b\}(= \{0, b, c\}) \nsubseteq \{b, c\}$.

Example 25. *Consider the hypergraphs* $(N, \mathcal{E})$ *and* $(G, \mathcal{E}^{'})$ *where* $\mathcal{E} = \{\{f_1, f_{12}\}, \{f_7, f_{15}\}\}$ *and* $\mathcal{E}^{'} = \{\{a, c\}, \{0, a, b\}\}$*, it is seen here that* $f_7\{f_1, f_{12}\}(= \{f_7, f_{15}\}) \in \mathcal{E}$*,* $\{f_1, f_{12}\}\{a\}(= \{a, c\}) \in \mathcal{E}^{'}$*;* $\{f_1, f_{12}\}\{f_1, f_{12}\} = \{f_4, f_{12}\}$*; for* $\{0, a, b\}(\in \mathcal{E}^{'})$ *containing* $a(= f_1(a))(\in G)$*, there exists no* $V(\in \mathcal{E})$ *containing* $f_1(\in N)$ *such* *that* $Va \subseteq \{0, a, b\}$. *Again for* $\{a, c\}, \{0, a, b\} (\in \mathcal{E}')$ *containing* $a(= f_7(a))(\in G)$ *we get* $f_7(\{a, c\}) = \{a, c\}$ *and* $f_7(\{0, a, b\})(= \{0, a\}) \subset \{0, a, b\}$ *but* $\{f_7, f_{15}\}\{a, c\}$ $= \{a, c\}$; $\{f_7, f_{15}\}\{0, a, b\}(= \{0, a, c\}) \nsubseteq \{0, a, b\}$.

It can be seen in the above examples *21, 22, 23, 24* and *25* that if f is the binary operation(multiplication) on the near-ring N and g is the scalar multiplication on the N-group G defined as in (*) and (**) above, the condition (i) does not hold good in Ex. *23*; condition (ii) does not hold good in Ex. *24*; also Ex. *25* does not follow the condition (iii). Thus, all these justify what we have evaluated.

3.2. Quasi regular cyclic and Nu-nilpotent hyper N groups.

3.2.1. *Quasi regular cyclic Ne-hyper open N-groups.* We consider here Q and Qe both N-hyper open and E-hyper open respectively. Together with zero annihilator of e which occurs in Lemma 2.6 as a necessity of near-ring group with so called Goldie character. In this hypergraph biased near-ring group with a.c.c. on annihilators we study ps-closedness of direct sum of maximal N-subgroups together with ps-closedness of the direct sum of group sum of ideals related to quasi regular left ideal of N. Moreover the radical of the N-group $E(= \bigoplus_{i=1}^{n} Ne_i)$ coincides with the radical subgroup if such type of direct sum is E-hyper dense in the radical. Also such type of conditions help us to get the cyclic character of E.

Suppose Be is a maximal N-subgroup(ideal) of Ne, then B is a maximal N-subgroup(ideal) of ${}_NN$. We claim $Be = \overline{Be}$

[We know that if B is an N-subgroup(ideal) of E, then $\overline{B}$ is also so etc.]

Theorem 3.2. *If $\bigoplus_{i=1}^{n} B_ie_i$ is a maximal N-subgroup (ideal) of $\bigoplus_{i=1}^{n} Ne_i$, then $\bigoplus_{i=1}^{n} B_ie_i$ is a ps-closed N-subgroup (ideal) of $\bigoplus_{i=1}^{n} Ne_i$.*

Proof. Clearly, it is sufficient to show the result for only one component. Suppose Be is a maximal N-subgroup(ideal) of Ne.

We claim $Be = \overline{Be}$.

As if Be is an N-subgroup(ideal) of Ne, then $\overline{Be}$ is also so etc.

Clearly, $Be \subseteq \overline{Be} \subseteq Ne$. Be being maximal, either $Be = \overline{Be}$ or $\overline{Be} = Ne$.

Case I: $\overline{Be} = Ne \implies e \in \overline{Be}$ as $e \in Ne$. Now by Note C(iv), $e - Qe$ is ps-open containing e also, by Note C(i) $(e - Qe) \cap Be \neq \phi \implies (1 - Q)e \cap Be \neq \phi$. As, $Ann(e) = 0$, we get $(1 - Q)e \cap Be = ((1 - Q) \cap B)e \neq \phi \implies (1 - Q) \cap B \neq$

$\phi \implies 1 - q = b$, for some $b \in B$, $q \in Q$. Hence, $1 \in B$, as for some $n \in N$, $n(1-q) = 1$ giving thereby $nb = 1$.

Thus, $Be = \overline{Be}$. Hence, Be is ps-closed.

Thus, $\bigoplus_{i=1}^{n} B_i e_i$ is a ps-closed N-subgroup (ideal) of $\bigoplus_{i=1}^{n} Ne_i$. □

As a corollary to the above and Zorn's lemma we have the following :

Corollary 3.2. *Let E be an N-group such that every maximal N-subgroup of E is of the form Be with $Ann(e) = 0$. If E contains no proper ps-closed N-subgroup then $Ne = E$.*

As Q and Qe are N-hyper open and E-hyper open respectively with $Ann(e) = 0$, and arbitrary intersection of ps-closed sets is ps-closed, we get both $J(Ne)$ and Ae as ps-closed.

Here, the set Se denotes the group sum of all ideals of Ne of the form Ie where I is a quasi-regular left ideal of N.

Theorem 3.3. *$\bigoplus_{i=1}^{n} Se_i$ is a ps-closed ideal of $E = \bigoplus_{i=1}^{n} Ne_i$.*

Proof. As above it is sufficient to show the result for one component. Now, if $I \nsubseteq A$, for some quasi-regular left ideal I of N, then $I + B = N$, for some maximal left N-subgroup B of N giving thereby $1 \in B$. Consequently, $S \subseteq A$, where S is the group sum of all quasi-regular left ideals of N. Thus, we get $Se \subseteq Ae \subseteq Qe$.

So, $\overline{Se} \subseteq Qe$, as Ae is ps-closed. Now, $\overline{Se} = \{qe | for\ some\ q \in Q\}$ and consider $J = \{p | p \in Q, pe \in \overline{Se}\}$. We have by Note E(i), that $\overline{Se}$ is an ideal of Ne of the form Je, where J is a quasi-regular left ideal of N. So, $\overline{Se} \subseteq Se$. □

As a corollary to the above and Lemma 2.7(ii) we get the following

Corollary 3.3. *If $\bigoplus_{i=1}^{n} Se_i$ is hyper dense in $J(E)$, ($E = \bigoplus_{i=1}^{n} Ne_i$) then $J(E) = \bigoplus_{i=1}^{n} Ae_i$.*

We note that the near-ring N [[5](4), p.340] of Klein's four group with the hypergraph $(N, \mathcal{E})$ and $\mathcal{E} = \{\{0, a\}, \{b, c\}\}$ has a maximal left N-subgroup $B(= \{0, a\})$. Again $Bb = \{0, a\}$ is a maximal left N-subgroup of N with $l(b) = 0$ such that $Nb = N$.

Thus the above example is sufficient to explain the point that the converse of the above theorem is not true.

In case of an E not necessarily of the type $\bigoplus_{i=1}^{n} Ne_i$ we have that hyper openness of Qe and hyper boundedness of N lead us to the following expected results, mainly on ps-openness of $J(E)$ together with ps-discrete and finiteness of E.

Theorem 3.4. *If N is hyper bounded and E is with fully radical character, then $J(E)$ is ps-open.*

Proof. As Qe is hyper open and N is hyper bounded, there exists a hyper open subset V of E such that $NV \subseteq Qe$. Thus, we can write $NV = \{qe | for\ some\ q \in Q\}$. Now, $V_1 = \{q \in Q : qe \in NV\}(\subseteq Q)$. And $NV = V_1e$, as $Ann(e) = 0$. Now, $N(V_1e) = N(NV) = N^2V \subseteq NV = V_1e$, which gives $NV_1 \subseteq V_1$, as $Ann(e) = 0$. So, $NV_1 = V_1$. Moreover, for each $x \in V_1, Nx$ is a quasi-regular left N-subgroup of N and as what have been stated above in case of A [8], we get $Nx \subseteq J(N)$ and hence $V_1 \subseteq J(N)$. Thus, we have $V = 1.V \subseteq NV = V_1e \subseteq J(N)e \subseteq J(N)E = J(E)$. If $y \in J(E), z \in V$, then $y + (-z) + V$(a ps-open subset of E containing y)$\subseteq J(E)$. So, there exists a hyper open set U containing y such that $U(\subseteq y + (-z) + V) \subseteq J(E)$. Hence, $J(E)$ is ps-open. □

Corollary 3.4. *If N is hyper bounded, E is with radical characer and $J(E) = 0$, then E is a ps-discrete.*

The following example reveals that the vanishing of radical is essential for the ps-discreteness of N-group E when N is hyper bounded.

Consider the hypergraph $(N, \mathcal{E})$ with the near-ring $N(= \mathbb{Z}_8)$[[5](127), p.344] and $\mathcal{E} = \{\{0, 4\}, \{1, 5\}, \{2, 6\}, \{3, 7\}\}$. Here N is hyper bounded, $J(N) = \{0, 4\}$, $l(e) = 0$, where $e = 1, 2, 3, 5, 6, 7$ but N is not ps-discrete.

Corollary 3.5. *If E is ps-compact having fully radical character, N is hyper bounded and $J(E) = 0$, then E is finite.*

Proof. We have by above corollary E is ps-discrete. If E is infinite, then E is not ps-compact. Hence E is finite. □

Note F:

(i) Suppose $C^nD' \subseteq Q$ for some left N-subset C of N, $n \in \mathbb{Z}^+$ and $D' \subseteq N$. If $C^nD' \nsubseteq B$, left ideal B of N maximal as N-subgroup, then $Nc_1c_2...c_nd' + B = N$, with, $c_1c_2...c_nd'(\notin B) \in C^nD'$, and $c_1, c_2, ..., c_n \in C$

and $d^{'} \in D^{'}$ giving thereby $nc_1c_2...c_nd^{'} + b = 1$, where some $n \in N$ and $b \in B$. And, $(nc_1)c_2...c_nd^{'} \in C^nD^{'}(\subseteq Q)$, we get $1 \in B$ a contradiction.

Hence, $C^nD^{'} \subseteq J(N)$.

(ii) Suppose $n > 1$ and $C^{n-1}D^{'} \nsubseteq J(N)$. Now, $nc+b = 1$ for some $n \in N, b \in B$ and $c(\notin B) \in C^{n-1}D^{'}$. Again for any $c_1c_2...c_{n-1}d^{'} \in C^{n-1}D^{'}$, with $c_1, c_2, ..., c_{n-1} \in C, d^{'} \in D$ we get $c_1c_2...c_{n-1}d^{'} \in B$, as C, a left N-subset giving thereby $c_1c_2...c_{n-1}d^{'}nc(\in C^nD^{'} \subseteq J(N)) \in B$. Hence $C^{n-1}D^{'} \subseteq B$, a contradiction. Therefore by induction, we get $CD^{'} \subseteq J(N)$ and by Lemma 2.7(ii) we get $CD \subseteq J(Ne)$.

(iii) When $C^nD \subseteq Qe$, we get $C^nD^{'} \subseteq Q$ as $Ann(e) = 0$.
By what we have got $CD \subseteq J(Ne)$.

Hence, we get the following theorem establishing the link with the hyper nilpotent notion of a subset of E and that of the radical of the N-group.

Theorem 3.5. *If $\bigoplus_{i=1}^n D_i(= \bigoplus_{i=1}^n D_i^{'}e_i)$ is a hyper nilpotent subset of E, then $\bigoplus_{i=1}^n C_iD_i \subseteq J(\bigoplus_{i=1}^n Ne_i)$, for some left N-subset $C_1 \cup C_2 \cup ... \cup C_n$ of N.*

3.2.2. *Quasi regular cyclic Ne-hyper closed N-groups.* On the other hand the hyper squeezed character of each of the Qe_i in Ne_i w.r.t. the given hypergraph $\mathcal{E}$, leads us to the ps-closedness of what has been stated above regarding the direct sum of the group sum of ideals related to quasi-regular left ideal of N, when zero is the only element of Ne_i that kills the e_i's.

Theorem 3.6. *If $Ann(e_i) = 0$ and each Qe_i is hyper closed in Ne_i for each i, then $\bigoplus_{i=1}^n Se_i$ is a ps-closed ideal of E.*

Proof. As above, the result for one component is sufficient. Now, by the proof of Theorem 3.2a.3, $Se \subseteq Qe$. So, $\overline{Se} \subseteq Qe$, as Qe is hyper closed. Again we have by the proof of the Theorem 3.2a.3, $\overline{Se} \subseteq Se$. Hence, Se is ps-closed. □

Theorem 3.7. *As in case of Theorem 4.2b.1 if $\bigoplus_{i=1}^n I_ie_i$ is the unique maximal N-subgroup of E, then $\bigoplus_{i=1}^n I_ie_i$ is ps-closed and each of $\bigoplus_{i=1}^n Ae_i$ and $J(E)$ is $\bigoplus_{i=1}^n I_ie_i$.*

Proof. Without loss of generality we see that, by hypothesis, Ie is the unique maximal N-subgroup of Ne and Qe being hyper closed, so $Ae = Ie \subseteq \overline{Ae} \subseteq Qe$. Again Qe is a proper subset of Ne, if not $Q = N$, as $Ann(e) = 0$ which is not true as $1 \notin Q$ and hence by Note E(i) we get Ie as ps-closed. Again, for any left ideal B that is maximal as N-subgroup of N, we have by Lemma 2.7(i) Be an ideal which is a maximal as N-subgroup of N. Thus, by uniqueness of Ie, we get $Be = Ie$. Hence, $J(Ne) = Ie$. Thus, $\bigoplus_{i=1}^{n} I_i e_i$ is ps-closed and each of $\bigoplus_{i=1}^{n} Ae_i$ and $J(E)$ is $\bigoplus_{i=1}^{n} I_i e_i$. □

3.2.3. *N_u-nilpotent Ne-hyper open N-groups.* The notion of N_u-nilpotent element in N-group E gives the following results, some of which are analogous to those obtained above. The results obtained here are on the assumption that each $Q_i^{'}(= the\ set\ of\ all\ N_u - nilpotent\ elements\ of\ Ne_i)$ is a hyper open proper N-subset of Ne_i with $Ann(e_i) = 0$.

Theorem 3.8. *If $\bigoplus_{i=1}^{n} B_i e_i$ is a maximal ideal of $\bigoplus_{i=1}^{n} Ne_i$, then $\bigoplus_{i=1}^{n} B_i e_i$ is ps-closed.*

Proof. Suppose $\overline{Be} = Ne$. Since $Q^{'}$ is hyper open, so, $(-Q^{'} + e)$ is ps-open and hence there is an element $q^{'}(= qe) \in Q^{'}$ and $b^{'}(= be) \in Be$ such that $-q^{'} + e = b^{'}$ which gives $-q + 1 = b$, as $Ann(e) = 0$. Now, as $q^{'} \in Q^{'}$ we have $N_u^t q^{'} = 0$ but $N_u^t \neq 0$ for some t(least)$\in \mathbb{Z}^+$ and hence $N_u^t q = 0$ as $Ann(e) = 0$. Now for any $n_1 n_2 ... n_t \in N_u^t$ we get $n_1 n_2 ... n_t = n_1 n_2 ... n_t (b + q) - n_1 n_2 ... n_t q \in B$ as $N_u^t q = 0$ and B is a left ideal of N and thus $N_u^t \subseteq B$. Again, $n_1 n_2 ... n_{t-1} = n_1 n_2 ... n_{t-1}(b + q) - n_1 n_2 ... n_{t-1} q + n_1 n_2 ... n_{t-1} q \in B$ as B being a left ideal of N and $n_1 n_2 ... n_{t-1} q \in N_u^t (\subseteq B)$ giving thereby $N_u^{t-1} \subseteq B$.

So, by induction, $N_u \subseteq B$ which is a contradiction, as by Lemma 2.7(i), B is maximal. But by Note E(i), Be is ps-closed. Thus it follows that $\bigoplus_{i=1}^{n} B_i e_i$ is ps-closed. □

Corollary 3.6. *$J(E) = J(\bigoplus_{i=1}^{n} Ne_i)$ is ps-closed.*

Corollary 3.7. *If N is hyper bounded, then $J(E) = J(\bigoplus_{i=1}^{n} Ne_i)$ is ps-open.*

Proof. As $Q^{'}$ is the hyper open subset of Ne containing 0 and N is hyper bounded, there exists a hyper open subset V of Ne such that $NV \subseteq Q^{'}$.
Now, for each $x \in V$, Nx is a N_u- nil N-subgroup of Ne and hence, by Note E(ii), $Nx \subseteq J(Ne)$, for each $x \in V$ and thus $V \subseteq J(Ne)$. Now, for $y \in V$ and $x \in J(Ne)$, we get $x + (-y) + V$ as a ps-open set containing x, so there exists a hyper open set $U(\subseteq x + (-y) + V)$ containing x that is contained in $J(Ne)$. □

Corollary 3.8. *If N is hyper bounded, then $E = \bigoplus_{i=1}^{n} Ne_i$ is ps-disconnected.*

Corollary 3.9. *If N is hyper bounded, $E = \bigoplus_{i=1}^{n} Ne_i$ and $J(E)$ is the zero ideal, then E is ps-discrete.*

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