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MODELLING MALAYSIAN GOLD PRICES USING GEOMETRIC BROWNIAN MOTION MODEL

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ABSTRACT. In this study, we use geometric Brownian motion, a mathematical model, to represent the future price path for Malaysian gold prices, namely *Kijang Emas*. We estimate the unknown parameters in the model, and we illustrate the accuracy of this method using a simulation method. The results obtained show that the proposed model is accurate by means of calculating the mean-absolute percentage error (MAPE) and forecast accuracy.

1. INTRODUCTION

Uncertain movements of stock prices which leads to interests of predicting the stock market, and many prediction models have been developed, such as regression method (Ismail et al., 2009) [7], Markov-Fourier Grey model (Hsu et al., 2009) [6], and artificial neural network (Atsalakis & Valavanis, 2009) [1]. Cajueiro et al. (2009) [3] on the other hand, predict stock market crashes using genetic algorithm.

Like stock prices, commodity prices also exhibit randomness that can be explained mathematically via a geometric Brownian motion (GBM) process. The GBM process is a stochastic process that assumes normally distributed and independent stock returns. Estember and Marana (2016) [5] show that GBM

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produces accurate and effective forecast for stock prices compared to the artificial neural network. Reddy & Clinton (2016) [9] use GBM for the simulation of stock prices paths and shows that the simulated prices are aligned with the actual prices. Djauhari et al. (2016) [4] prove that using GBM in time series modeling provides a simple and straightforward method for a lower cost and higher computational speed.

The modelling of *Kijang Emas* future price paths is useful for investment purposes in Malaysia (Miswan et al., 2013) [8]. Hence, this study aims to model *Kijang Emas* prices with the GBM mathematical approach for different time intervals. The remainder of this paper is organized as follows. Section 2 provides a brief description of the simulation model that is implemented in the study, and Section 3 provides some numerical results of the approach. Section 4 concludes the study.

2. METHODOLOGY

Consider that gold price G_t is modeled through its relative change in time, and follows a geometric Brownian motion (GBM) process as follows:

$$dG_t = \mu G_t dt + \sigma G_t dW_t, \tag{2.1}$$

where W_t is a normally distributed Brownian motion with mean 0 and standard deviation $\sqrt{dt}$. Adapted from Abidin & Jaffar (2014), let $F_t = \ln G_t$ describes the gold price as a lognormal random walk. Hence, by Ito's lemma, we have:

$$\begin{aligned} dF_t = {} & \frac{dF_t}{dG_t}\mu G_t dt + \frac{dF_t}{dG_t}\sigma G_t dW_t + \frac{1}{2}\frac{d^2F_t}{dG_t^2}\mu^2 G_t^2 dt^2 \\ & + \frac{d^2F_t}{dG_t^2}\mu\sigma G_t^2 dt dW_t + \frac{1}{2}\frac{d^2F_t}{\mathrm{dG_t^2}}\sigma^2 G_t^2 dW_t^2, \end{aligned}$$

which can be written as such:

$$dF_t = \frac{dF_t}{dG_t}dG_t + \frac{1}{2}\sigma^2 G_t^2 \frac{d^2F_t}{dG_t^2}dt, \tag{2.2}$$

by taking:

$$\begin{aligned} dW_t \cdot dW_t &= dt \\ dt \cdot dt = dW_t \cdot dt &= 0. \end{aligned} \tag{2.3}$$

Solving $\frac{dF_t}{dG_t}$ and $\frac{d^2F_t}{dG_t^2}$ then substituting dG_t yields the following:

$$dG_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_t. \tag{2.4}$$

Integrating both sides of Equation (2.4) yields:

$$G_t = G_{t-1}\text{exp}\left[\left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_t\right], \tag{2.5}$$

which is a log-normally distributed GBM that is used to simulate the gold prices.

3. Result and discussion

The data consists of 246 daily observations of the 1 oz. *Kijang Emas* prices from 4 January 2016 until 30 December 2016 that is available from the official website of *Bank Negara Malaysia* (2020) [2]. Daily returns are computed as logarithmic price relatives.

Following Wilmott (2000) [10], the daily logarithmic returns, R_i for $i = 1, \ldots, k$ over time interval τ is defined as follows:

$$R_i = \ln\left(\frac{G_i}{G_{i-1}}\right), \tag{3.1}$$

where G_i is the closing price at the end of i^{th} trading day. Hence, the standard deviation of f_i is estimated as such:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{k}\left(R_i - \overline{R}\right)^2}{k-1}}, \tag{3.2}$$

where

$$\overline{R} = \frac{\sum_{i=1}^{k} R_i}{k}, \tag{3.3}$$

is an unbiased estimator of the log returns. The mean and standard deviation for the gold prices is estimated using historical daily gold prices at fixed time intervals. T he estimated mean and standard deviation are tabulated in Table 1. Figures 1-5 plot the actual and modelled prices for 14, 28, 42, 56, and 246 daily observations, respectively.

In order to evaluate the accuracy of the gold futures prices simulated using GBM, we use the common relative measure that is the mean-absolute percentage error (MAPE) values.

TABLE 1. Mean and standard deviation

No. of daily observations	Mean	Standard deviation
14	0.003345573	0.01436483
28	0.003478028	0.01665236
42	0.003170129	0.01526641
56	0.001211178	0.01459915
246	0.000525765	0.07009403

Given k trading days, actual gold prices at time Y_i and simulated gold prices G_i , MAPE is defined as:

$$MAPE = \frac{\sum_{i=1}^{k} \left| \frac{Y_i - G_i}{Y_i} \right|}{k} \times 100\%. \tag{3.4}$$

FIGURE 1. GBM-modelled prices for 14 daily observations

FIGURE 2. GBM-modelled prices for 28 daily observations

FIGURE 3. GBM-modelled prices for 42 daily observations

FIGURE 4. GBM-modelled prices for 56 daily observations

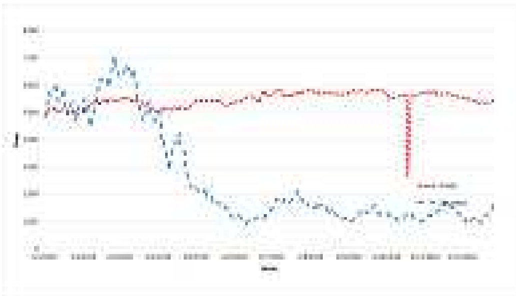

FIGURE 5. GBM-modelled prices for 246 daily observations

Other than the MAPE values of the modelled prices, we also compute the accuracy of the modelled prices over a 100%, FA, which is calculated as follows:

$$\text{FA} = \max\left(0,\ 100\% - \left(\left|\frac{Y_i - G_i}{Y_i}\right| \times 100\%\right)\right).$$

Table 2 documents the MAPE values for every number of observations, and its forecast accuracy.

TABLE 2. MAPE and FA

No. of daily observations	MAPE	FA
14	8.48968	91.51
28	8.20954	91.79
42	11.19248	88.81
56	11.29736	88.70
246	55.62915	44.37

A MAPE value that is very small (less than 10%) indicates a very accurate forecast, while a MAPE value that is very large (more than 50%) indicates a very inaccurate forecast. Thus, based on the MAPE-scale in Table 2, the modelled prices are highly accurate for 14 and 28 daily observations. The forecast accuracy for these two observations are also close to 100%. As the number of observations increases to 42 and 56, the accuracy of the modelled prices changed to good, and the forecast accuracy decreases below 90%. However, the accuracy becomes inaccurate when the prices are modelled for one year (246 observations), and the forecast accuracy is less than 50% accurate.

4. CONCLUSION

In this study, we model gold prices with the geometric Brownian motion (GBM) process. The simulated prices produce a low MAPE value for a period up to one month which means that GBM may produce accurate forecast gold prices for a period of one month. As the number of observations increases above a month period, the MAPE value increases as well which indicates inaccurate forecast. On that account, it means that investors and traders may use GBM to produce accurate gold price forecast for a short time period. Future direction of this study is to incorporate mean-reversion and jumps in developing a forecast model that captures the behavior of gold prices that may exhibit mean-reversion and jumps in its price path. Another future work to consider is to increase the number of forecast periods.

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